A progressive method to solve large-scale AC Optimal Power Flow with discrete variables and control of the feasibility

**Manuel Ruiz**, Jean Maeght, Alexandre Marié, Patrick Panciatici and Arnaud Renaud

manuel.ruiz@artelys.com
MOTIVATIONS
The iTesla project

- Pan-European R&D project:
  - Security assessment on large scale power networks by means of security rules
  - Coordinated by RTE (Réseau de Transport d’Electricité)
  - Includes 6 European TSOs, 13 R&D companies and academic partners
  - Official website: http://www.itesla-project.eu/

**Offline platform**
- Sampling of network states
- Infeasible states detected through steady-state optimization
- Unstable states detected through dynamic optimization
- Data mining on the results

**Every week**

**Online platform**
- Data acquisition from European TSOs (24 hour forecasts)
- Data merging
- Security assessment
- Recommendation for the operators

**Every day**
• Power System network with 2 loads (NORTH and SOUTH)

• Generated from historical data, sampled cases are to be analyzed
  – Sampled data are loads or fixed injections (including renewable generation)
  – Steady state, Unstable state or Unknown

• Screening rules describing the boundary between stable and unstable are then extracted from the analysis of the sampled cases
• Monte Carlo simulations provide us with thousands of samples (partial network situations)

• Our aim is to build up the complete state for each sample:
  – To assess feasibility
  – To identify the sample parameters that cause infeasibilities
  – While having a realistic and detailed representation of the network

• Specificity of the mathematical problem
  – Non linear
  – Without any guarantee of feasibility (due to sampling)
  – With discrete aspects (rigorous description of the power system network)
  – Large scale network (target is European network)
MATHEMATICAL FORMULATION
EXTENDED OPTIMAL POWER FLOW
• The power system network is described using buses and branches:
  – Loads and production units are set up on buses.

• AC formulation using voltage and production levels:
  – Induces active and reactive non linear power balances,
  – Allows computation of active power losses.

• Security constraints:
  – Bounds on voltage magnitude,
  – Upper bounds on currents (thermal limits).
• Phase-Shifting Transformers are used to adjust the voltage ratio and the difference in the phase angle.

• Operating points of PST are to be chosen in a set of discrete configurations defining:
  – Reactance,
  – Voltage ratio and the difference in the phase angle.

• Using the admittance matrix $Y_j$ of operating point $j$, the model simply reads:
  – $Y = \Sigma_j \lambda_j Y_j$,
  – $\Sigma_j \lambda_j = 1, \lambda_j \in \{0; 1\}$. 
• Commitment of production unit
  – Active and reactive power injection are bounded when the unit is switched on
  – Given set production levels are to be reached

• Commitment constraints for unit $g$
  – $p_g, q_g$: active and reactive injections
  – $is_{-}on_g \in \{0; 1\}$
  – $is_{-}on_g \left[\frac{p_g^{min}}{Q_g^{min}}\right] \leq \left[\frac{p_g}{q_g}\right] \leq is_{-}on_g \left[\frac{p_g^{max}}{Q_g^{max}}\right]$

• Definition of the over active injection level:
  – $P_g^c$: set value of active injection computed using data mining
  – $p_g^{add}$: active over injection defined by
  – $p_g \leq P_g^c + p_g^{add}$ pour chaque groupe $g$

• The criterion is defined as $\sum_{g \in Units}(p_g + p_g^{add})$
Other aspects

• Compensation units with similar attributes can be activated at some nodes
  - $-V_n^2 \cdot n_{b_n}^{shunt} \cdot B_n^{shunt}$: reactive injection in power balance
  - $n_{b_n}^{shunt}$: number of activated devices
  - $B_n^{shunt}$: value of the attribute for one unit

• The main objective is to build a realistic feasible solution to be able to launch dynamic simulation
  – Reduce active power losses
  – Reduce the deviation to set production levels

• The network topology is given as an input
  – Ongoing work to optimize the topology
SOLVING APPROACH
CUSTOM DECOMPOSITION STRATEGY
Diagnosis of feasibility

• Sampling may have produced infeasible situations in terms of loads and fixed injections:
  – Fixed injection or loads too high on a node,
  – Current intensity level too low on a line.

• Optimal power flows:
  – Slack variables in power balances or constraints related to thermal limits
  – Continuous relaxations of discrete aspects using \( x \in \{0; 1\} \Rightarrow x \in [0,1] \):

• The resulting NLP are solved with KNITRO using an interior point method for non linear programming.

• When necessary, fixed injections can be modified:
  – Production curtailment of fatal production unit (PC)
  – Load shedding (LS)

• Whenever LS is used the instance is considered as not feasible
  – In such a case, results analysis can be used to correct the sampling (high loads or renewable productions)
<table>
<thead>
<tr>
<th>STEP</th>
<th>slack_Q</th>
<th>slack_P</th>
<th>PC</th>
<th>LS</th>
</tr>
</thead>
<tbody>
<tr>
<td>BALANCE_Q</td>
<td>MIN 1</td>
<td>MIN 0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BALANCE_P</td>
<td>= 0</td>
<td>MIN 1</td>
<td></td>
<td>ALLOWED</td>
</tr>
<tr>
<td>BALANCE_P_PC</td>
<td>= 0</td>
<td>MIN 1</td>
<td>ALLOWED</td>
<td>ALLOWED</td>
</tr>
<tr>
<td>BALANCE_P_PC_LS</td>
<td>= 0</td>
<td>MIN 1</td>
<td>ALLOWED</td>
<td>ALLOWED</td>
</tr>
</tbody>
</table>

**NOT FEASIBLE REACTIVE BALANCE FAILED WITHOUT CURRENT**

**FEASIBLE FOR ACTIVE AND REACTIVE BALANCES WITHOUT CURRENT**
<table>
<thead>
<tr>
<th>STEP</th>
<th>$slack_Q$</th>
<th>$slack_P$</th>
<th>PC</th>
<th>LS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CURRENT_Q</td>
<td>MIN 1</td>
<td>MIN 0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CURRENT_P</td>
<td>= 0</td>
<td>MIN 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CURRENT_P_PC</td>
<td>= 0</td>
<td>MIN 1</td>
<td>ALLOWED</td>
<td></td>
</tr>
<tr>
<td>CURRENT_P_PC</td>
<td>= 0</td>
<td>= 0</td>
<td>MIN 1</td>
<td></td>
</tr>
<tr>
<td>CURRENT_P_PC_LS</td>
<td>= 0</td>
<td>MIN 1</td>
<td>ALLOWED</td>
<td>ALLOWED</td>
</tr>
<tr>
<td>CURRENT_PC_LS</td>
<td>= 0</td>
<td>= 0</td>
<td>MIN 0.1</td>
<td>MIN 1</td>
</tr>
</tbody>
</table>

The diagram shows the flow of logic for determining the feasibility of reactive balance with current conditions. The decision points are based on the slack values for $Q$ and $P$, and the feasibility of meeting PC and LS limits.
Building up a feasible solution

• Discrete aspects are handled separately.

• Resolution of MINLP is based on a MPEC reformulation that can be directly handled by KNITRO.
  \[ x \in \{0; 1\} \iff x = 0 \text{ or } 1 - x = 0 \iff 0 \leq x \perp 1 - x \geq 0 \]

• KNITRO then treats MPEC as a NLP.
  – Defining the constraint \( x \geq 0 \) and \( 1 - x \geq 0 \)
  – Adding a penalty term \( \Pi \cdot x(1 - x) \) in the objective
  – Iteratively updating the penalty weight \( \Pi \) to converge to a locally optimal solution.
Decomposition strategy

Discrete aspects
- MPEC reformulation
- Continuous relaxation
- Fixed

Feasibility phase
- PST configuration
- Unit commitment
- Shunt commitment
- Detection of infeasibilities using slack variables

PST setting
- PST setting
- Unit commitment
- Shunt commitment
- Starting from the solution of the feasibility phase

Unit commitment
- PST setting
- Unit commitment
- Shunt commitment
- Starting from the solution of the PST phase

Shunt commitment
- PST setting
- Unit commitment
- Shunt commitment
- Starting from the solution of unit commitment
COMPUTATIONAL RESULTS
• FR-THT: Very High Voltage (VHV) French transmission network.

• 1600 substations, 1900 buses, 2700 branches.

• On average 7000 variables and 7500 constraints after presolve. Jacobian size is over 43 000 non zero elements and the hessian matrix 15 000.

• MPEC reformulations consist in 83(PST step), 112(UNIT step) and 1384(SHUNT step) complementarities.

• Each problem is solved by KNITRO with less then 10s.
• FR-THT-HT-M: Very High Voltage French transmission network, High Voltage (HV) transmission area of Marseilles.

• 2400 substations, 2800 buses, 4060 branches.

• On average 13K variables and 12K constraints after presolve. Jacobian size is over 60 000 non zero elements and the hessian matrix 20 000.

• MPEC reformulations consist in 130(PST), 135(UNIT) and 6681(SHUNT) complementarities.

• During the feasibility phase each problem is solved by KNITRO with less then 10s. MPEC reformulation are solved within 1 or 2 minutes.
• FR-THT-HT-full: Very High Voltage network and the whole High Voltage (HV) transmission network for France; a guard ring is added, with a few buses representing a simplified version of the neighborhood of France.

• 5857 substations, 6471 buses, 9831 branches.

• On average 36K variables and 30K constraints after presolve. Jacobian size is over 200K non zero elements and the hessian matrix 50K.

• MPEC reformulations consist in 355(PST), 174(UNIT) and 24018(SHUNT) complementarities.

• During the feasibility phase each problem is solved by KNITRO with less then 40s. MPEC reformulation are solved within less than 3 minutes.
Remarks on nominal voltage

• Each instance address a very different kind of power system with different voltage levels
Conclusion

- The problem of solving extended OPF with no guarantee on feasibility is addressed.

- A custom methodology is designed. It includes feasibility diagnosis and resolution of several OPF with discrete variables.

- The MPEC reformulation of MINLP is successfully applied and computational results obtained are promising.

- We are currently working on experimentations on European scale.
QUESTIONS ?