

# Robust Policies for Unit Commitment

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# Designing a policy

## □ Dealing with uncertainty

- » We have to design policies to manage the different forms of uncertainty.
- » We do this by looking for *robust policies*, which are rules for making decisions.
- » We write our optimization problem in the form:

$$\min_{\pi} E^{\pi} \left\{ \sum_{t=0}^T \gamma^t C \left( S_t, X^{\pi} (S_t) \right) \right\}$$

Search for the best policy

Averaging over multiple samples

Day-ahead, hour-ahead and real-time decisions

Objective function where

$$S_{t+1} = S^M (S_t, X^{\pi} (S_t), W_{t+1})$$

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$$\min_{\pi} E^{\pi} \left\{ \sum_{t=0}^T \gamma^t C(S_t, X^{\pi}(S_t)) \right\}$$

- » We refer to this as the *base model* which is typically calculated as a simulation:

$$\min_{\pi} \bar{F}^{\pi} = \frac{1}{N} \sum_{n=1}^N \sum_{t=0}^T \gamma^t C(S_t(\omega^n), X_t^{\pi}(S_t(\omega^n)))$$

$$\text{where } S_{t+1} = S^M(S_t, x_t, W_{t+1}(\omega))$$

# Four classes of policies

## 1) Policy function approximations (PFAs)

» Lookup tables, rules, parametric functions

## 2) Cost function approximation (CFAs)

$$\text{» } X^{CFA}(S_t | \theta) = \arg \min_{x_t \in \bar{X}_t(\theta)} \bar{C}^\pi(S_t, x_t | \theta)$$

## 3) Policies based on value function approximations (VFAs)

$$\text{» } X_t^{VFA}(S_t) = \arg \min_{x_t} \left( C(S_t, x_t) + \gamma \bar{V}_t^x(S_t^x(S_t, x_t)) \right)$$

## 4) Lookahead policies

» ***Deterministic lookahead:***

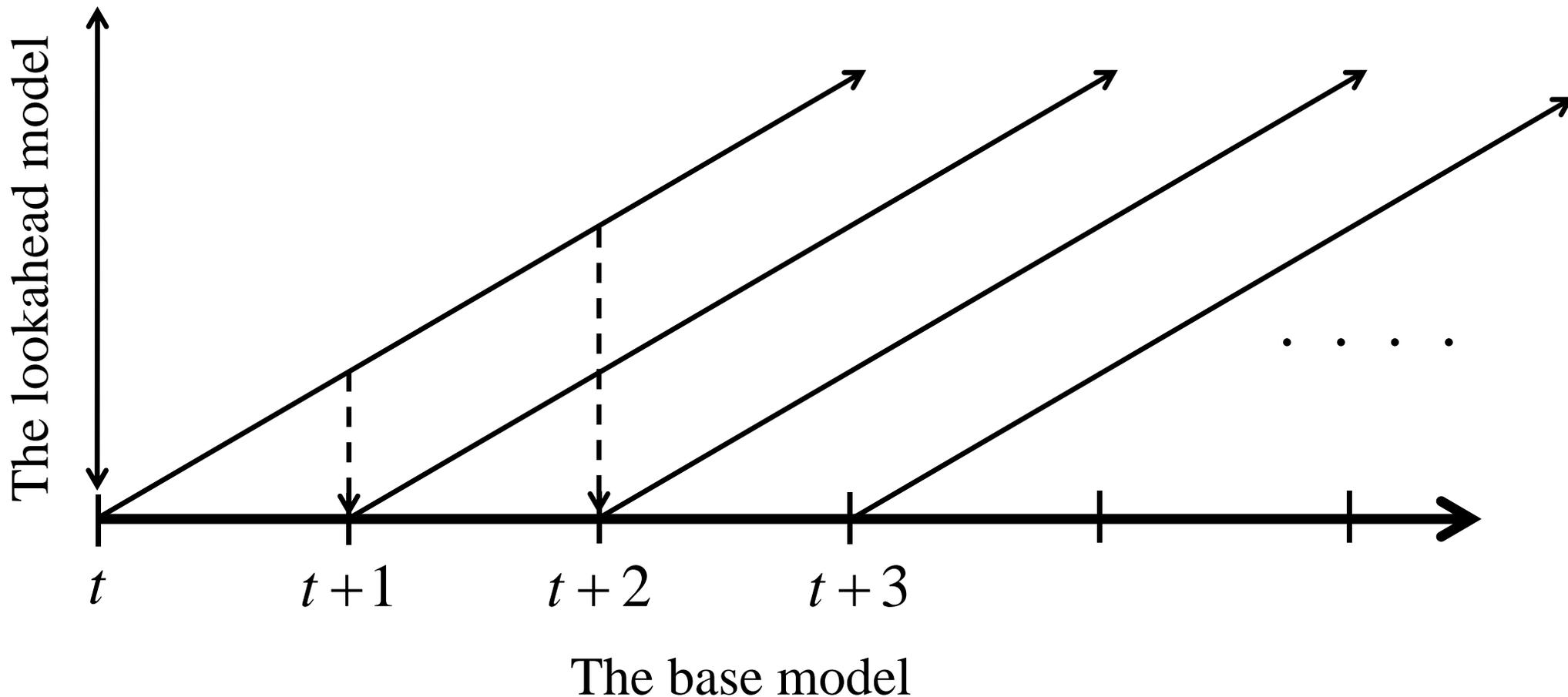
$$X_t^{LA-D}(S_t) = \arg \min_{\tilde{x}_t, \tilde{x}_{t,t+1}, \dots, \tilde{x}_{t,t+T}} C(\tilde{S}_t, \tilde{x}_t) + \sum_{t'=t+1}^T \gamma^{t'-t} C(\tilde{S}_{t'}, \tilde{x}_{t'})$$

» ***Stochastic lookahead (e.g. stochastic trees)***

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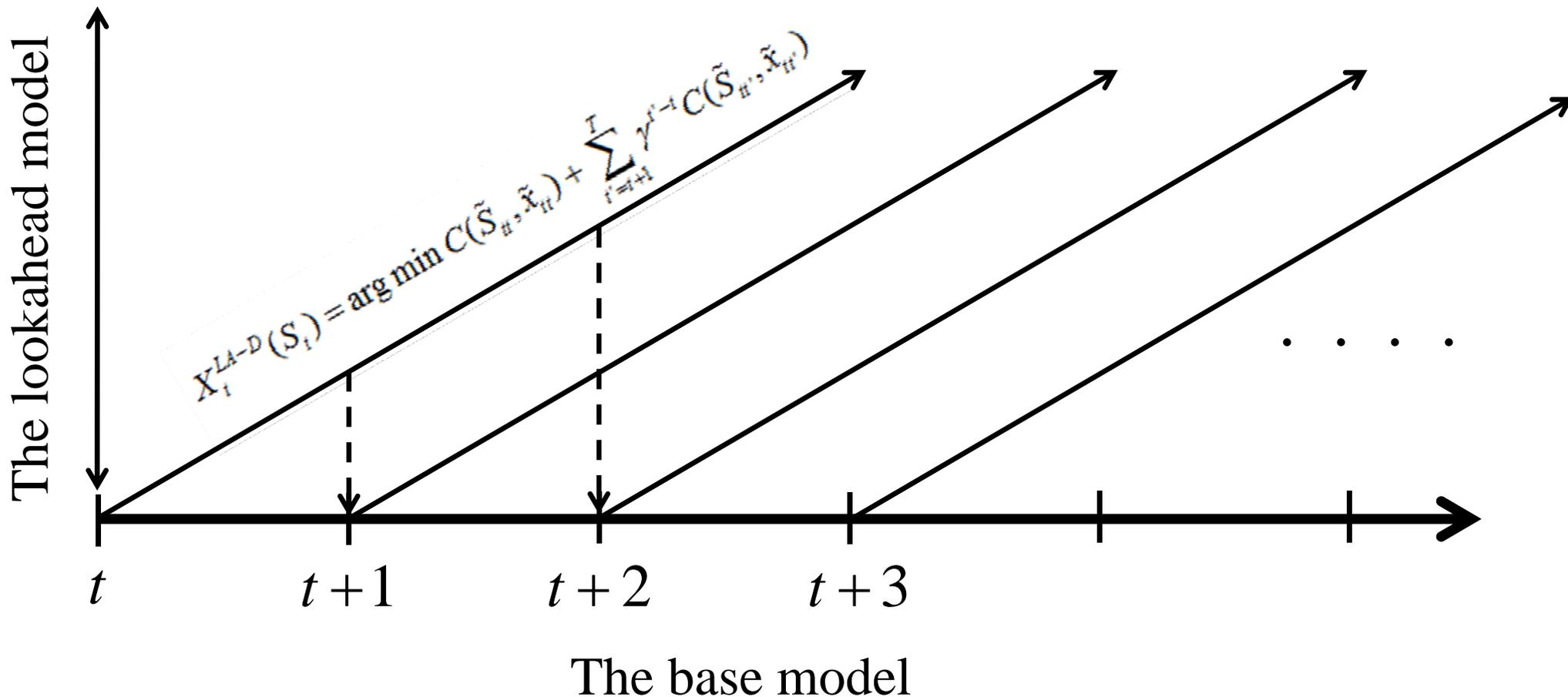
# Lookahead policies

- Lookahead policies peek into the future
  - » Optimize over deterministic lookahead model



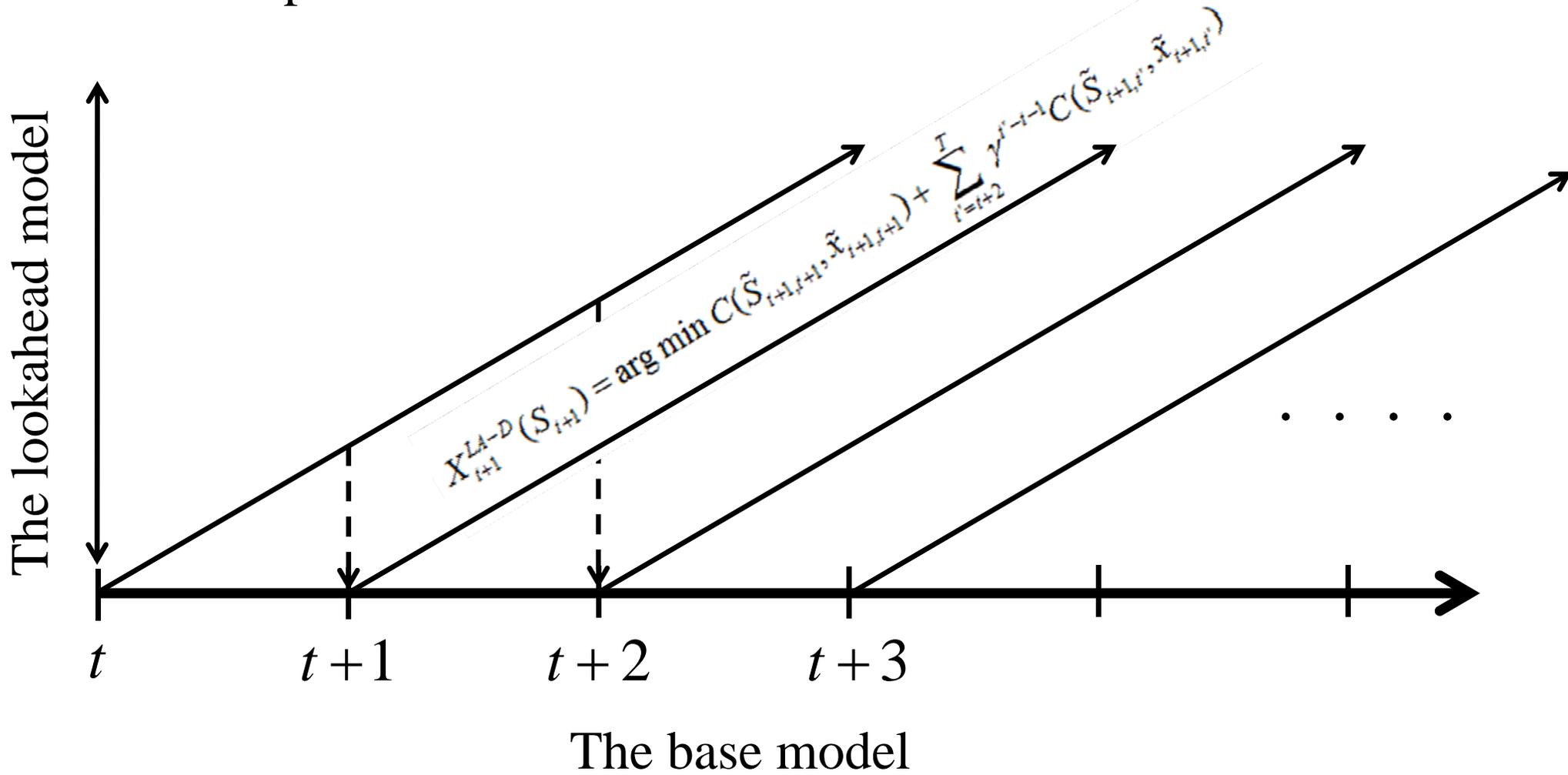
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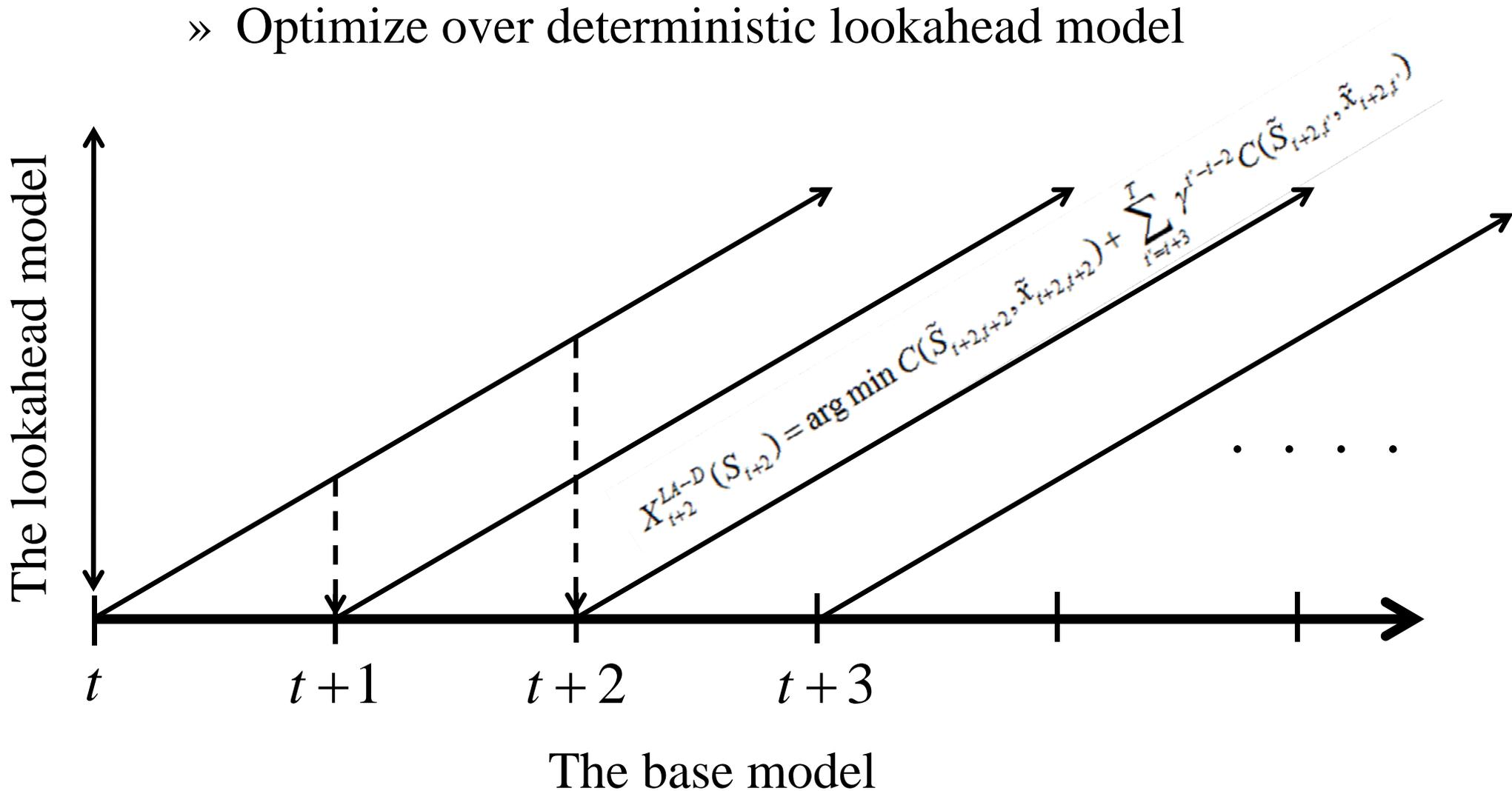
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# Stochastic lookahead policies

- The optimal policy requires solving

$$X^*(S_t) = \arg \min_{x_t} C(S_t, x_t) + \mathbb{E} \left\{ \min_{\pi \in \Pi} \sum_{t'=t+1}^T C(S_{t'}, X_{t'}^\pi(S_{t'})) \mid S_t \right\}$$

Expectation that we cannot compute

Minimization that we cannot compute

# Stochastic lookahead policies

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- We use a series of approximations:
  - » Stage aggregation – Replacing multistage problems with two-stage approximations.
  - » Outcome aggregation/sampling – Simplifying the exogenous information process
  - » Discretization – Of time, states and decisions
  - » Horizon truncation – Replacing a longer horizon problem with a shorter horizon
  - » Dimensionality reduction – We may ignore some variables (such as forecasts) in the lookahead model that we capture in the base model (these become *latent* variables in the lookahead model).

# Stochastic lookahead policies

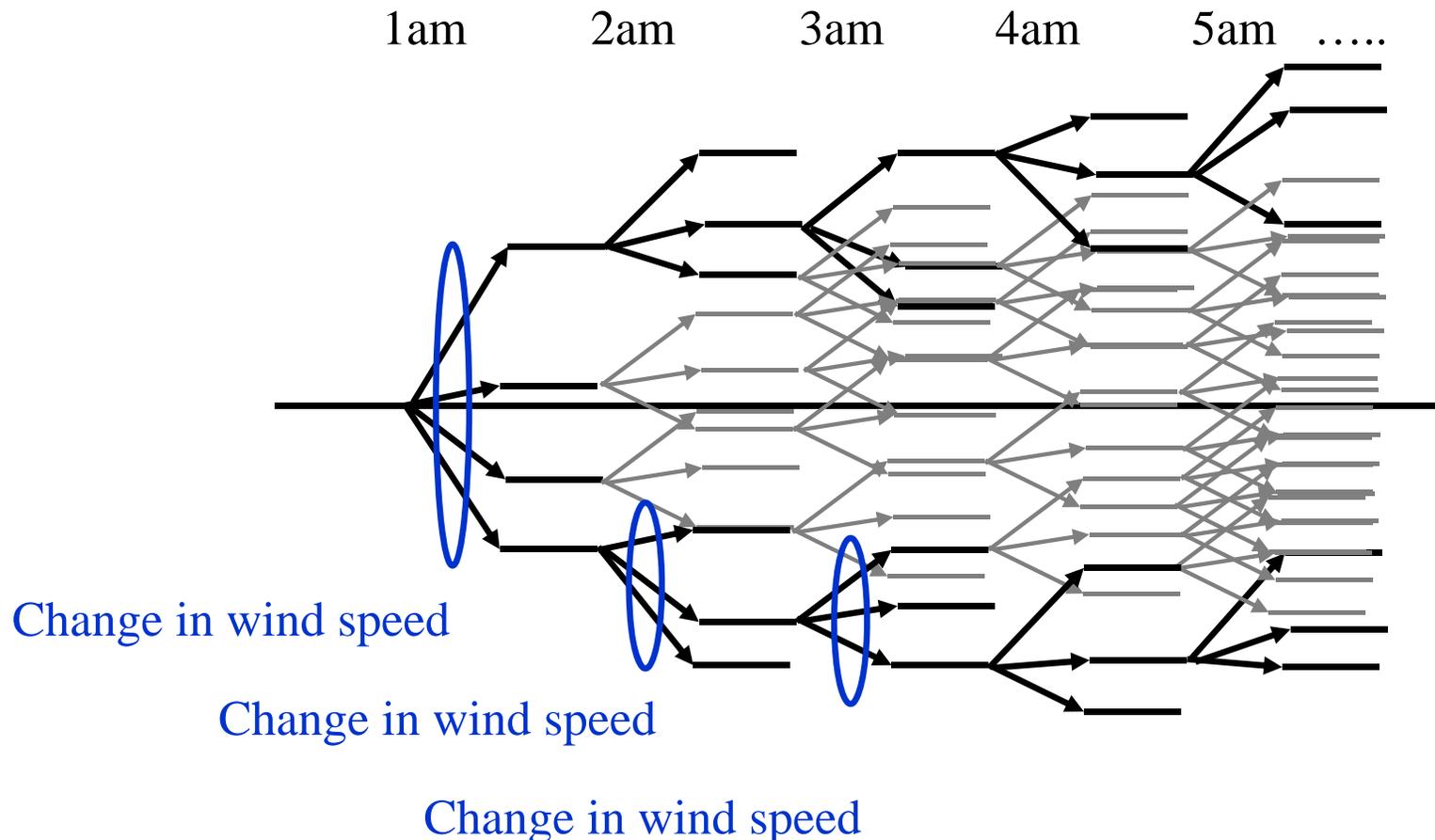
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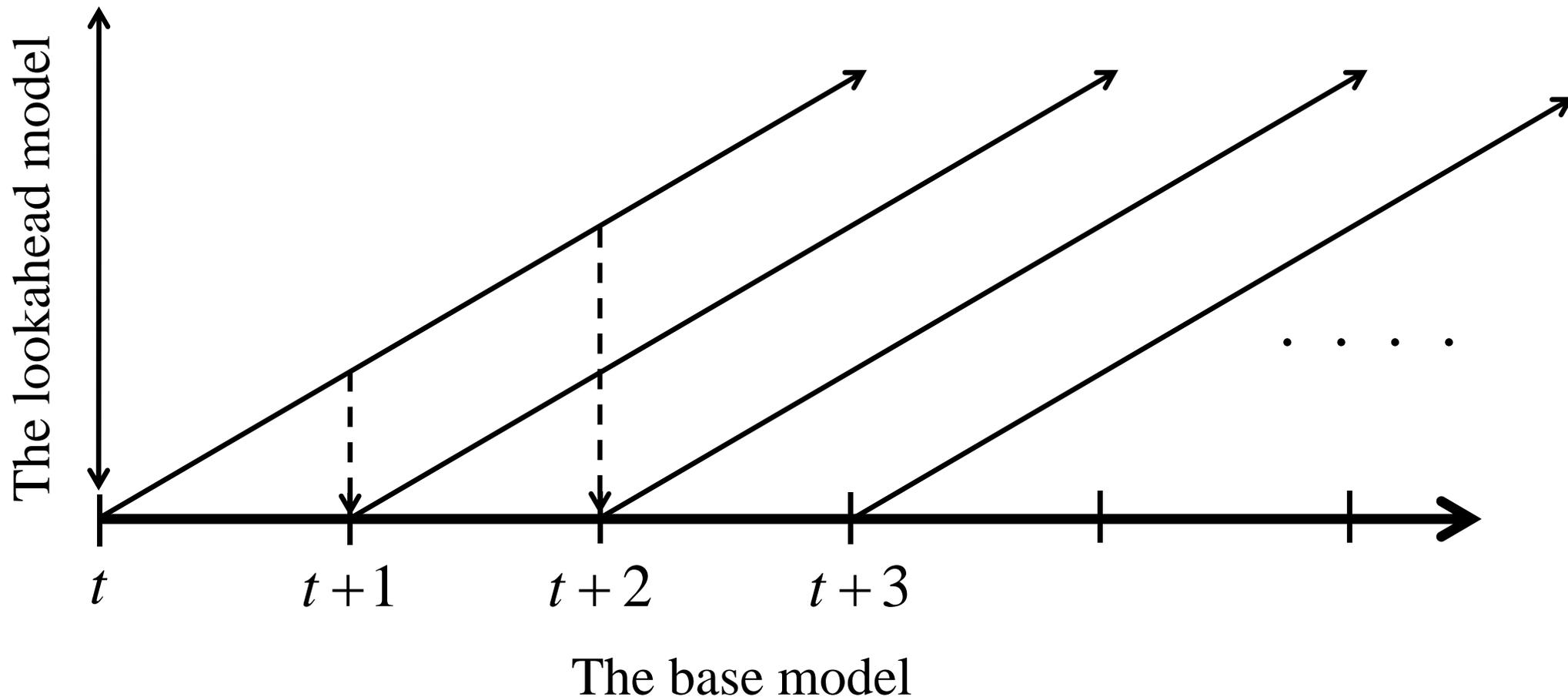
## □ Stochastic lookahead

- » Here, we approximate the information model by using a Monte Carlo sample to create a scenario tree:



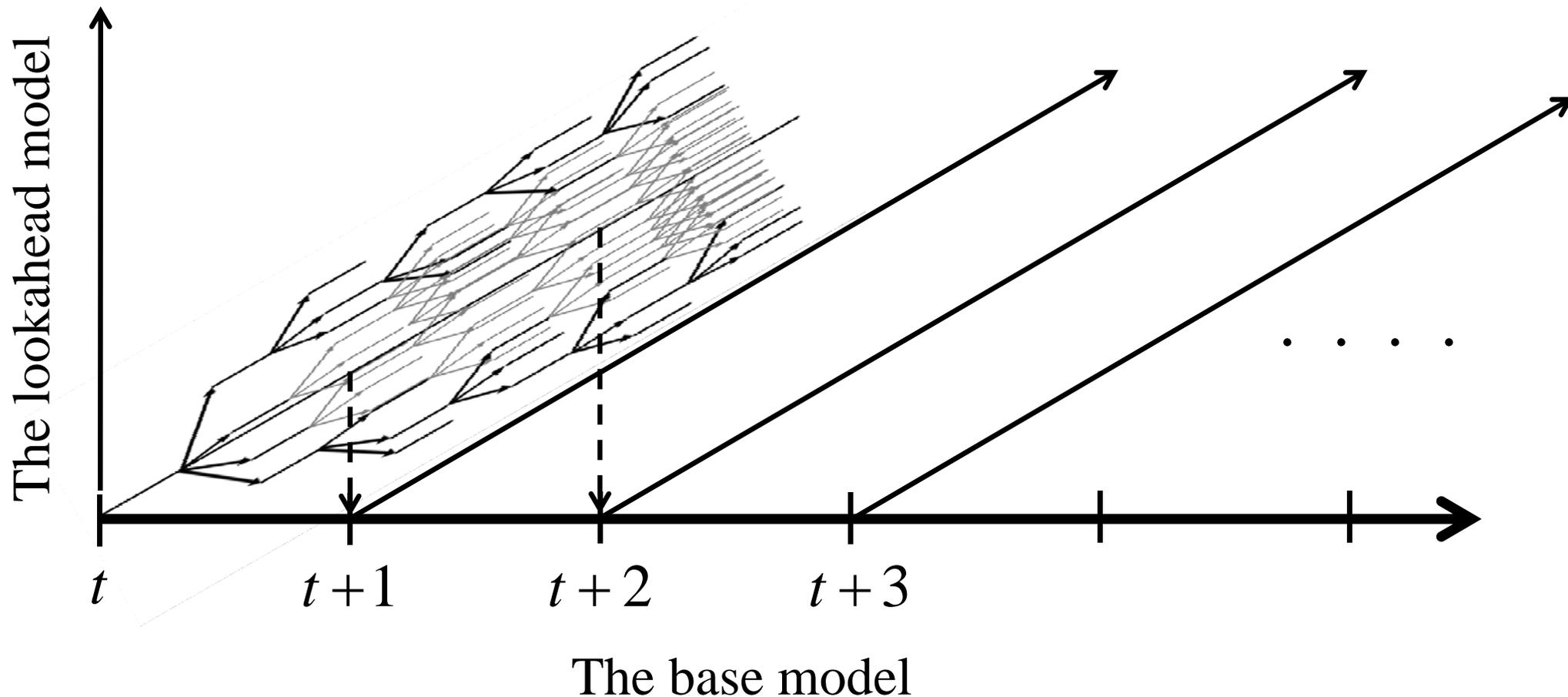
# Stochastic lookahead policies

- We can then simulate this *lookahead policy* over time:



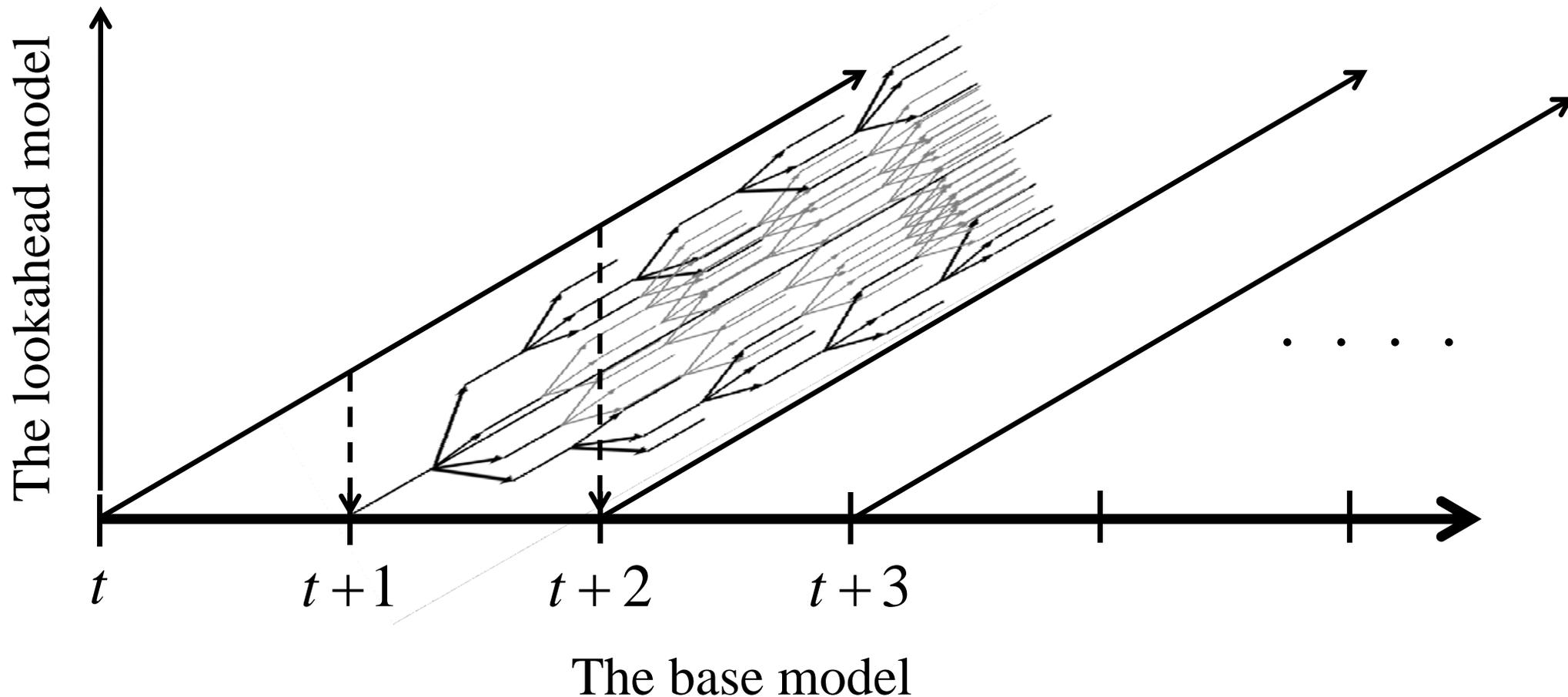
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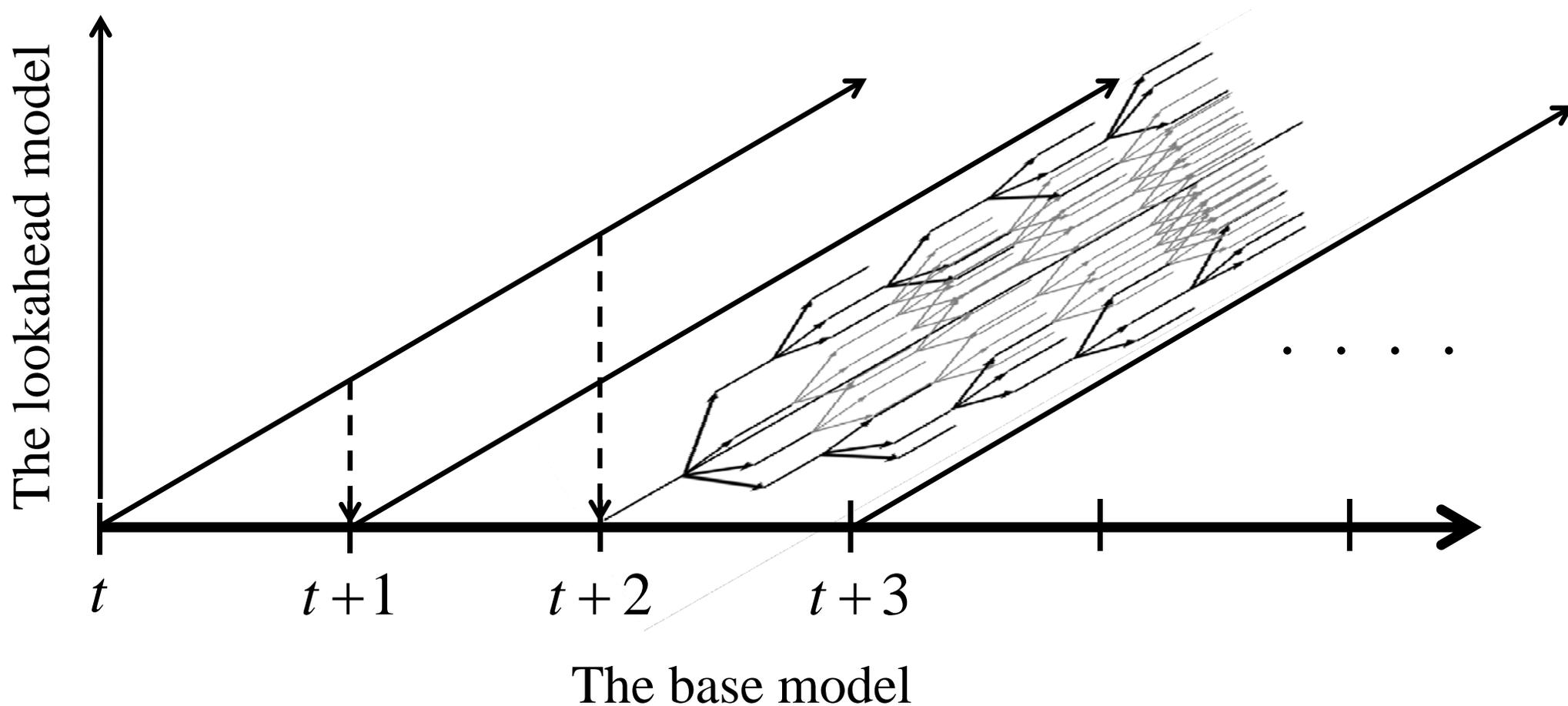
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# Stochastic lookahead policies

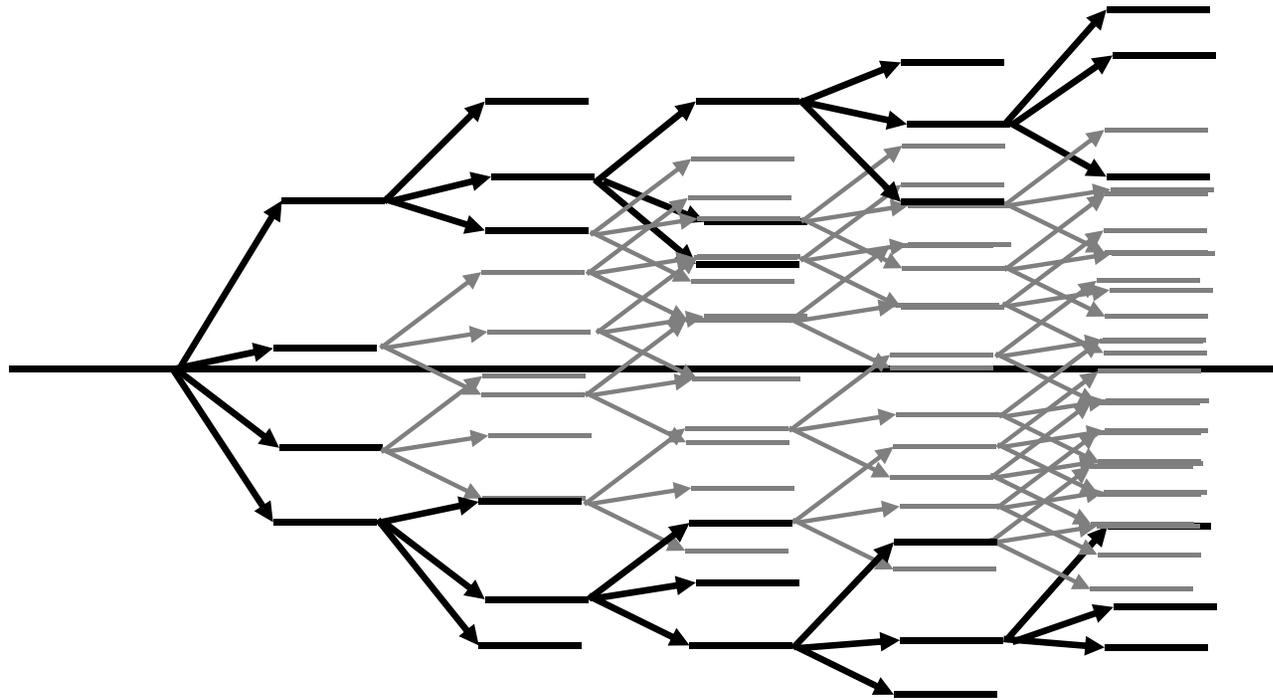
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# Stochastic lookahead policies

## □ Multistage lookahead approximation

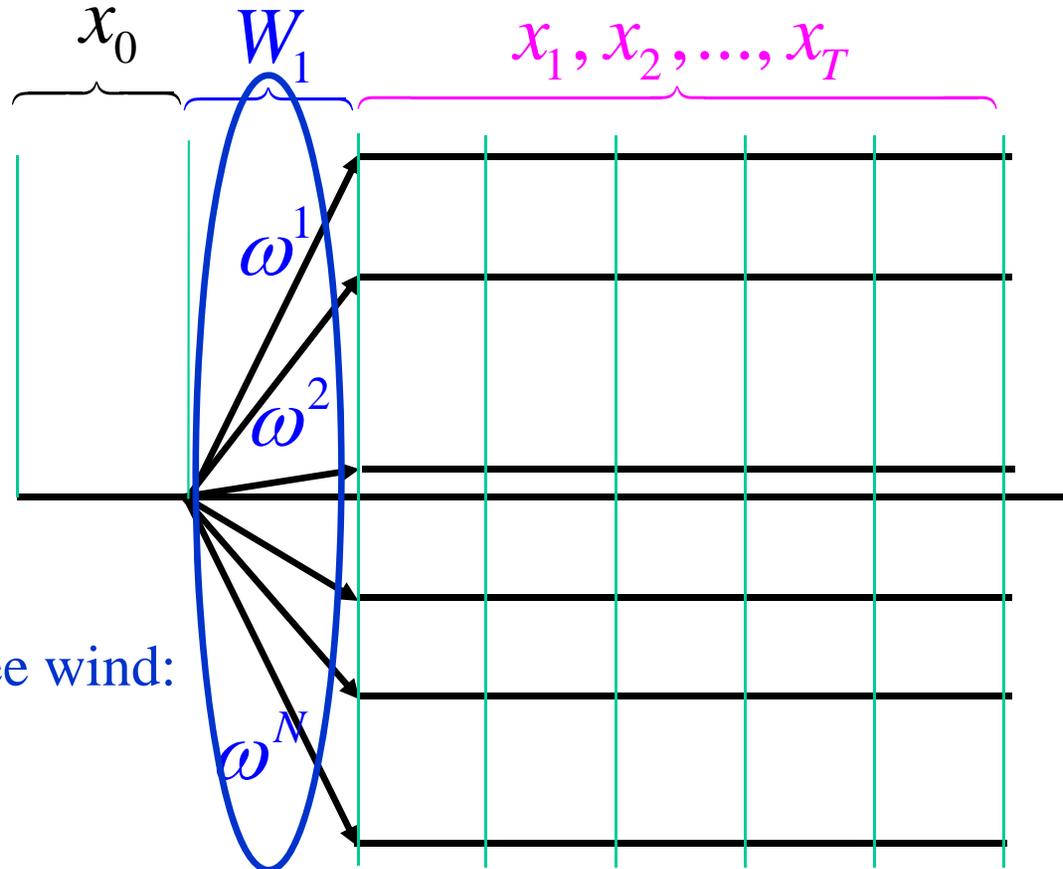
- » The problem with multistage trees is that even with sparse sampling, they are too expensive to compute:



# Stochastic lookahead policies

## □ Two stage lookahead approximation

1) Schedule steam

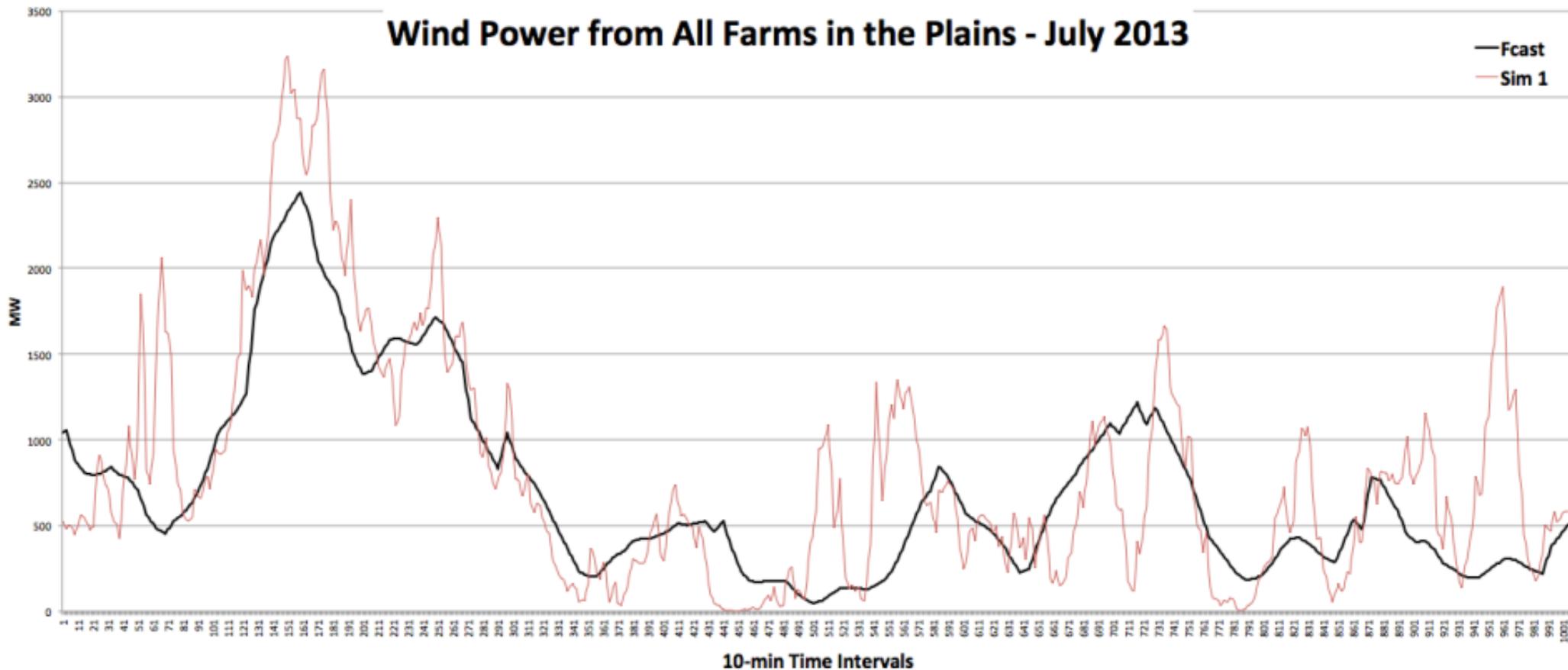


3) Schedule turbines

2) See wind:

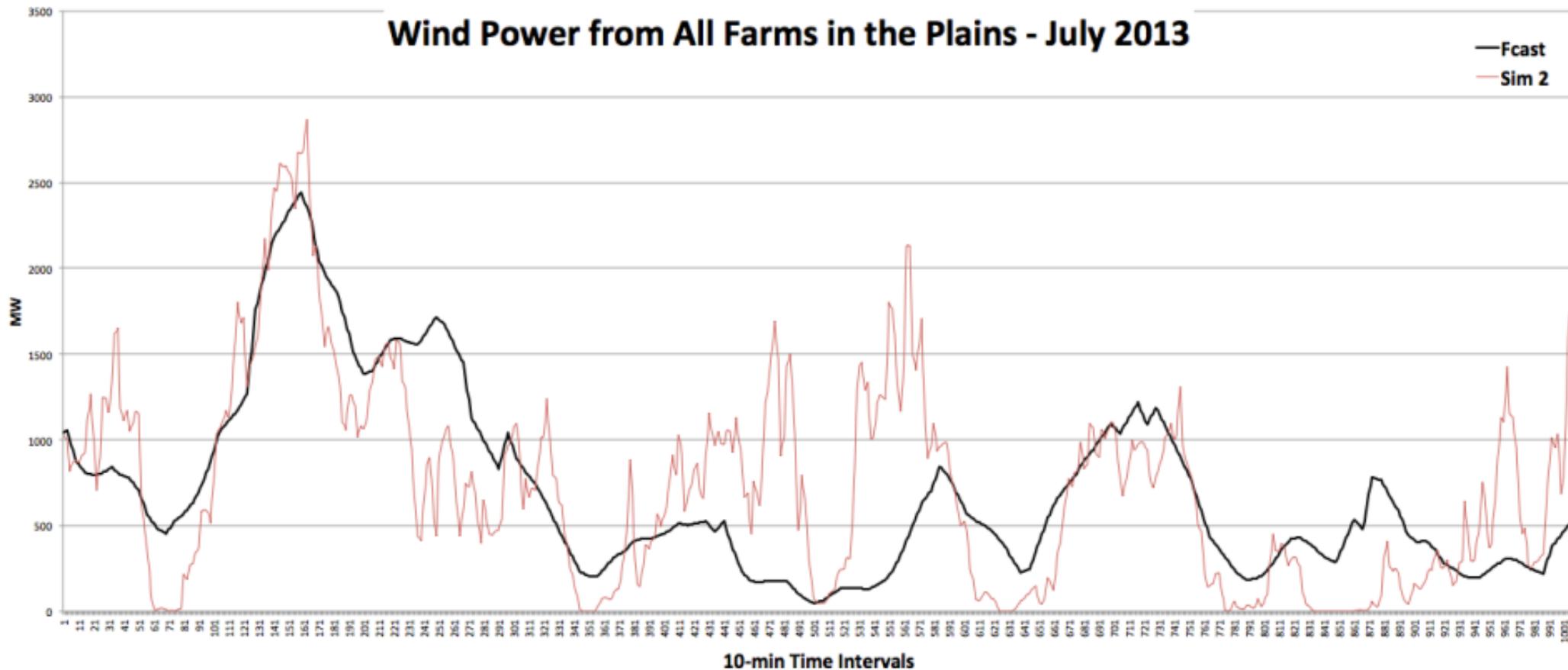
# Stochastic lookahead policies

- ❑ Creating wind scenarios (Scenario #1)



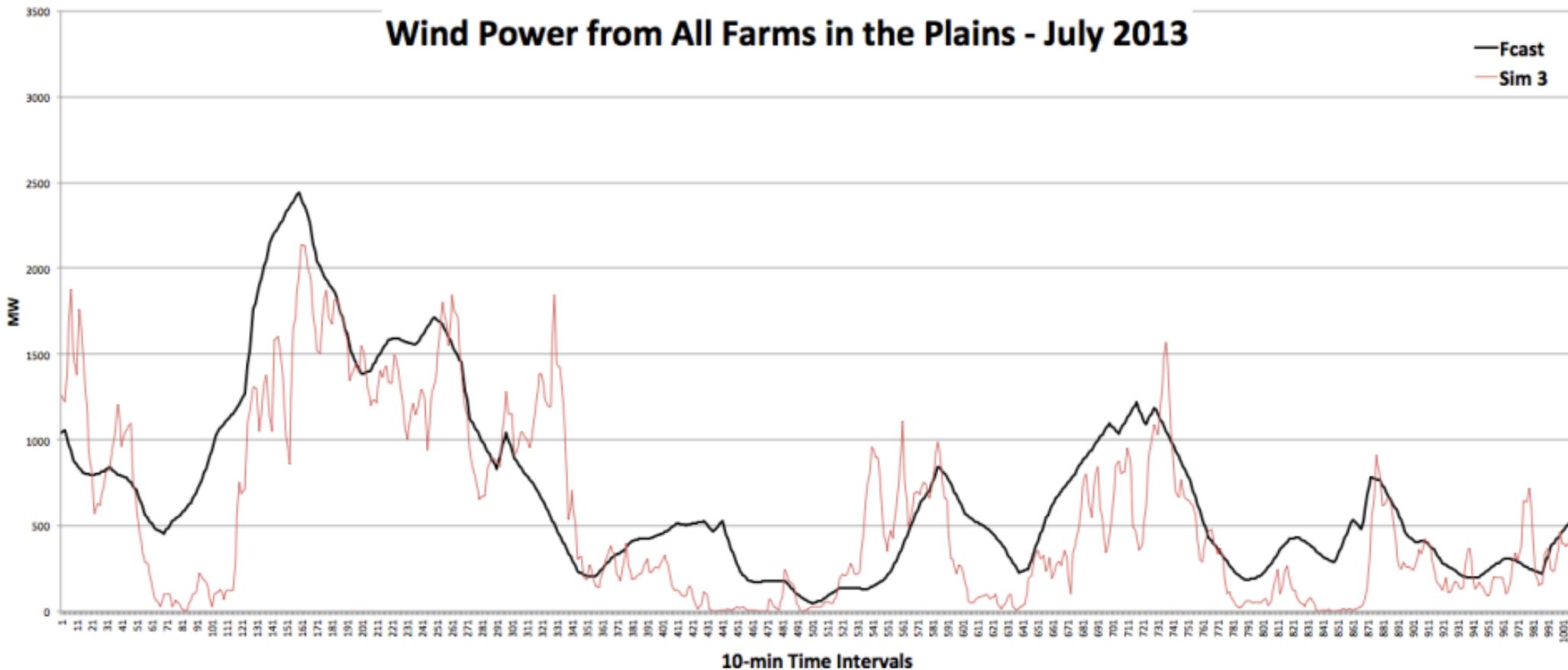
# Stochastic lookahead policies

- ❑ Creating wind scenarios (Scenario #2)



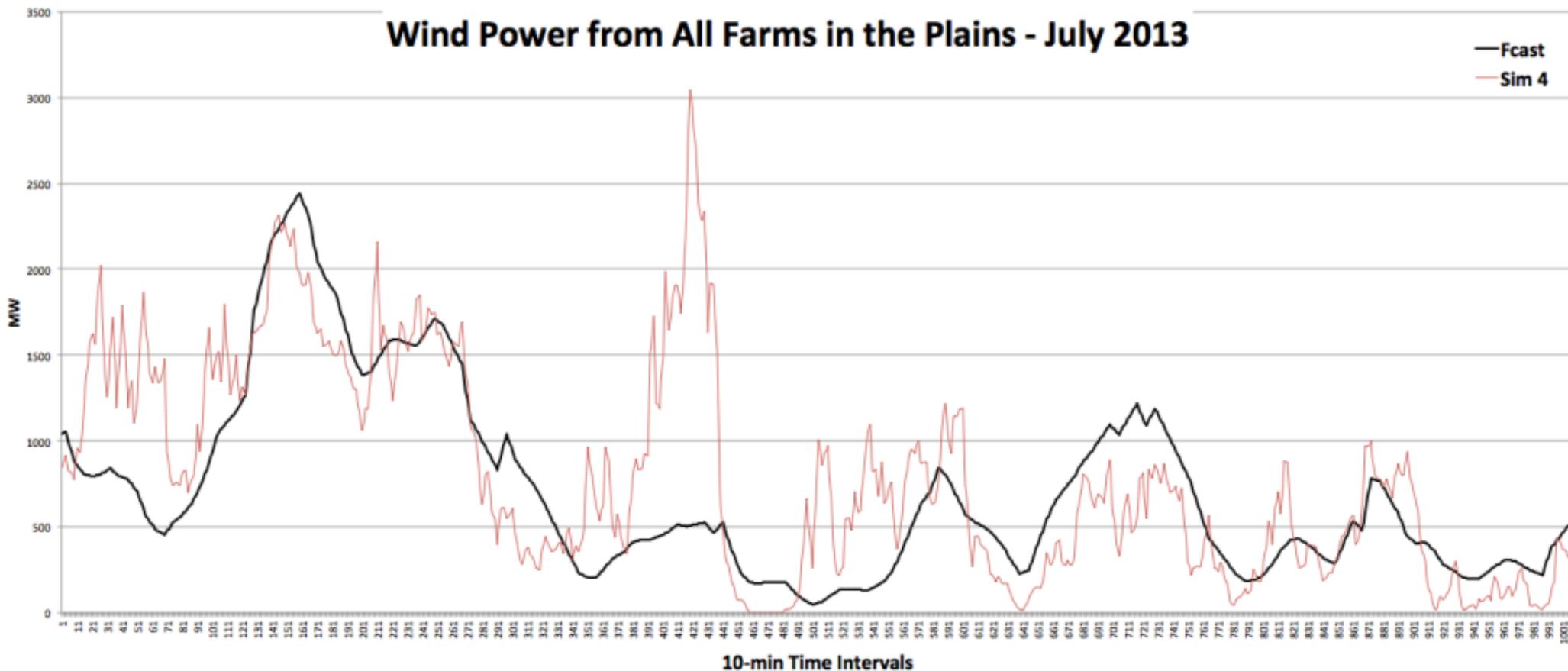
# Stochastic lookahead policies

- ❑ Creating wind scenarios (Scenario #3)



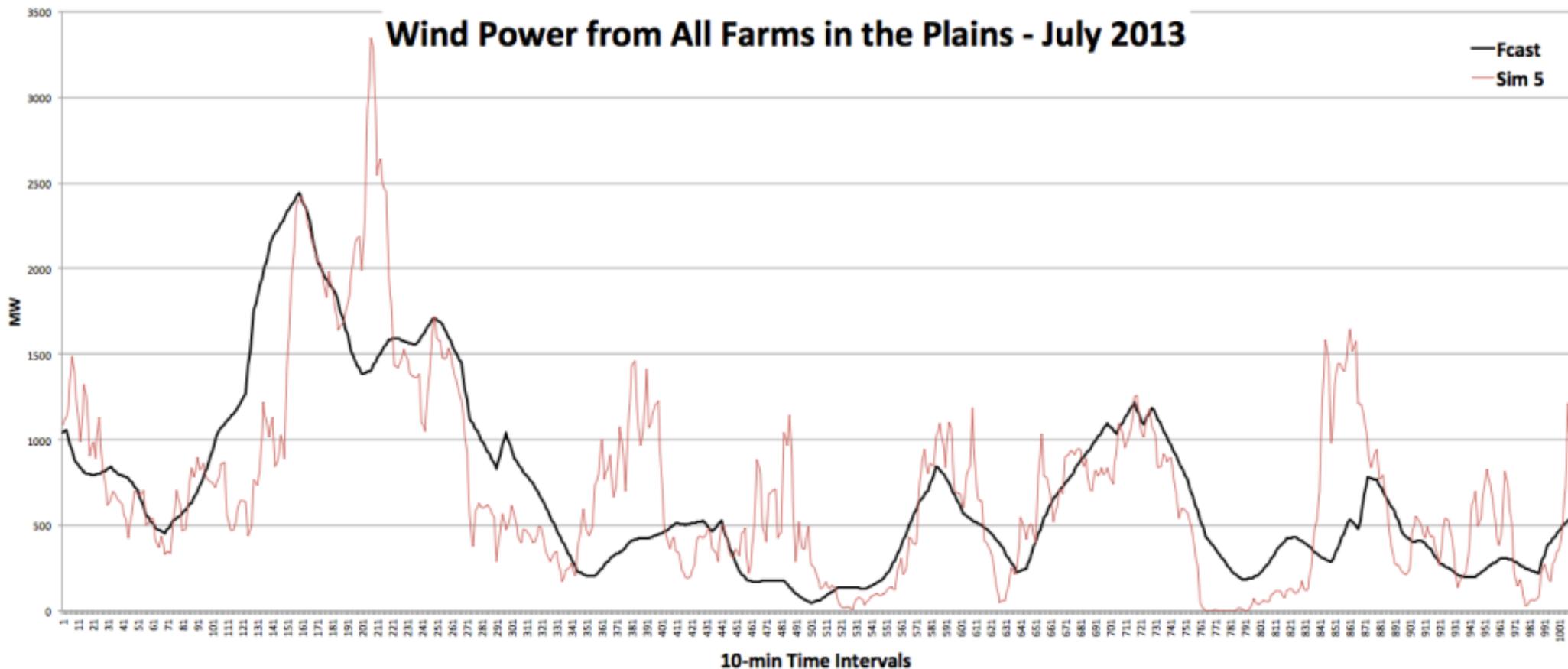
# Stochastic lookahead policies

- ❑ Creating wind scenarios (Scenario #4)



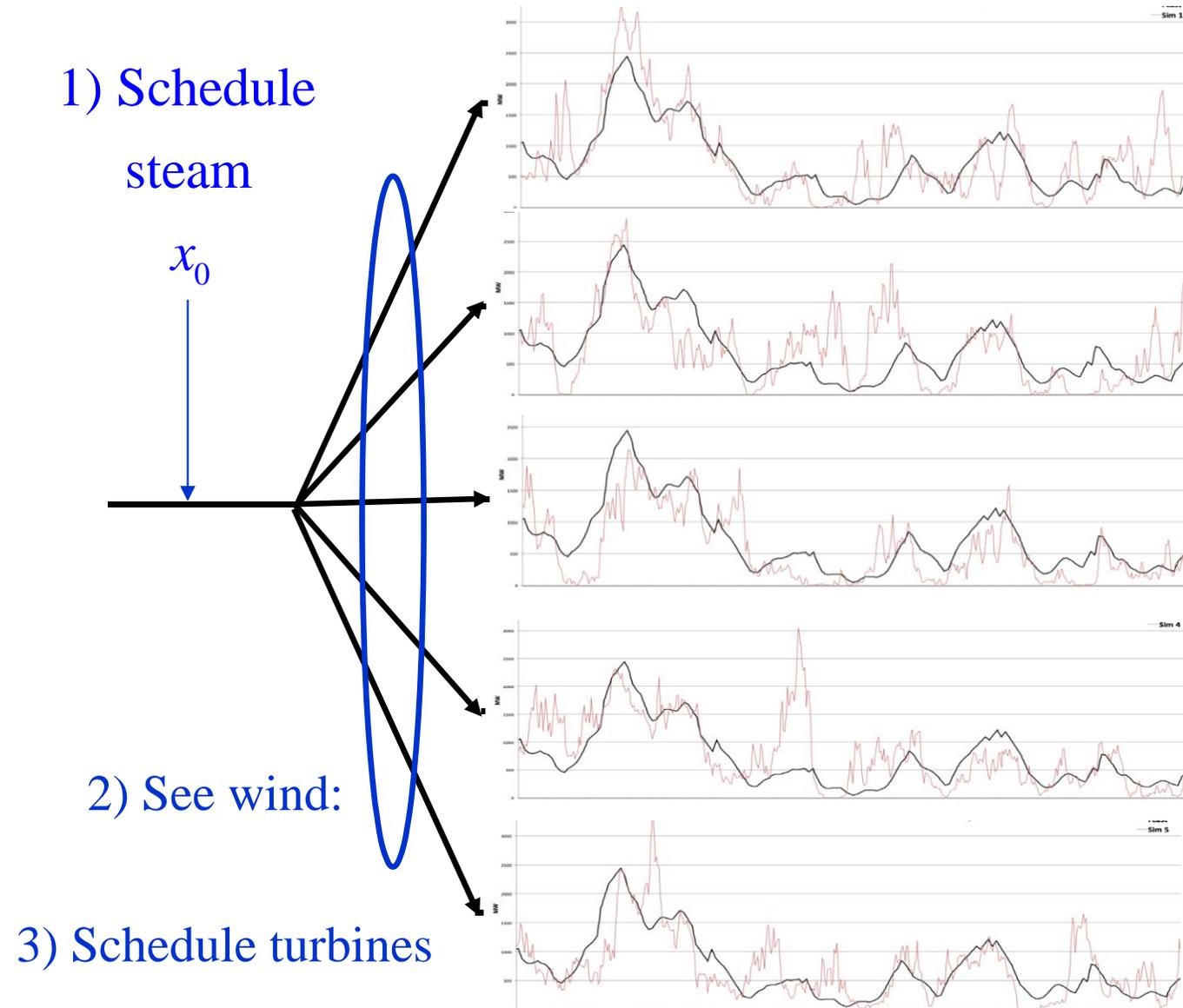
# Stochastic lookahead policies

- ❑ Creating wind scenarios (Scenario #5)



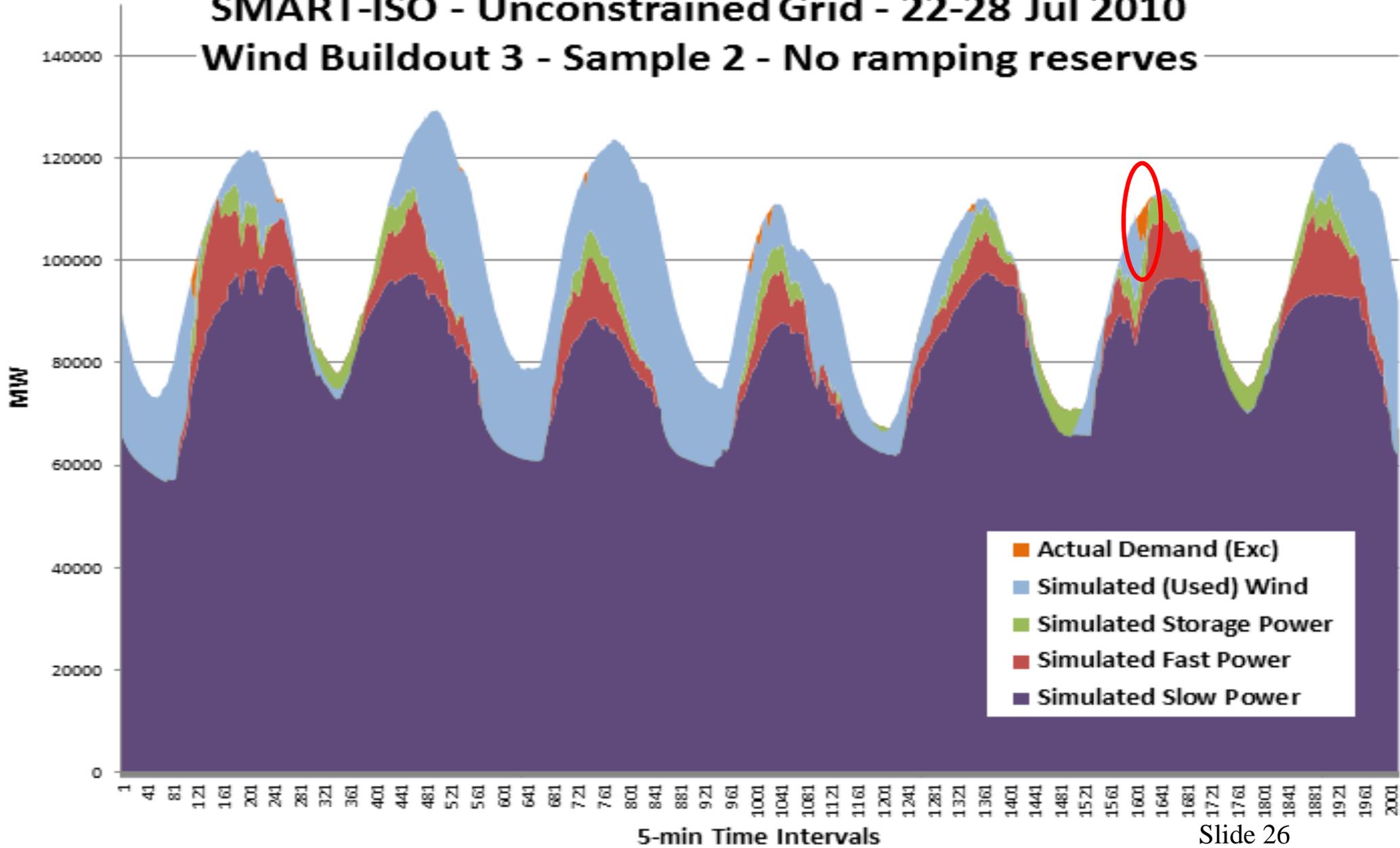
# Stochastic lookahead policies

## □ The two-stage approximation

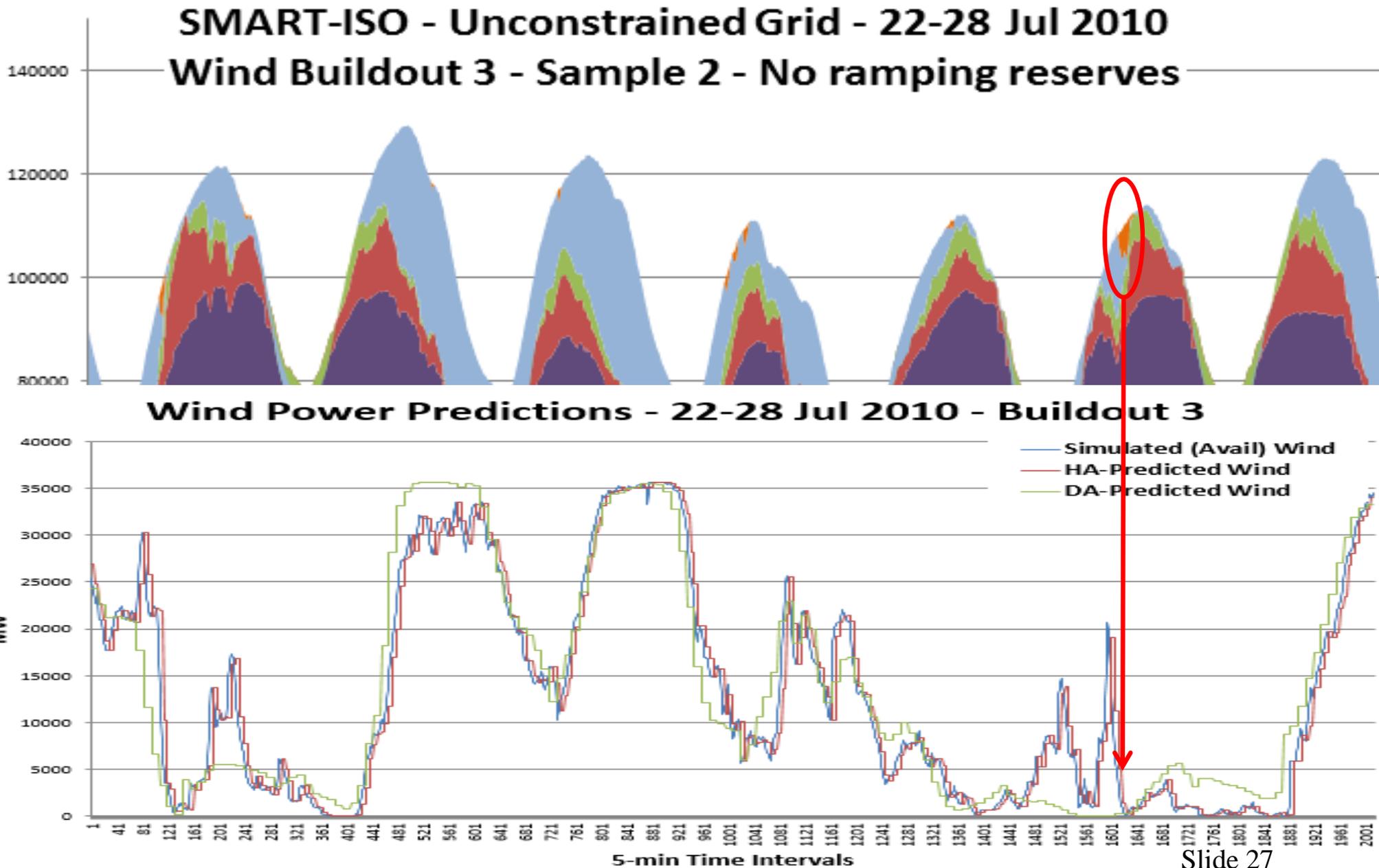


# Stochastic lookahead policies

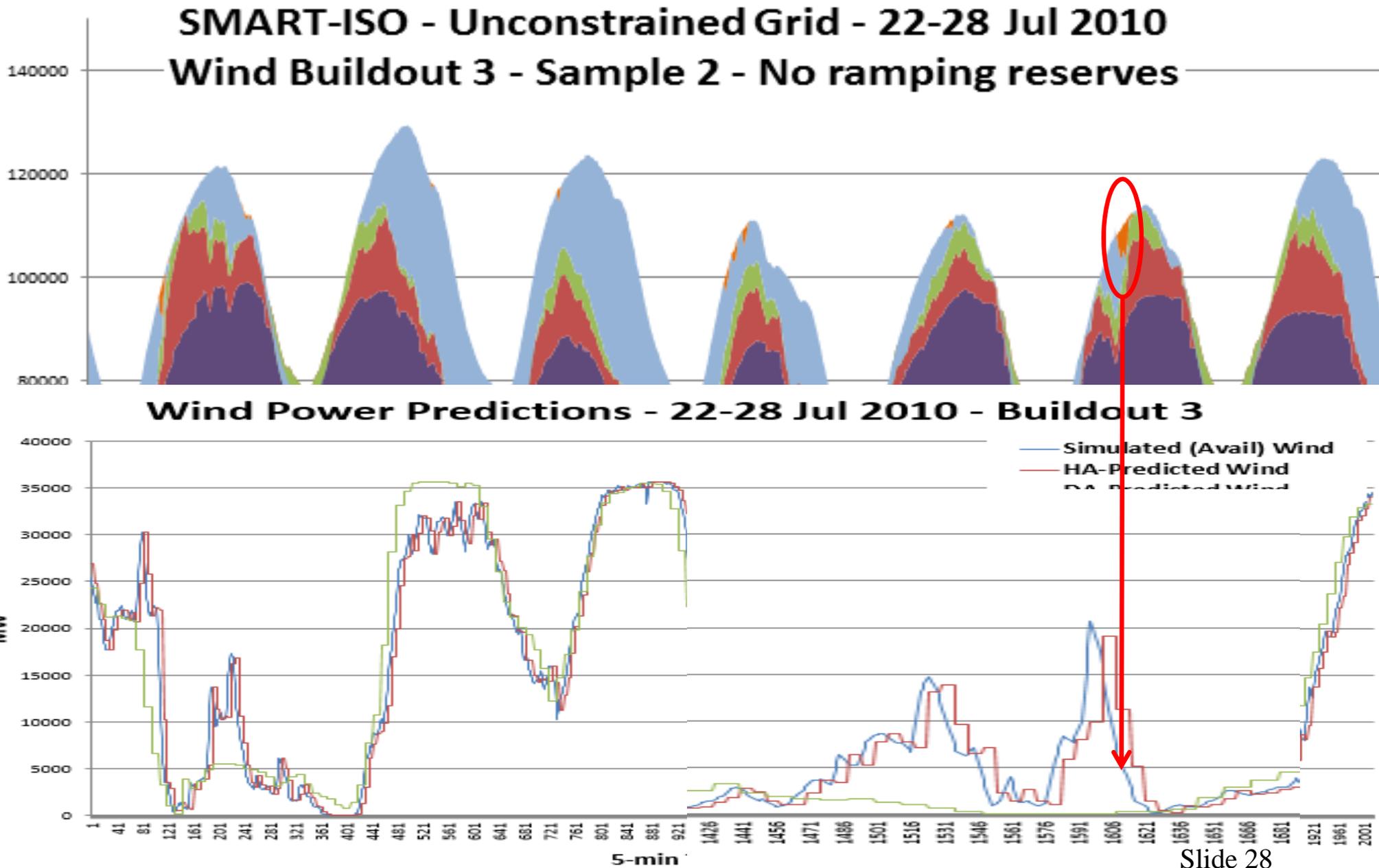
**SMART-ISO - Unconstrained Grid - 22-28 Jul 2010**  
**Wind Buildout 3 - Sample 2 - No ramping reserves**



# Stochastic lookahead policies

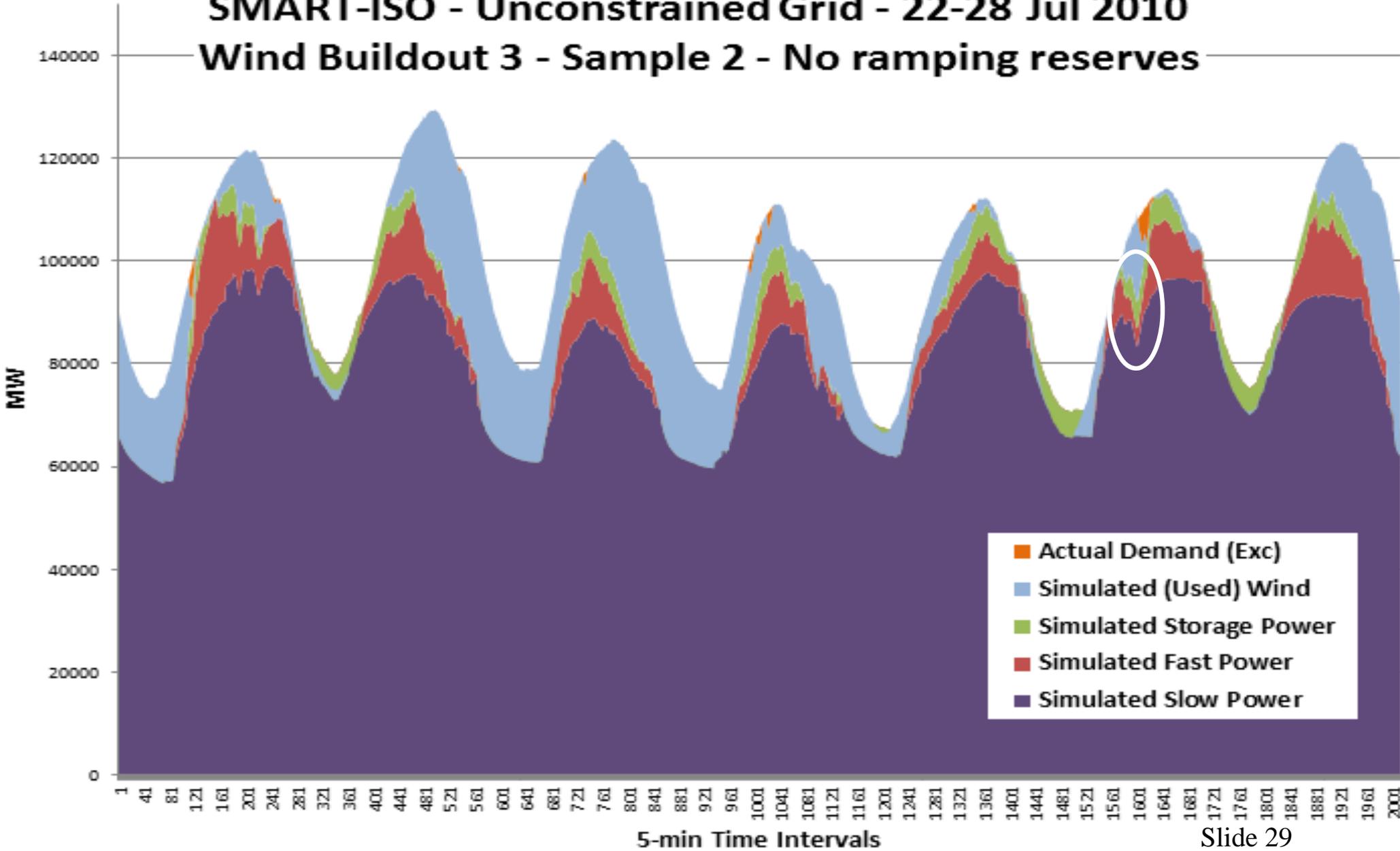


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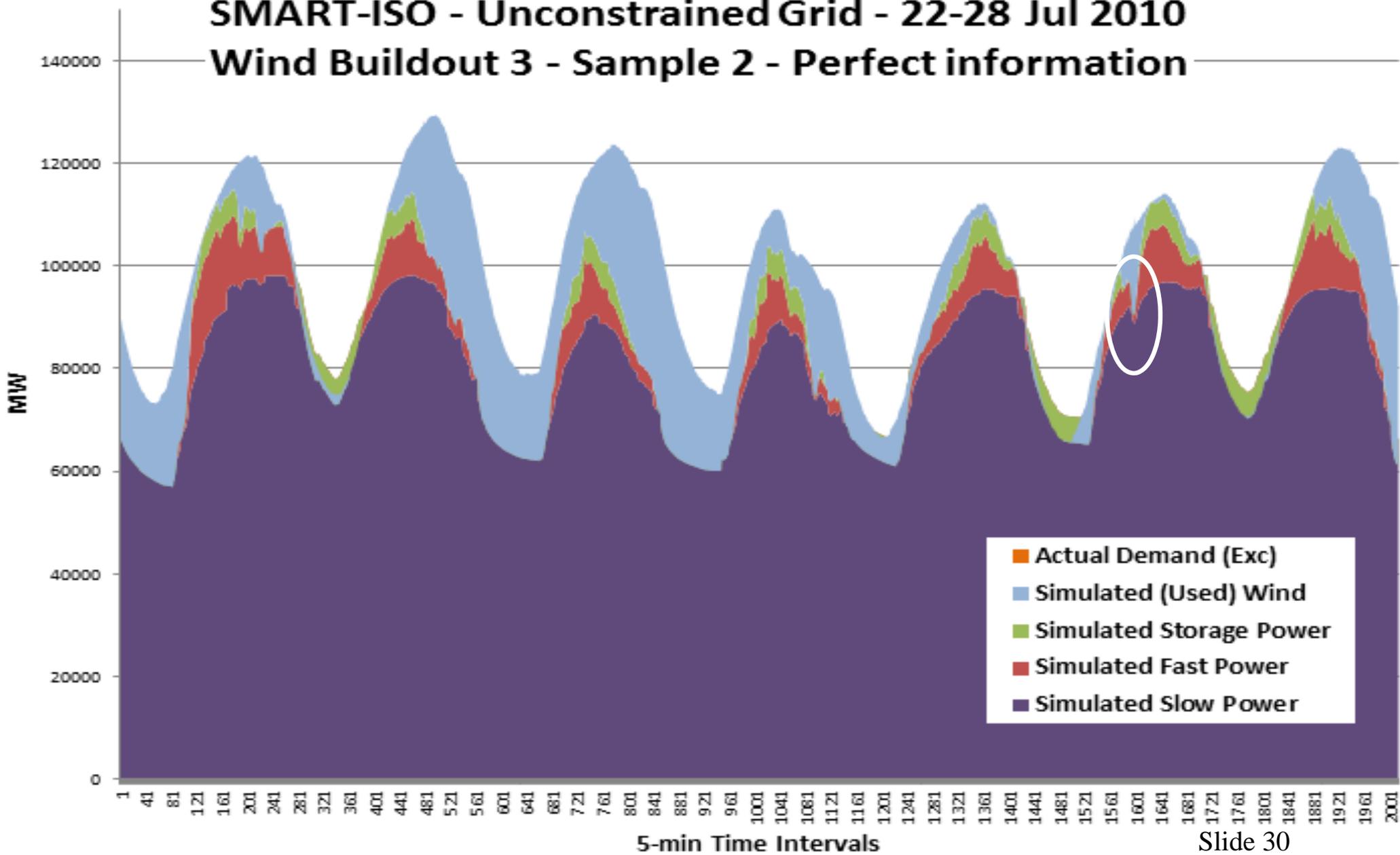
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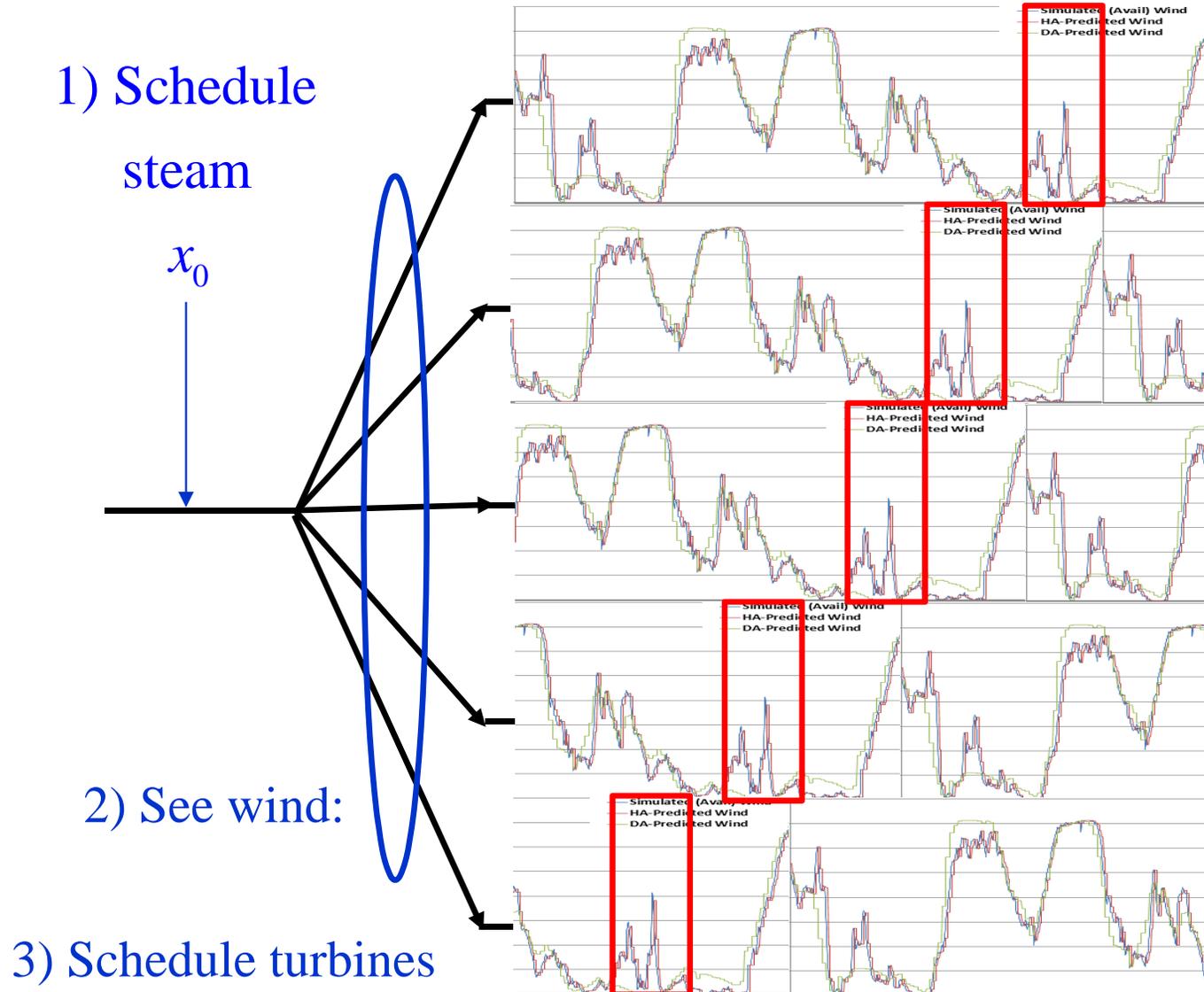
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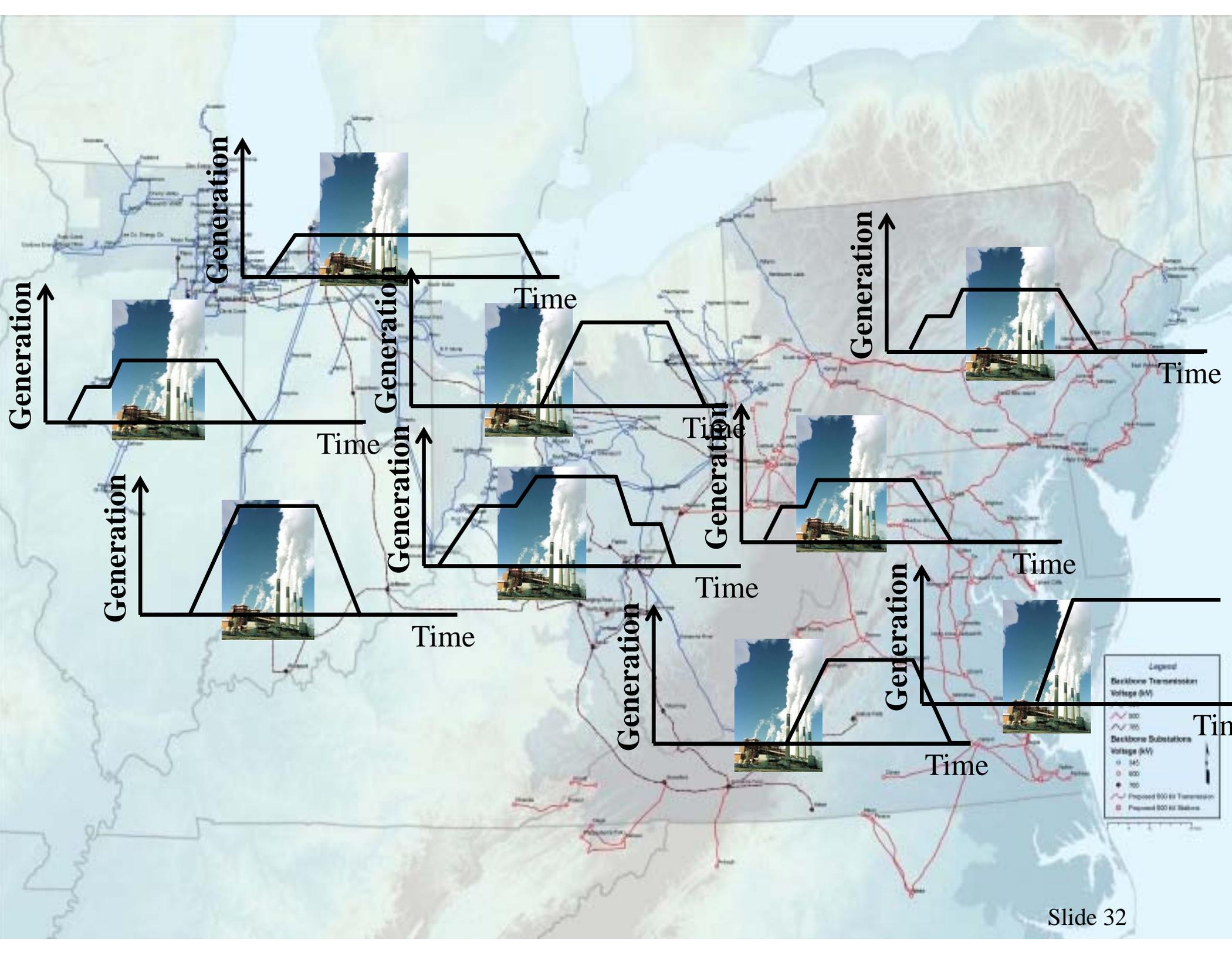
**SMART-ISO - Unconstrained Grid - 22-28 Jul 2010**  
**Wind Buildout 3 - Sample 2 - Perfect information**



# Stochastic lookahead policies

## □ The two-stage approximation





# Stochastic lookahead policies

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## □ Observations

- » As a general rule, we do not need a large number of scenarios to produce a robust policy.
- » The problem here is that the day-ahead decision (steam generation) is a very high-dimensional vector. We need to schedule steam generators:
  - Across the grid (spatially)
  - Over the entire day (temporally)
- » Scenarios have to create robust behavior across space, and across time.
- » *It appears for this application, we may need a dramatically larger number of scenarios than is required in applications with lower-dimensional first stage decisions.*

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# A hybrid lookahead-CFA policy

- A deterministic lookahead model
  - » Optimize over all decisions at the same time

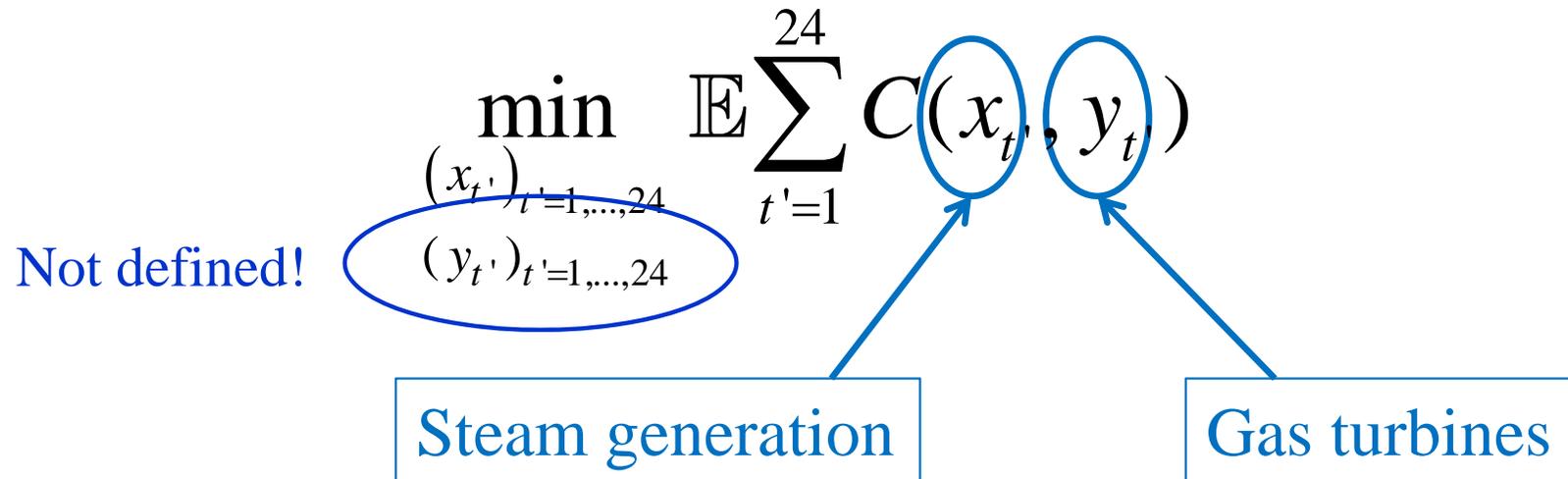
$$\min_{\substack{(x_{t'})_{t'=1,\dots,24} \\ (y_{t'})_{t'=1,\dots,24}}} \sum_{t'=1}^{24} C(x_{t'}, y_{t'})$$

The diagram illustrates the cost function  $C(x_t, y_t)$  where  $x_t$  and  $y_t$  are decision variables. The variable  $x_t$  is associated with 'Steam generation' and  $y_t$  is associated with 'Gas turbines'. Both variables are circled in blue, and blue arrows point from the corresponding boxes below to the circled variables in the cost function.

- » These decisions need to be made with different horizons
  - Steam generation is made day-ahead
  - Gas turbines can be planned an hour ahead or less

# A hybrid lookahead-CFA policy

- A stochastic lookahead model
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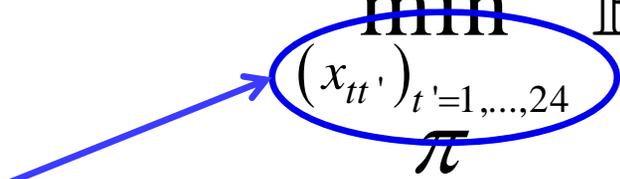
# Designing a policy

## □ A robust lookahead-CFA policy

- » We imbed a policy for fast-response adjustments within a lookahead model for planning steam:

$$\min_{\pi} \mathbb{E} \sum_{t'=t}^{t+48} C(x_{tt'}, Y^{\pi}(S_{tt'}))$$

Steam generation



- $x_{t,t'}$  is determined at time  $t$ , to be implemented at time  $t'$
- $y_{t',t'}$  is determined at time  $t'$  by the policy  $Y^{\pi}(S_{tt'})$

- » The challenge now is to adaptively estimate the ramping constraints  $\theta = (\theta^{up}, \theta^{down})$  and the policies  $Y^{\pi}(S_{tt'})$ .

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$$\min_{(\pi, (x_{tt'})_{t'=1, \dots, 24})} \mathbb{E} \sum_{t'=t}^{t+48} C(x_{tt'}, Y^\pi(S_{tt'}))$$

Real-time adjustment of hydro and thermal plants.  
 $\pi$  is the type of policy.

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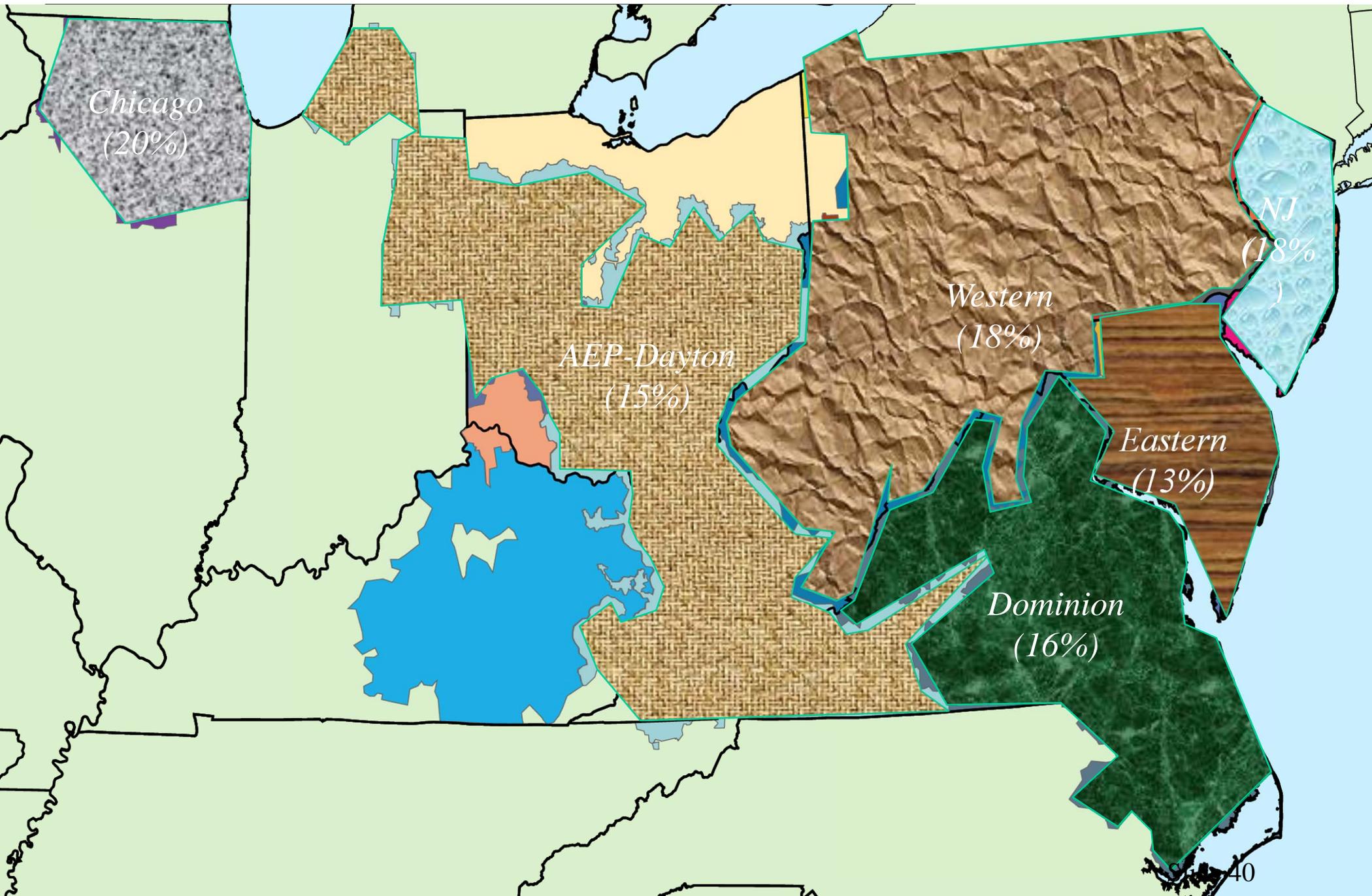
$$x_{t,t'}^{\max} - x_{t,t'} \geq \theta^{up} L_{tt'} \quad \text{Up-ramping reserve}$$

$$x_{t,t'} - x_{t,t'}^{\max} \geq \theta^{down} L_{tt'} \quad \text{Down-ramping reserve}$$

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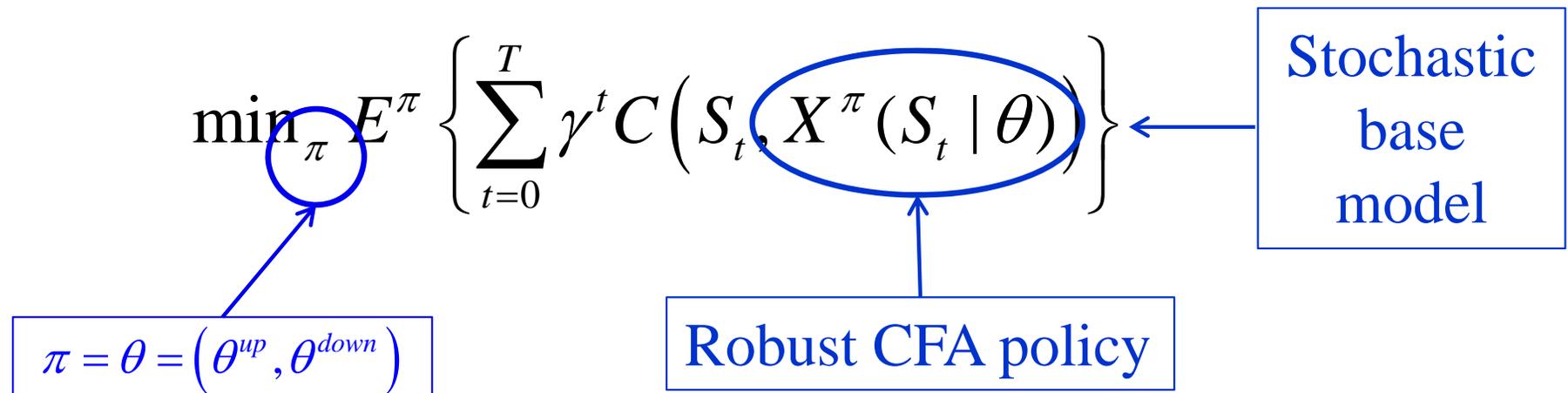
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# SMART-ISO “Regions”



# Designing a policy

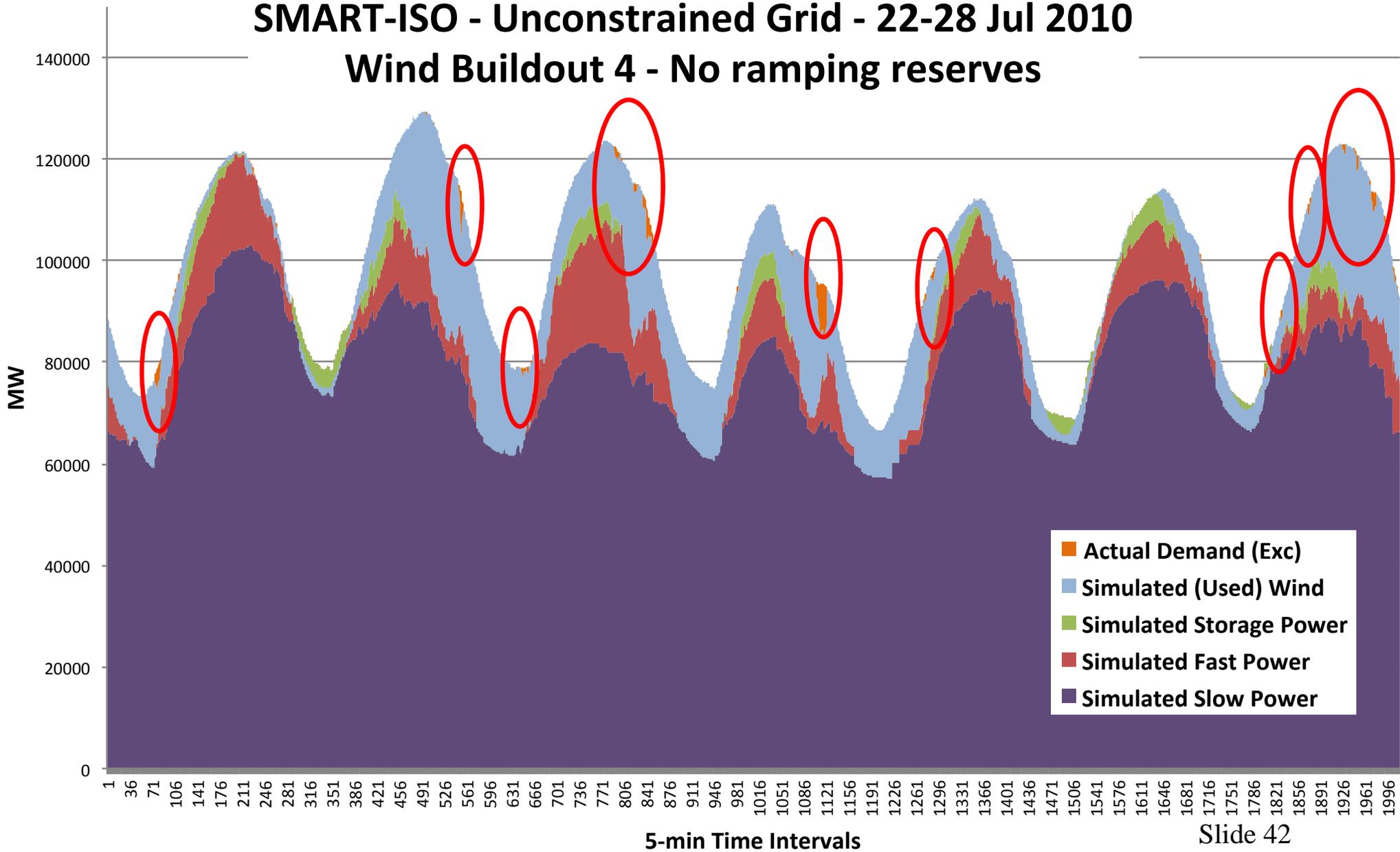
- We then have to tune the parameters of this policy in our *stochastic base model*.



- » The challenge now is to adaptively estimate the ramping constraints  $\theta = (\theta^{up}, \theta^{down})$ .

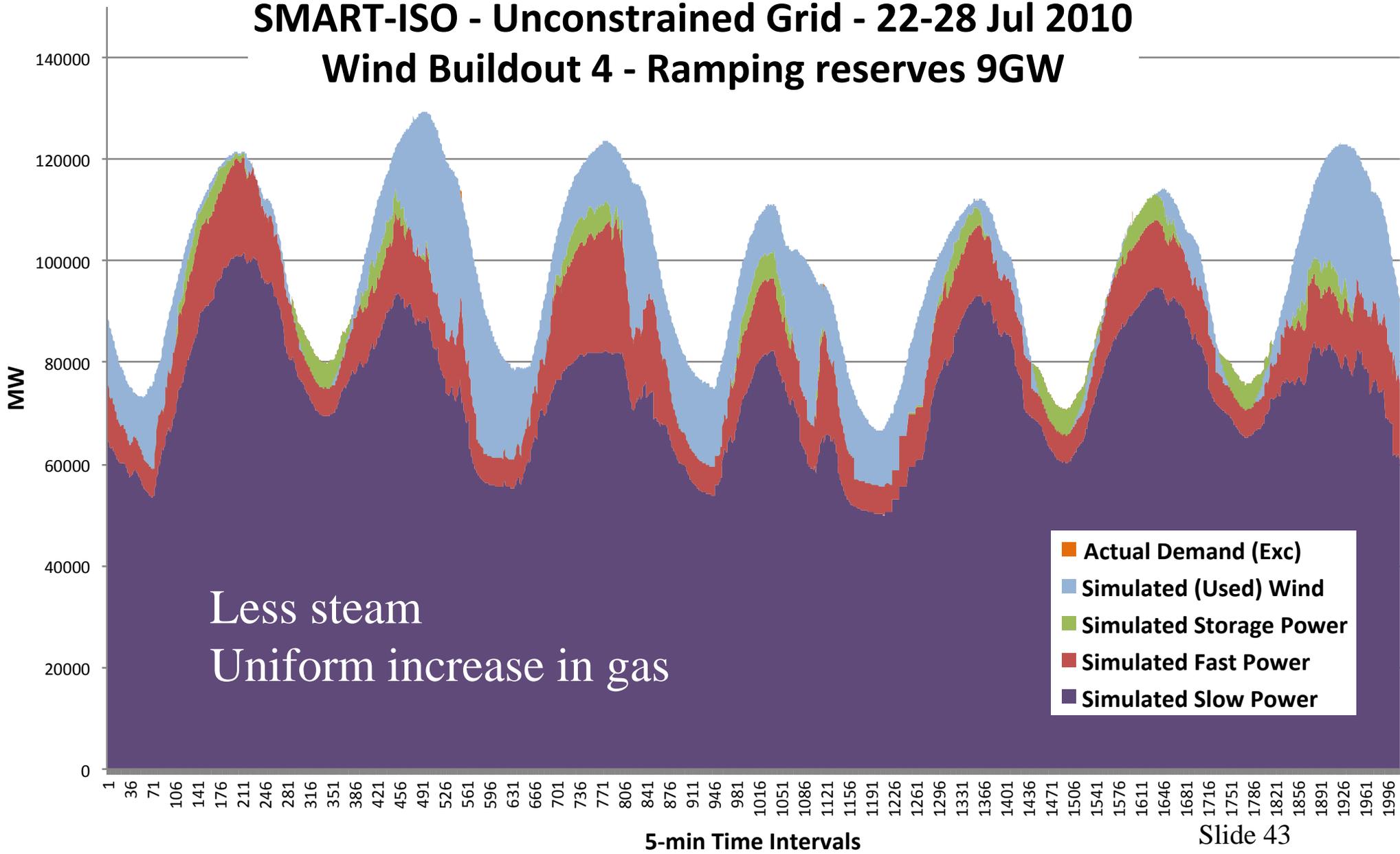
# SMART-ISO: Offshore wind study

**SMART-ISO - Unconstrained Grid - 22-28 Jul 2010**  
**Wind Buildout 4 - No ramping reserves**



# SMART-ISO: Offshore wind study

**SMART-ISO - Unconstrained Grid - 22-28 Jul 2010**  
**Wind Buildout 4 - Ramping reserves 9GW**



# Lecture outline

- Perspectives on robust policies

# Comparison of policies

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## ❑ Robust cost function approximation

- » Requires solving modified deterministic lookahead model
- » Parameters have to be tuned, ideally with a highly realistic *base model* (“simulator”)
- » User captures domain knowledge when specifying the structure of the CFA

## ❑ Stochastic lookahead

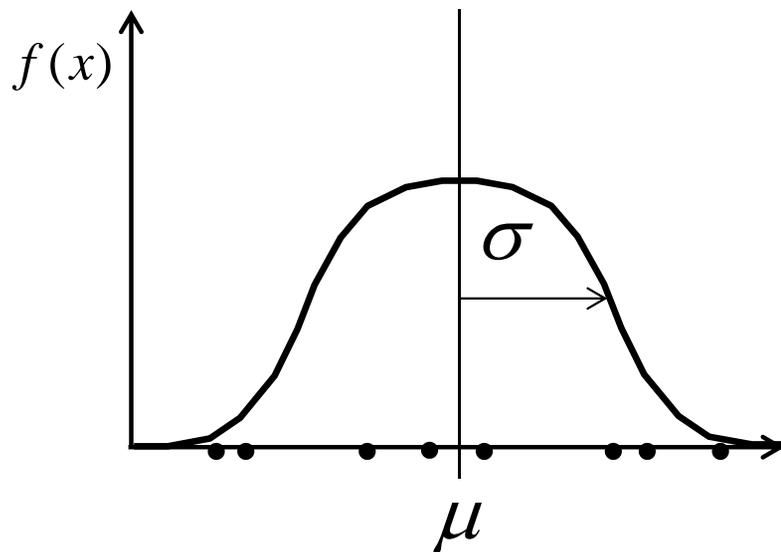
- » Requires solving approximate stochastic lookahead model
- » Tuning is generally not done (although in theory possible, but it is very expensive)
- » No need for user-specified parametric approximation (and no ability to incorporate domain knowledge)

# Approximating distributions

## □ Stochastic lookahead model

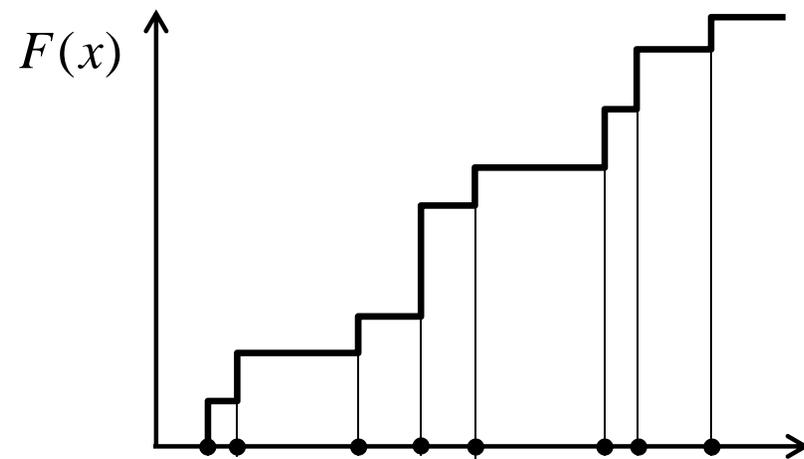
- » Uses approximation of the *information process* in the lookahead model

Parametric distribution (pdf)



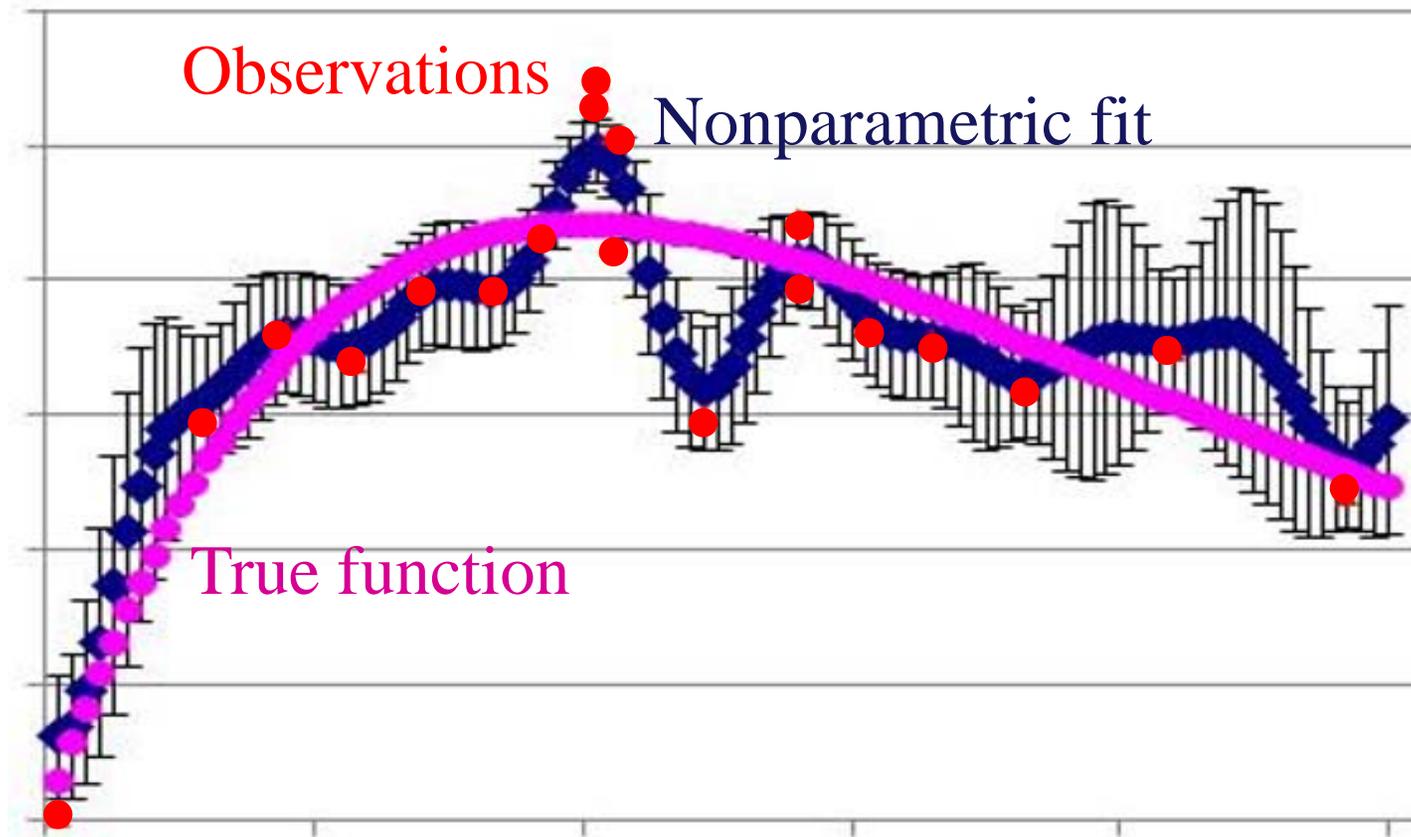
$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left(\frac{(x-\mu)^2}{\sigma^2}\right)}$$

Nonparametric distribution (cdf)



# Approximating a function

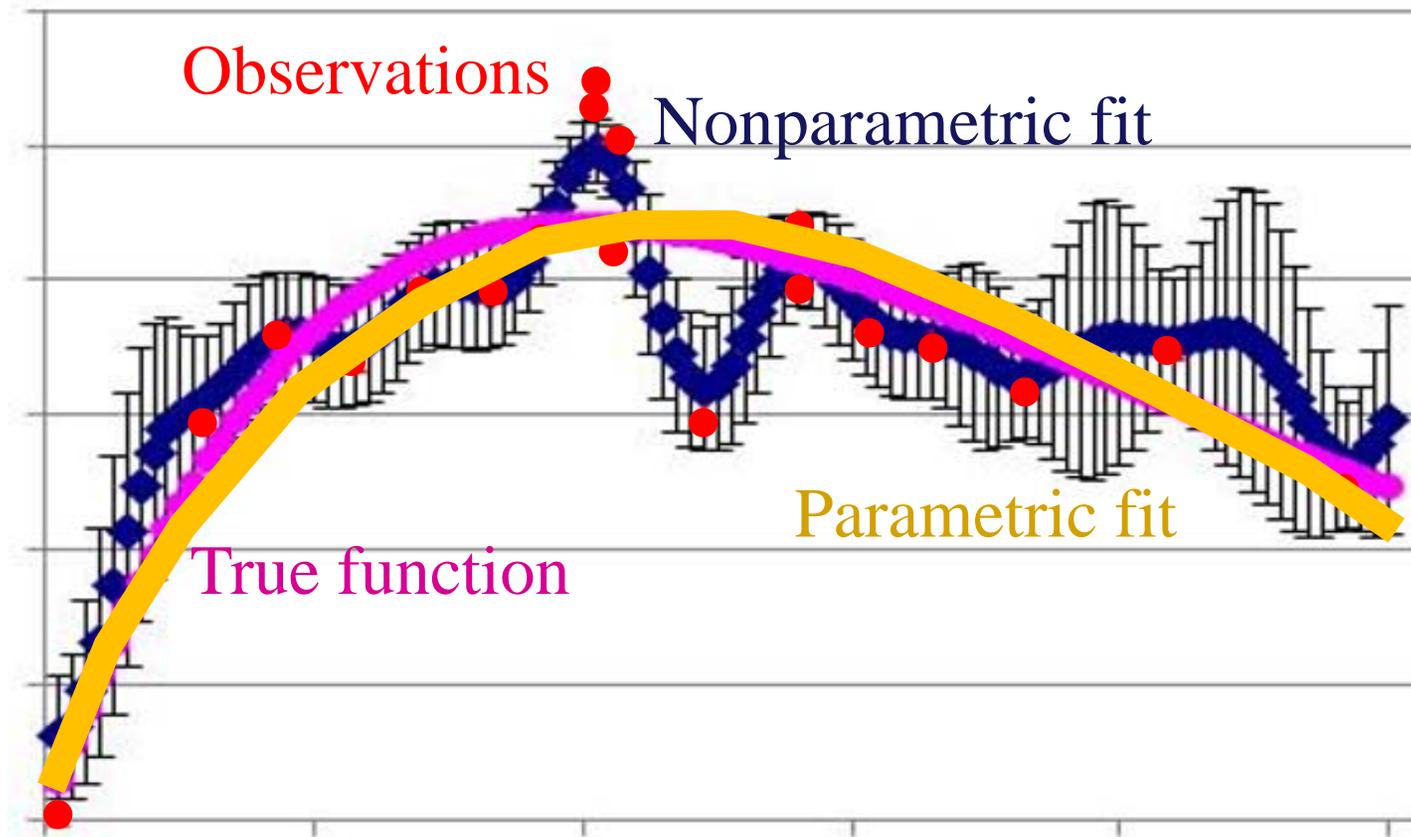
## □ Parametric vs. nonparametric



» We can use our understanding of the function to impose a shape.

# Approximating a function

## □ Parametric vs. nonparametric



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# Designing a policy

## □ A robust lookahead-CFA policy

- » We imbed a policy for fast-response adjustments within a lookahead model for planning steam:

$$F_t(S_t, \theta) = \min_{(x_{tt'})_{t'=1, \dots, 24}} \mathbb{E} \sum_{t'=t}^{t+48} C(x_{tt'}, Y^\pi(S_{tt'}))$$

$x_{t,t'}^{\max} - x_{t,t'} \geq \theta^{up} L_{tt'}$       Up-ramping reserve

$x_{t,t'} - x_{t,t'}^{\max} \geq \theta^{down} L_{tt'}$       Down-ramping reserve

- » It is easy to tune this policy when there are only two parameters  $\theta = (\theta^{up}, \theta^{down})$
- » But perhaps the ramping reserves should depend on other information?

# Designing a policy

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$\pi$

$$x_{t,t'}^{\max} - x_{t,t'} \geq \theta^{up}(S_t)L_{t,t'} \quad \text{Up-ramping reserve}$$
$$x_{t,t'} - x_{t,t'}^{\max} \geq \theta^{down}(S_t)L_{t,t'} \quad \text{Down-ramping reserve}$$

- » The parameters might depend on a state variable that captures
- Weather forecast (esp. change in weather)
  - Load forecast (indicates how close to capacity)
- » Now the ramping parameters are *functions*. ☹

# Designing a policy

## □ How do we compare policies?

- » Let's agree on a base model (a simulator) and compare different policies!

$$\bar{F}^{\pi} \approx \sum_{n=1}^N p(\omega^n) \sum_{t=0}^T \gamma^t C\left(S_t(\omega^n), X^{\pi}(S_t(\omega^n))\right)$$

where  $\omega$  represents a sample path capturing season, type of meteorology (stormy, calm), and stochastic variations around this base.

- » Now compare
  - $\bar{F}^{Robust-CFA}(\theta^{Robust-CFA})$
  - $\bar{F}^{Stoch-LA}(\theta^{Stoch-LA})$

# Designing a policy

- ❑ So we have a choice:
  - » Stochastic lookahead model?
  - » Robust cost function approximation tuned on a *stochastic base model* (the “simulator”)?
- ❑ Stochastic lookahead model:
  - » Lookahead model introduces several approximations:
    - Finite horizon, two-stages, limited scenarios, coarser temporal decomposition, unable to handle adaptive hedging
    - Computationally very hard to solve
    - Produces stochastic LMPs
  - » Robust cost function approximation
    - Tuned using very realistic *base model* using parametric cost function approximation which captures domain knowledge.
    - Computationally comparable to current models
    - LMPs computed using existing practices.

