# Quantifying the trade-off between secure and economic operation of power systems under uncertainty

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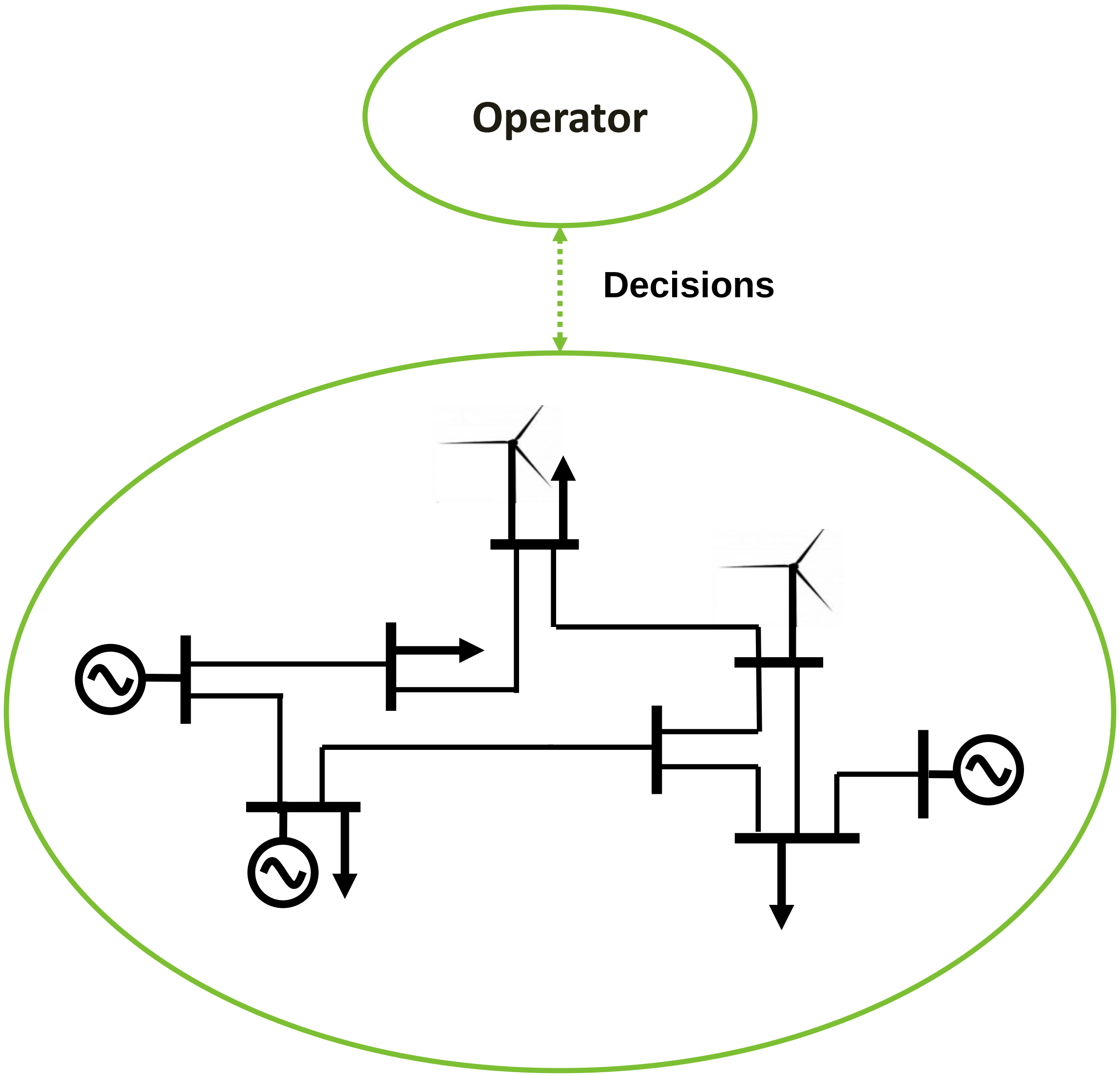
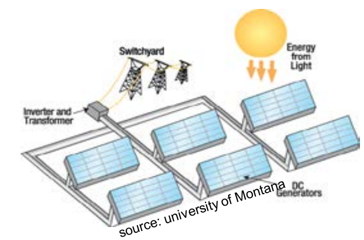
Joint work with
K. Margellos (UC Berkeley),
and J. Lygeros, G. Andersson (ETH Zurich)

# Decisions under uncertainty

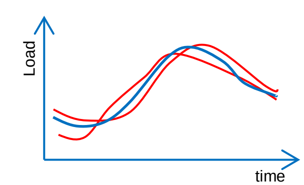



**Wind power**

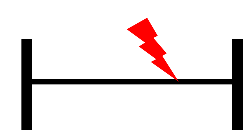
**PV power**




**Demand**




**Contingencies**



# Decisions under uncertainty

Main tasks of the Transmission System Operator (TSO):

**1. Ensure ''N-1 security''**

N-1 security criterion:

No operational limit violation after any single component outage

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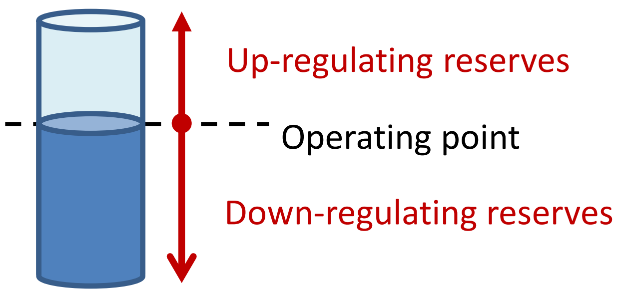
No operational limit violation after any single component outage

**2. Maintain power balance**

Generation active power capacity



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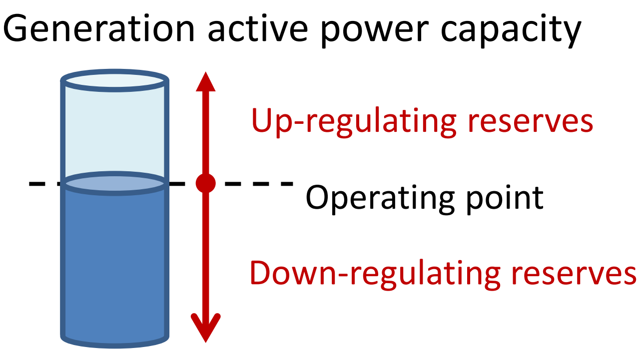
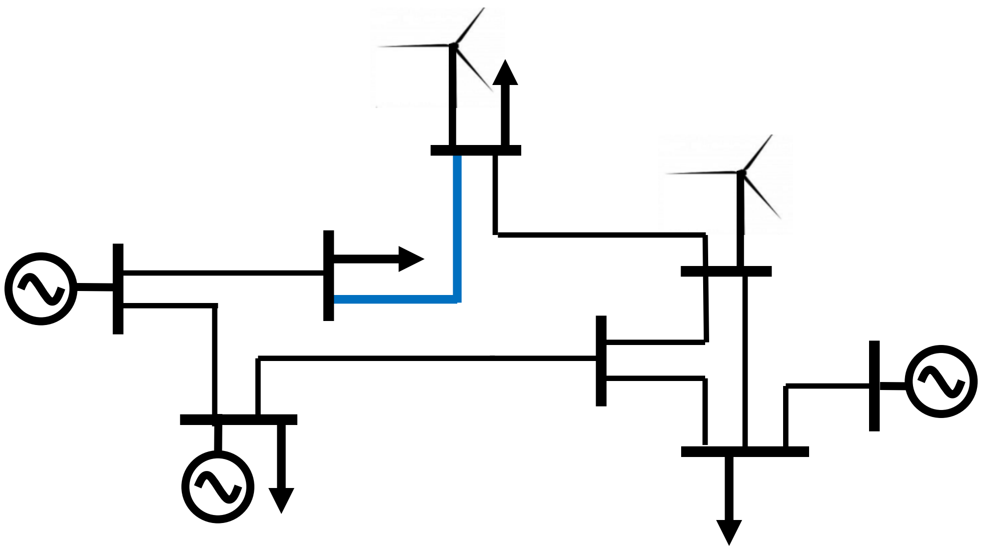


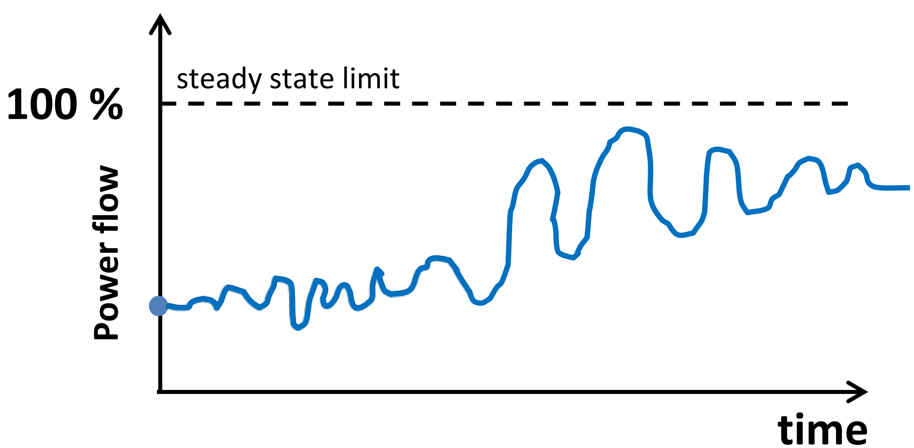

**Optimal component setpoints**
for *preventive* and *corrective* control

**Optimal reserve capacity and allocation**

..but an optimal and secure operation under **uncertainty** is a challenging problem!

# Decisions under uncertainty



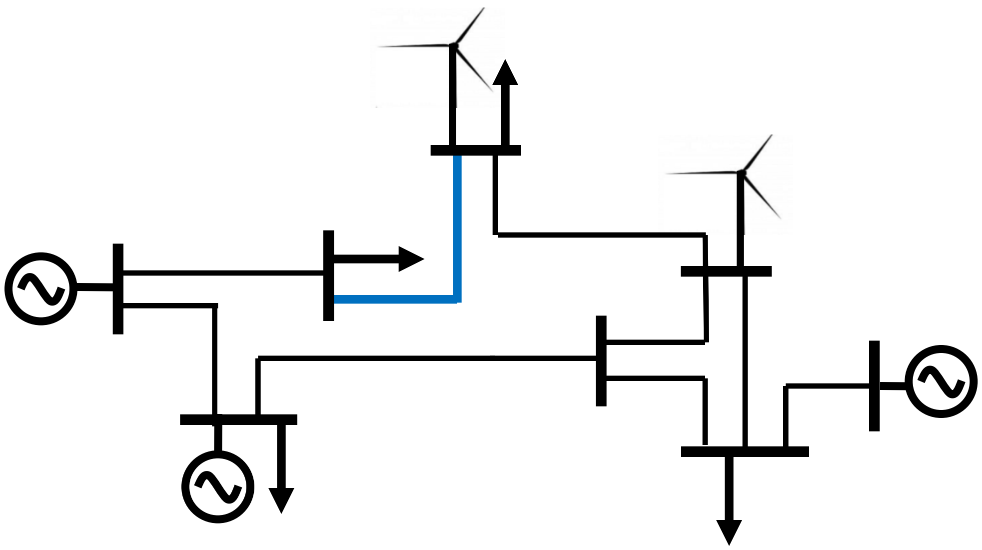


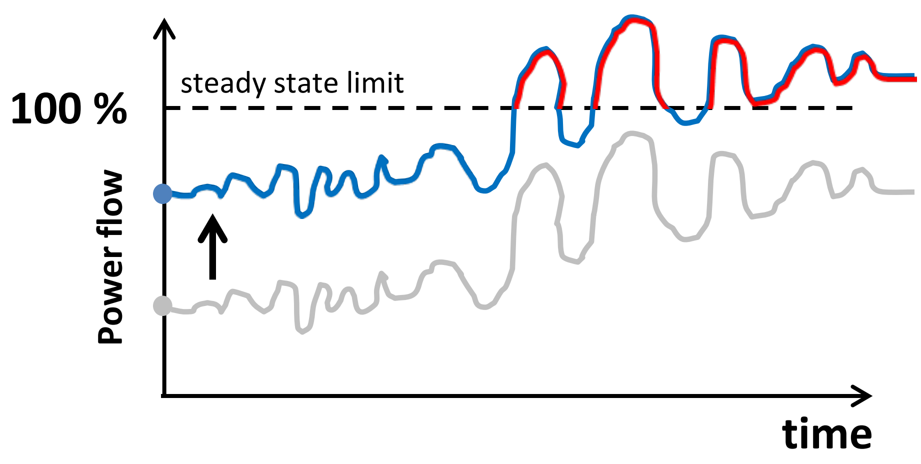

**In the past:**

- Not heavily loaded system
- Low share of Renewable Energy Sources (RES)

➡ **Enough margin to withstand uncertainty**

# Decisions under uncertainty

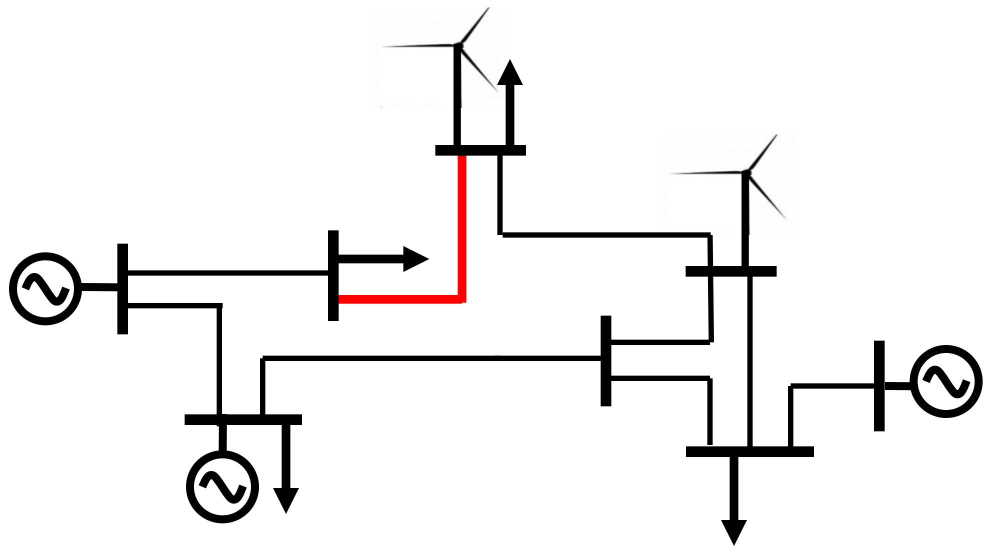


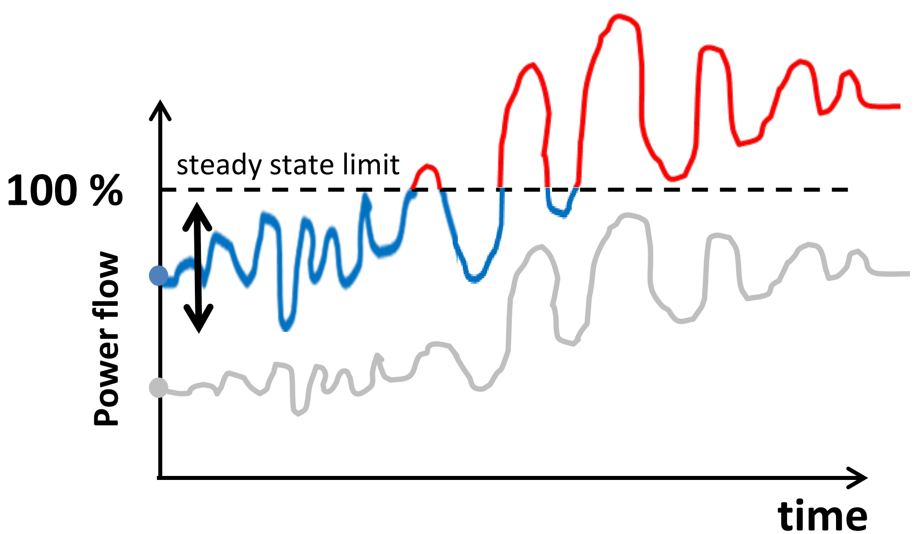



**But there are trends of:**

⬆ demand
(higher loading on the system)

# Decisions under uncertainty

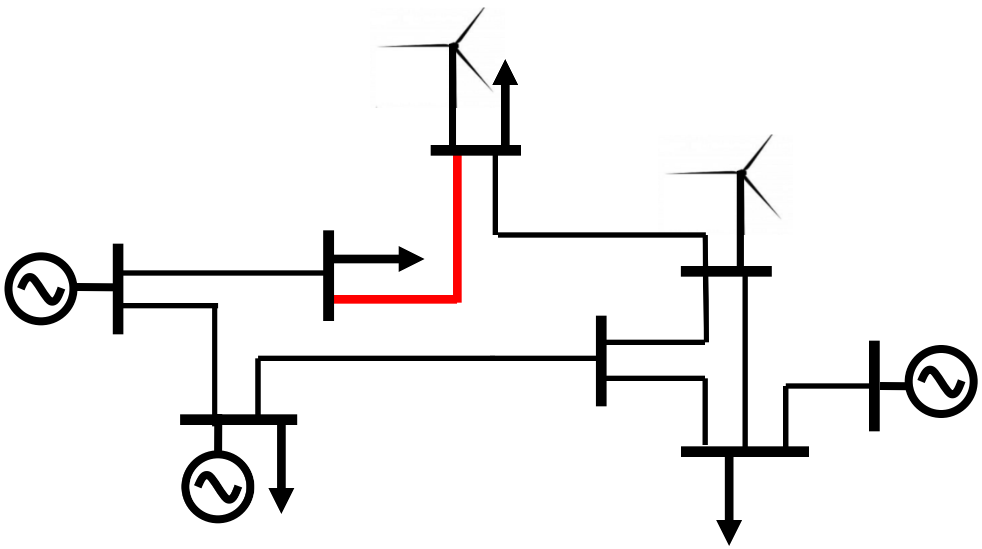


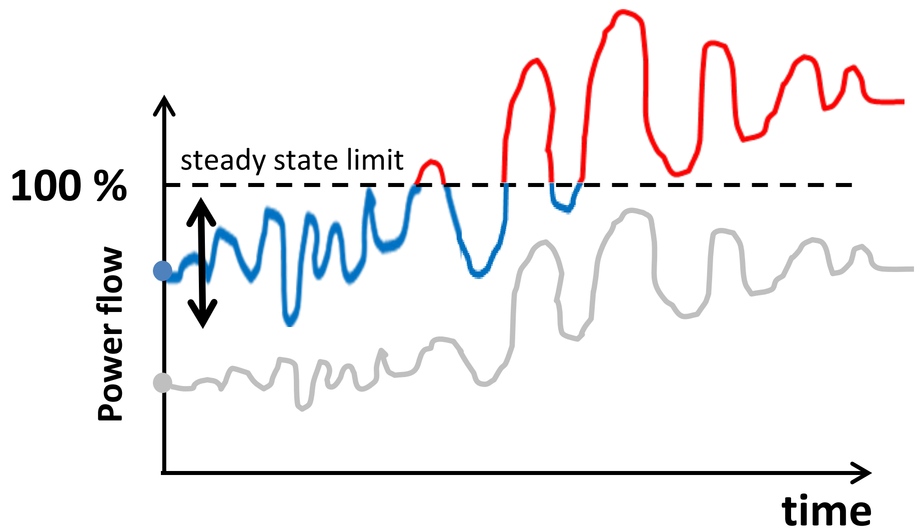



**But there are trends of:**

- ⬆ demand (higher loading on the system)
- ⬆ penetration of RES (higher uncertainty)

# Decisions under uncertainty

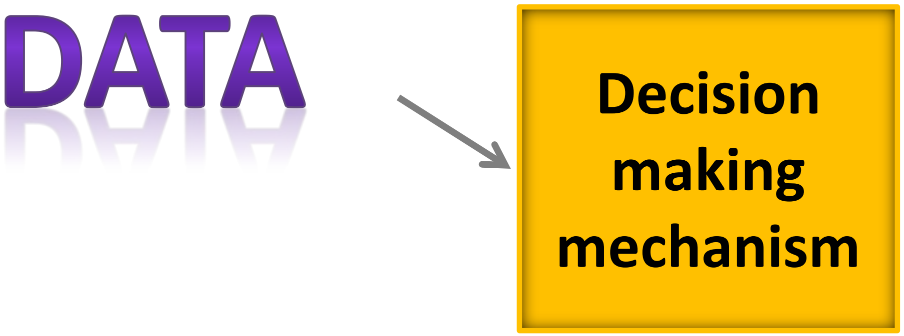





- Operating closer to the margins
- Maybe not possible to withstand uncertainty

# Motivation

## Planning



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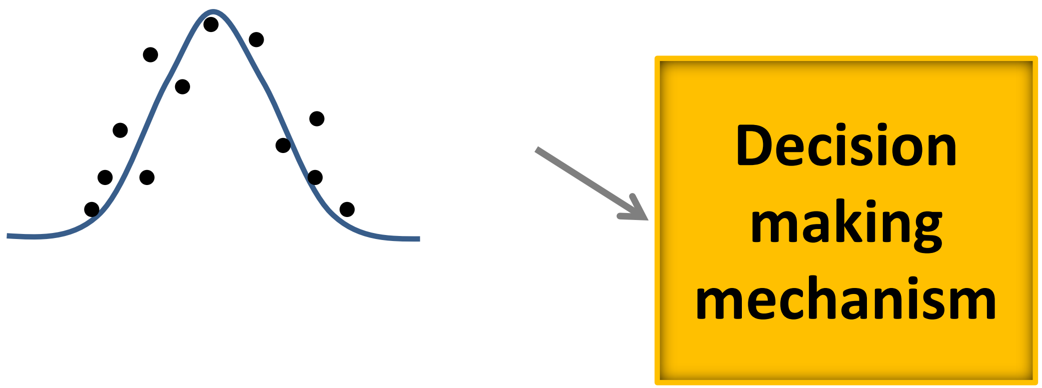

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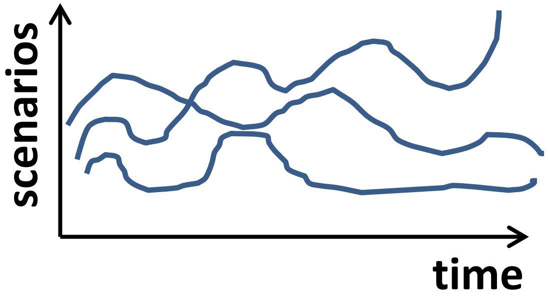
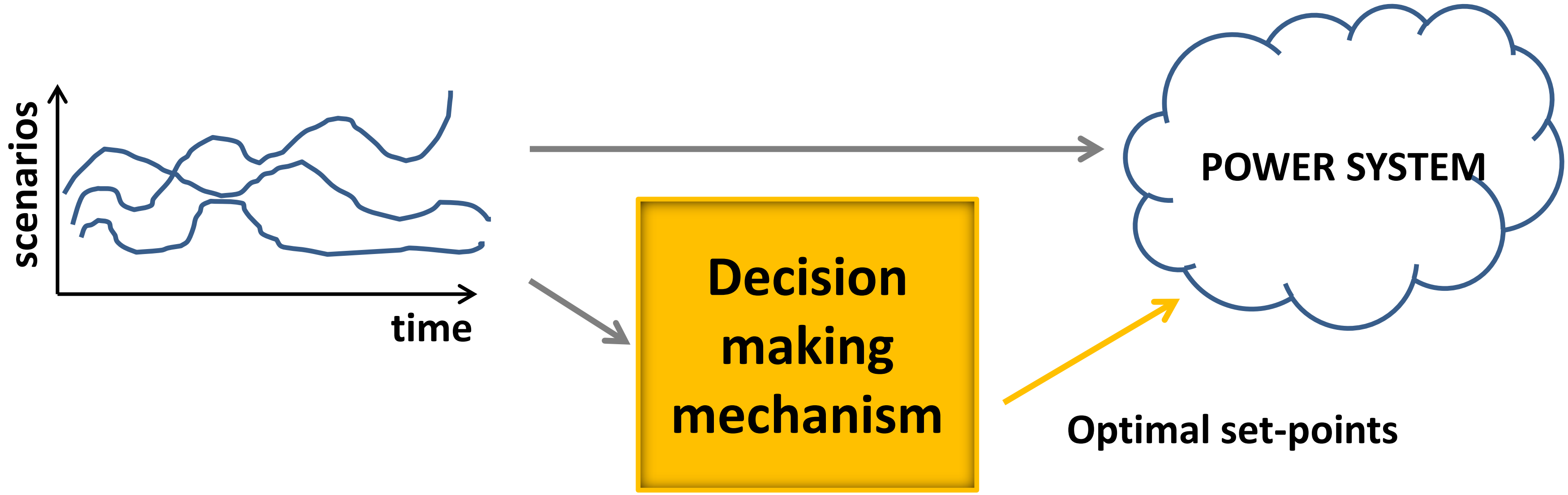



**Decision making mechanism**

# Motivation

## Planning



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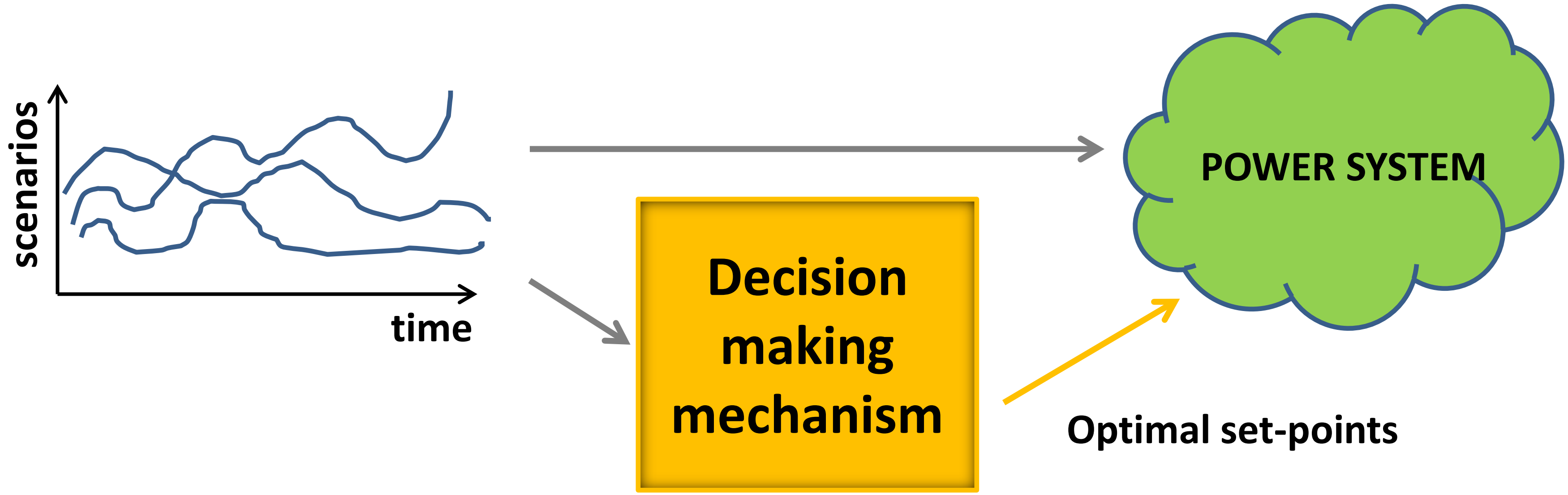

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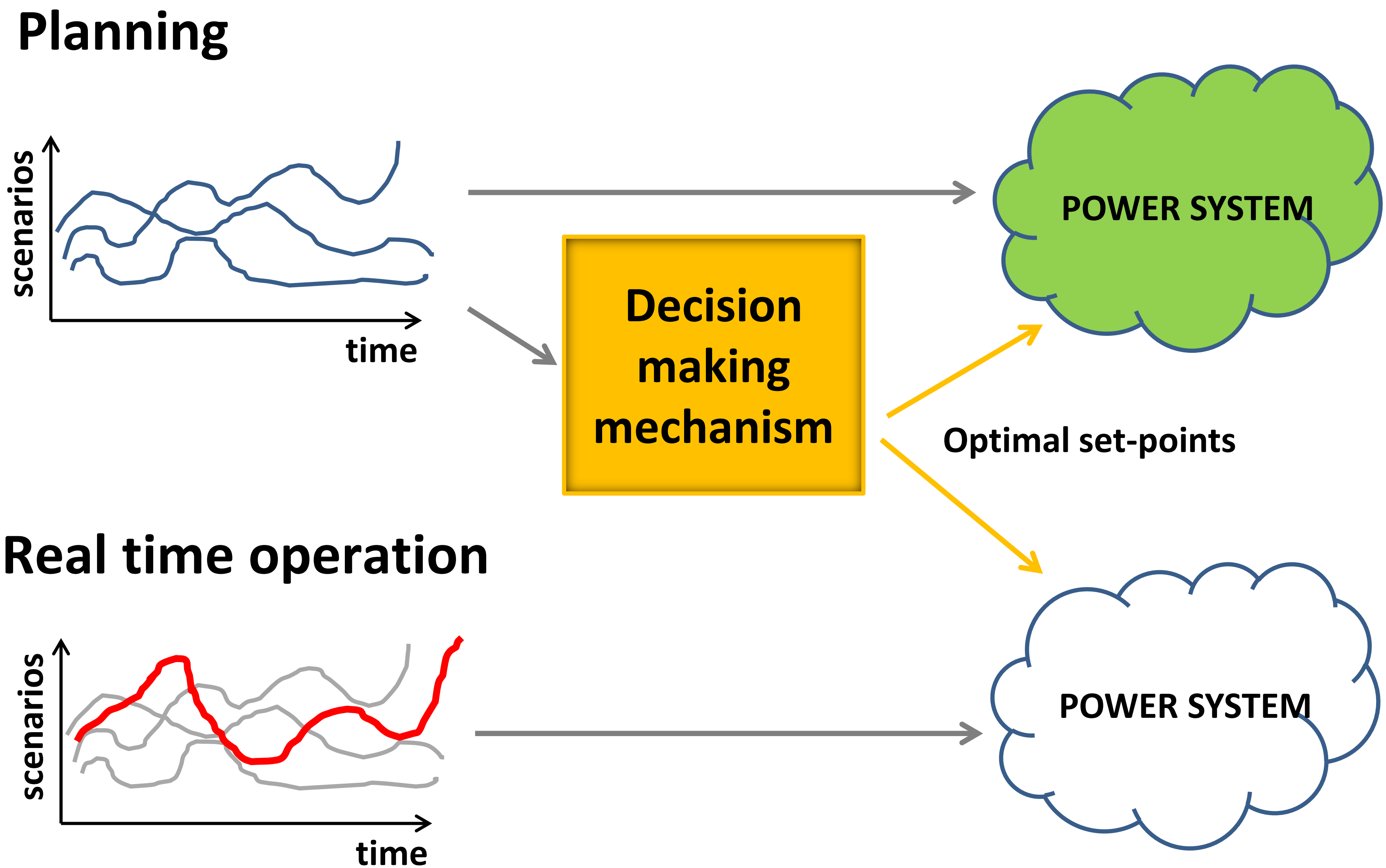
Planning
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POWER SYSTEM
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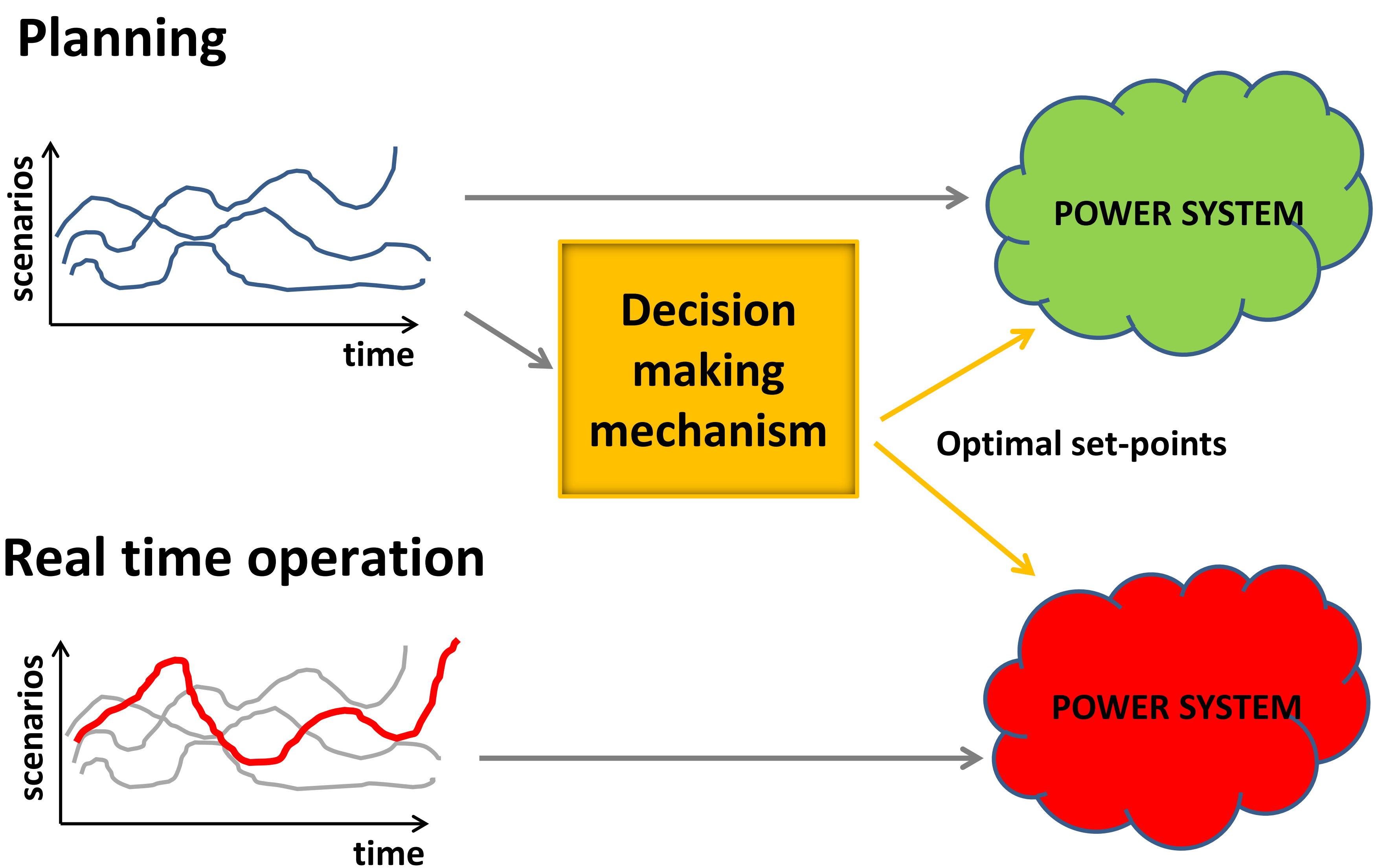
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# Objectives

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- Probabilistic Optimal Power Flow formulations
  - N-1 Security
  - Reserve provision

# Outline

1. Probabilistic DC based SC-OPF
2. Probabilistic AC based SC-OPF
3. Exploiting component controllability
4. Co-optimization of energy and reserves

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1. **Probabilistic DC based SC-OPF**

2. Probabilistic AC based SC-OPF

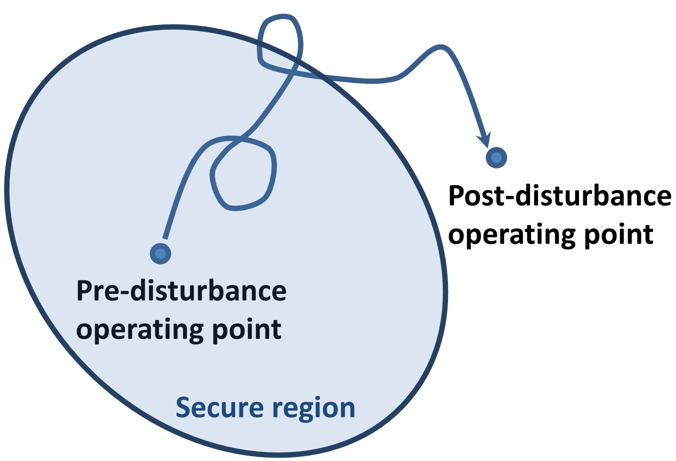
3. Exploiting component controllability

4. Co-optimization of energy and reserves

# 1. Probabilistic DC based SC-OPF

## a) Problem set-up

- DC power flow $\longrightarrow$ linearized network equations
- Uncertainty: wind power $P_w$
- Preventive control: generation dispatch $P_G$
- Security for the post-disturbance steady state operating point after the Secondary Frequency Control



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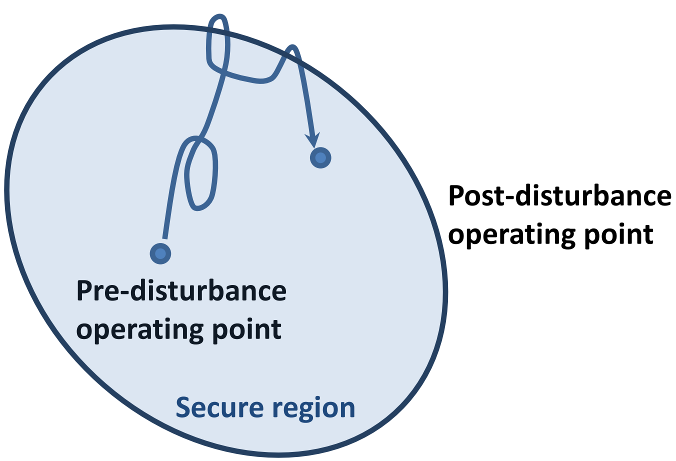

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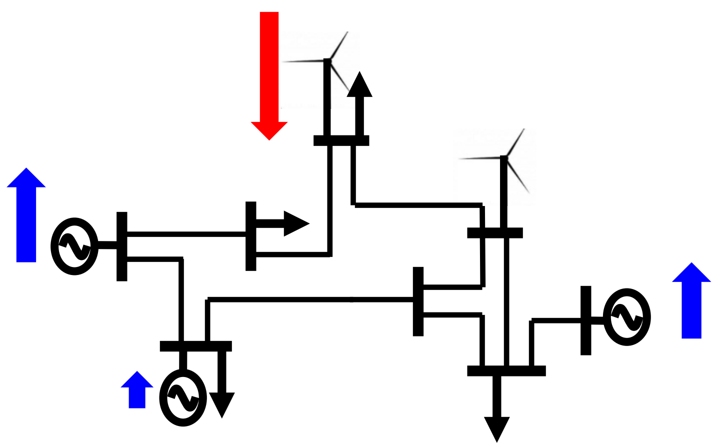
Generation-load mismatch $P_{mismatch}$ is compensated by the generators



$$P_{G,post} = P_G - d \cdot P_{mismatch}$$

$P_{mismatch}$: linear function of $P_G$ and $P_w$

$d$: "distribution vector"

# 1. Probabilistic DC based SC-OPF

## b) Optimization problem

**Deterministic problem**

$$\min_{\boldsymbol{x}} J(\boldsymbol{x})$$

subject to

$$F_{eq}\boldsymbol{x} + f_{eq} + H_{eq}\boldsymbol{\delta}^{\boldsymbol{f}} = 0$$

$$F\boldsymbol{x} + f + H\boldsymbol{\delta}^{\boldsymbol{f}} \leq 0$$

Decision variables: $\boldsymbol{x} = P_G$

Uncertain variables: $\boldsymbol{\delta} = P_w$

# 1. Probabilistic DC based SC-OPF

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**Chance constrained problem**

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$$\mathbb{P}(F\boldsymbol{x} + f + H\boldsymbol{\delta} \leq 0) \geq 1 - \varepsilon$$

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**Trade-off between security and cost:** $\varepsilon \in (0,1)$

# 1. Probabilistic DC based SC-OPF

## c) Dealing with the chance constraint

**Probabilistically robust design** [1]

- Mixture of randomized and robust optimization
- No assumptions on the underlying distribution of the uncertainty
- Guarantees that the chance constraint will be satisfied with a certain confidence

**Other methods used in the thesis**

- The scenario approach [2]
- Trading feasibility to optimality - Sampling and discarding [3]

[1] Margellos, Goulart, Lygeros, Trans. Aut. Con 2013.
[2] Calafiore, Campi, Trans. Aut. Con., 2006
[3] Campi , Garatti, Journal of Optimization Theory and Applications, 2011.

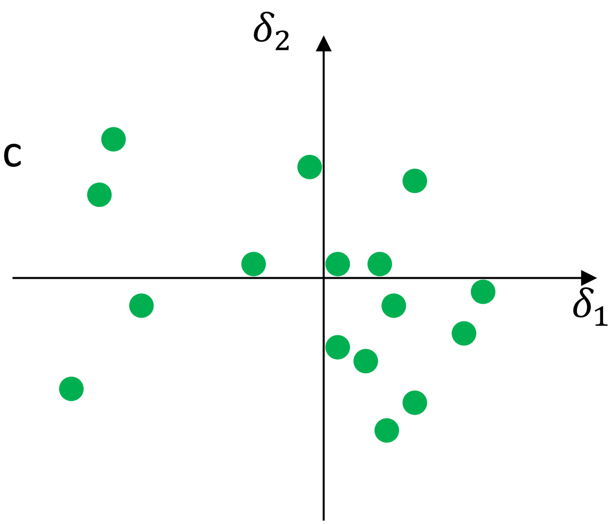
# 1. Probabilistic DC based SC-OPF

## c) Dealing with the chance constraint

**Probabilistically robust design (two-step approach) [1]**

### Step 1

- Use **the scenario approach** to find '**bounds**' of the uncertainty elements
- How many scenarios do we need to provide probabilistic guarantees?




$$N \geq \frac{1}{\varepsilon}\frac{e}{e-1}\left(\ln\frac{1}{\beta} + 2N_\delta - 1\right)$$

number of scenarios

violation level

confidence

number of uncertainty elements

[1] Margellos, Goulart, Lygeros, Trans. Aut. Con 2013.

# 1. Probabilistic DC based SC-OPF

## c) Dealing with the chance constraint

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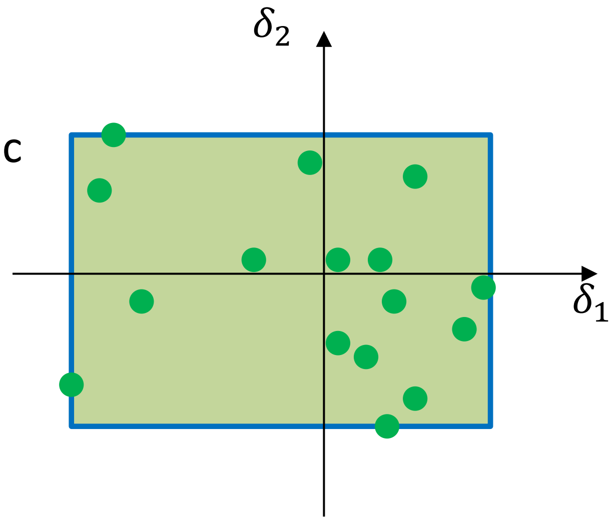


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# 1. Probabilistic DC based SC-OPF

## c) Dealing with the chance constraint

**Probabilistically robust design (two-step approach)** [1]

- **Step 2**

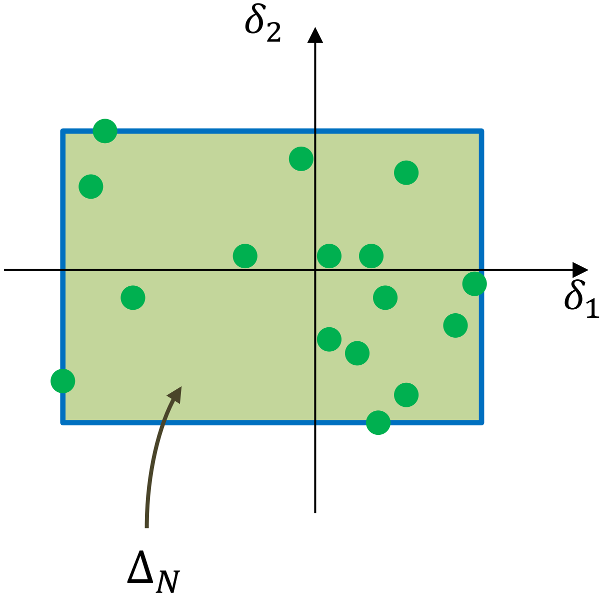
  - Solve a **robust formulation** of the initial chance constrained problem

  - Any solution of the robust problem is feasible for the chance constrained with confidence at least $1-\beta$

$$\mathbb{P}(F\boldsymbol{x} + f + H\boldsymbol{\delta} \leq 0) \leq 1-\varepsilon$$

$$\Downarrow$$

$$F\boldsymbol{x} + f + H\boldsymbol{\delta} \leq 0 \quad \text{for all} \quad \boldsymbol{\delta} \in \Delta_N$$




[1] Margellos, Goulart, Lygeros, Trans. Aut. Con 2013.

# 1. Probabilistic DC based SC-OPF

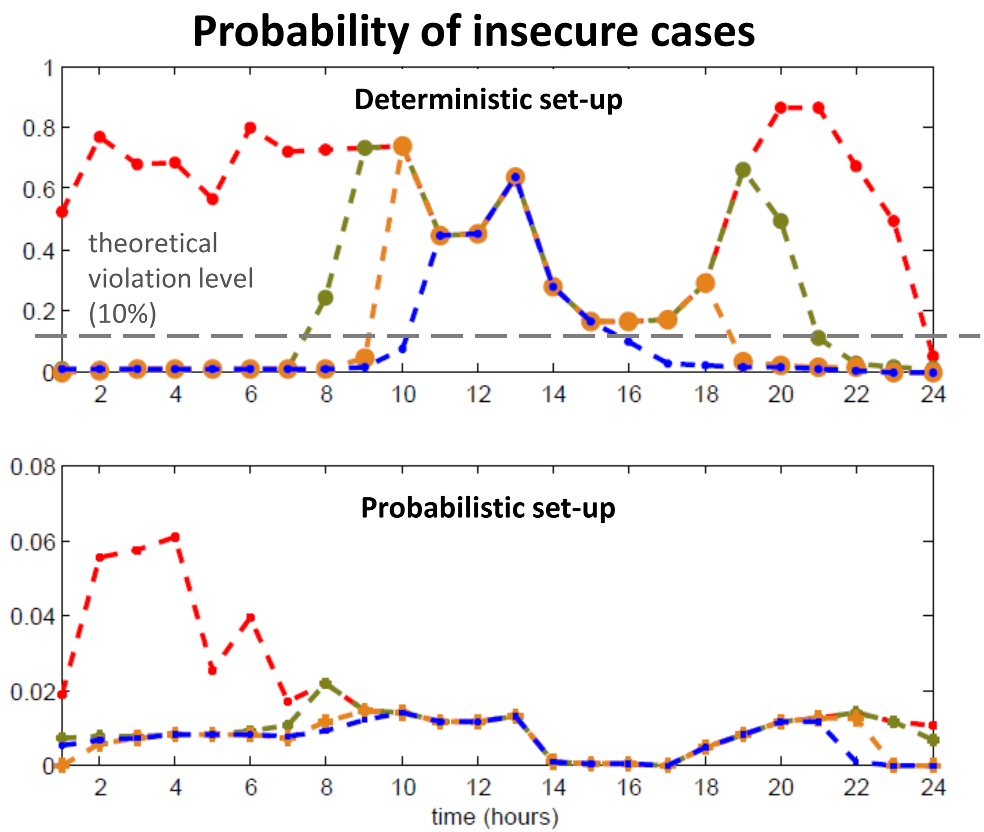
## d) Case study

- IEEE 30-bus network
- 4 load profiles


- Desired violation level 10%
- Monte Carlo evaluation for 10000 wind power realizations

**Fails to satisfy the desired violation level**

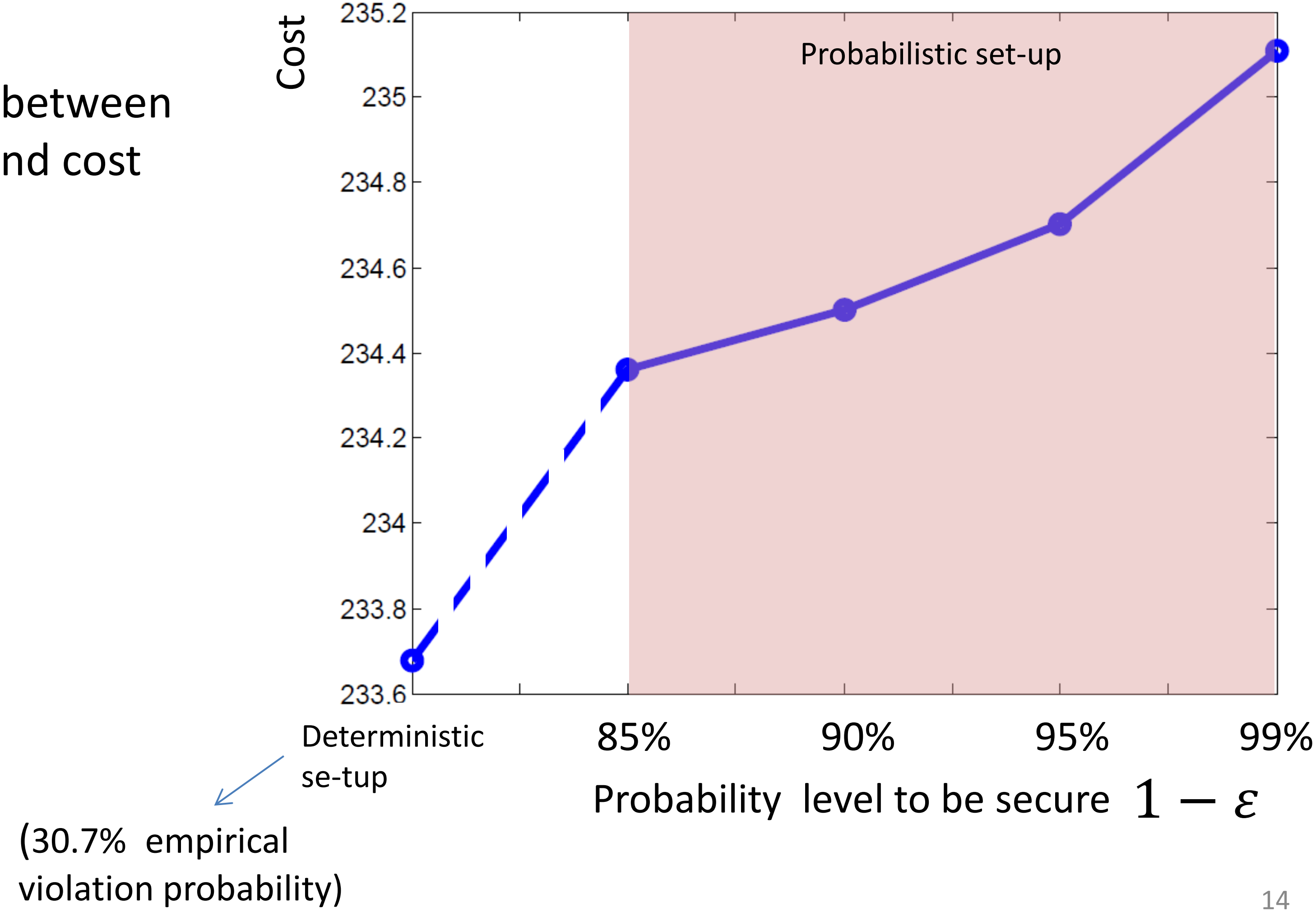
**Satisfy the desired violation level**



# 1. Probabilistic DC based SC-OPF

## d) Case study

- Trade-off between security and cost



# Outline

1. Probabilistic DC based SC-OPF
2. **Probabilistic AC based SC-OPF**
3. Exploiting component controllability
2. Co-optimization of energy and reserves

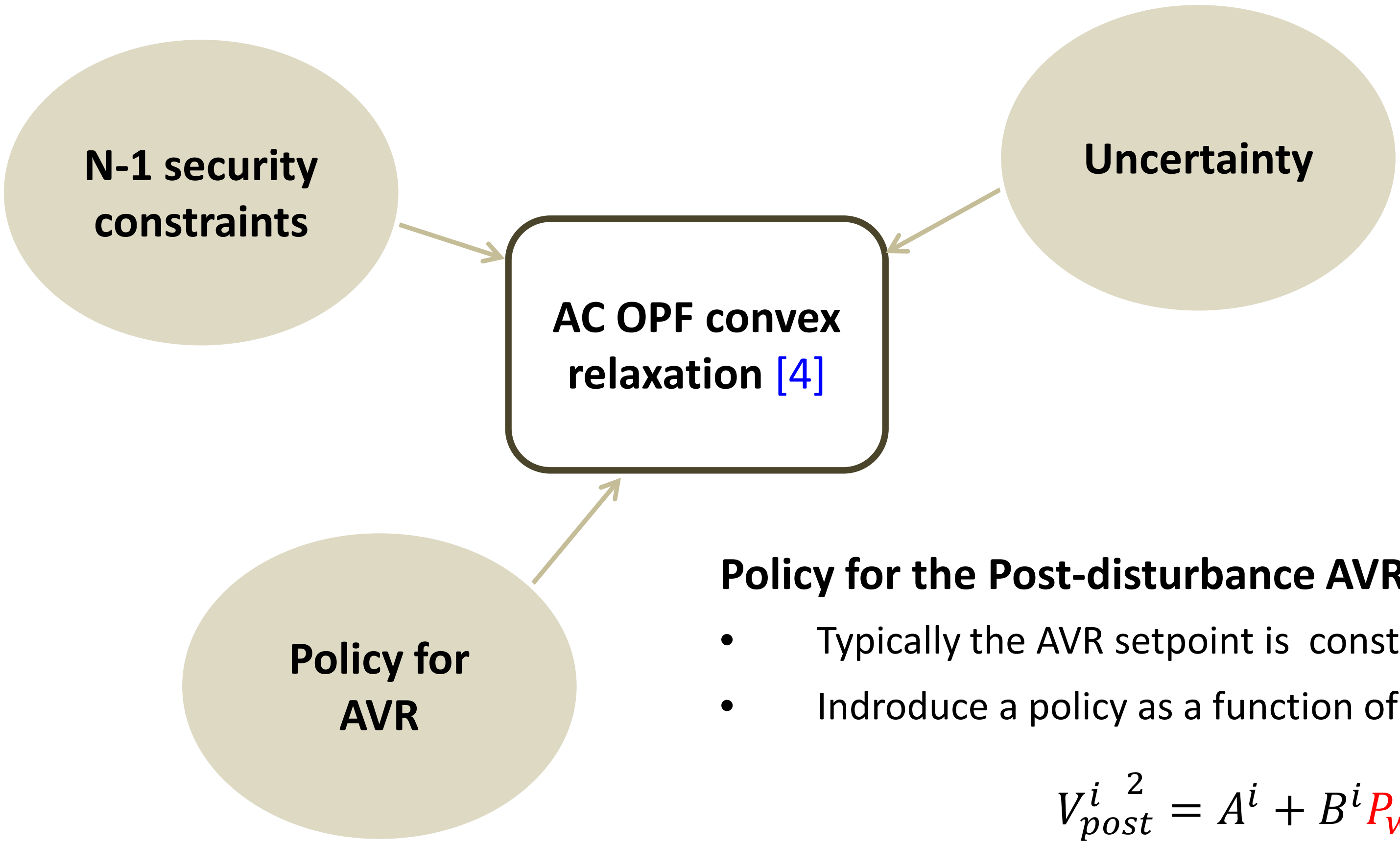
# 2. Probabilistic AC based SC-OPF

## a) Problem set-up

- AC power flow $\longrightarrow$ non-convex network equations
- Uncertainty: wind power $P_w$
- Security for the post-disturbance steady state operating point after the Secondary Frequency Control (or AGC)
- Preventive control: generation dispatch
- Corrective control: Automatic Voltage Regulation (AVR) set-point

# 2. Probabilistic AC based SC-OPF

## b) Add-ons to AC OPF




**Policy for the Post-disturbance AVR setpoint**

- Typically the AVR setpoint is constant.
- Indroduce a policy as a function of uncertainty

$$V_{post}^{i\;2} = A^i + B^i P_w$$

- Optimize over the coefficients of this policy: $A^i$, $B^i$

[4] Lavaei, Low, Transactions on Power Systems 2012

# 2. Probabilistic AC based SC-OPF

## c) Problem formulation

- **We use the convex OPF reformulation proposed in [4]**

$$\min_{W} J(W)$$

subject to

$$f(W) \leq 0$$
$$Rank(W) = 1$$

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where $f(\cdot)$ represents the power balance equations and typical limits on

- active and reactive power production
- active and apparent power flow
- voltage magnitude
- magnitude of the voltage difference between two neighboring buses

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and $W = XX^T$ with $X$ consisting of the real and imaginary part of the bus voltages

Note: $f(\cdot)$ includes semidefinite constraints!

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- **We use the convex OPF reformulation proposed in [4]**

$$\min_{W} J(W)$$

subject to

$$f(W) \leq 0$$
$$\cancel{Rank(W) = 1}$$

$$W(k,k) = 0$$

sets the bus that corresponds to k as the reference bus

where $f(\cdot)$ represents the power balance equations and typical limits on

- active and reactive power production
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# 2. Probabilistic AC based SC-OPF

## c) Problem formulation

- **Augment the convex OPF reformulation to include security constraints**

$$\min_{W} J(W_0)$$

subject to

$$f_i(W^i) \leq 0$$

$$W_i\,(k,k) = 0$$

$$g_i\,(W_i) = g_0\,(W_0) - d \cdot p(W_0)$$

for all $i = 0, \ldots, N_{out}$

no outage case

number of possible single outages

**AVR corrective action**

different $W^i$ for each contigency

**AGC action**

$P_G^{new} = P_G - d \cdot P_{mismatch}$

# 2. Probabilistic AC based SC-OPF

## c) Problem formulation

- **Augment the SC-OPF to include the uncertainty**

$$\min_{W} J(W_i(P_{w,f}))$$

subject to

$$\mathbb{P}\Bigg( \begin{array}{l} f_i(W_i(P_w)) \leq 0 \\ W_i\,(k,k) = 0 \\ g_i\,(W_i(P_w)) = g_0\,(W_i(P_{w,f})) - d \cdot p(W_i(P_w)) \\ \text{for all } \; i = 0, \dots, N_{out} \end{array} \Bigg) \geq 1 - \varepsilon$$

violation level

+ deterministic constraints for $P_w = P_{w,f}$

# 2. Probabilistic AC based SC-OPF

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+ deterministic constraints for $P_w = P_{w,f}$

- **Optimization over functions ?**
  - typically intractable!
  - To acieve tractability we select $W_i$ to be an affine function of the uncertainty:

$$W_i(P_w) = A^i + B^i P_w$$

# 2. Probabilistic AC based SC-OPF

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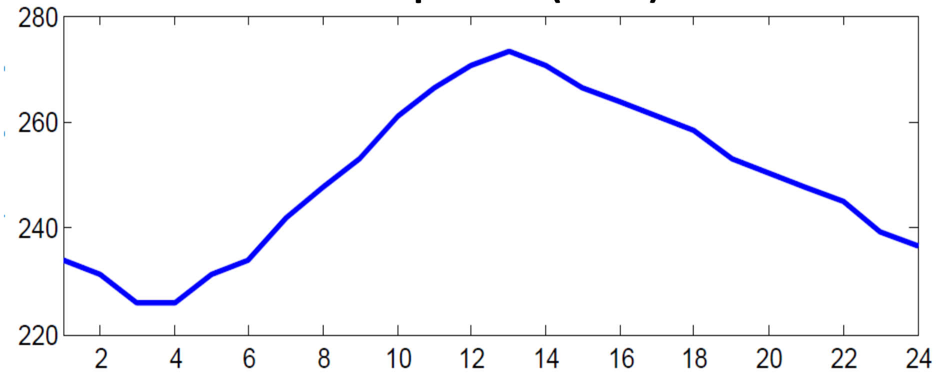
**Practically:** AVR function of the uncertainty

# 2. Probabilistic AC based SC-OPF

## d) Case study

- IEEE 14-bus network
- Monte Carlo evaluation for 10000 wind power realizations

Load profile (MVA)

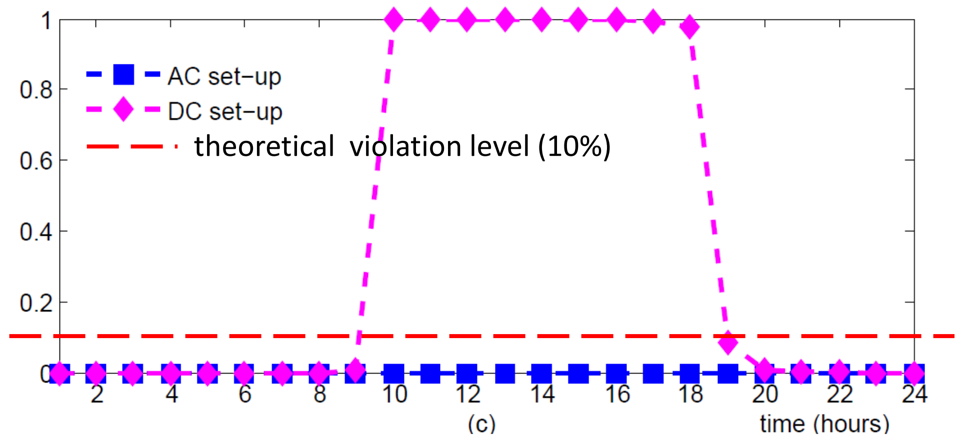



**Empirical probability of constraint violation**

DC set-up: fails to satisfy the violation level

AC set-up : satisfy the violation level

Probability of insecure cases




DC set-up: Poor representation of the real AC model

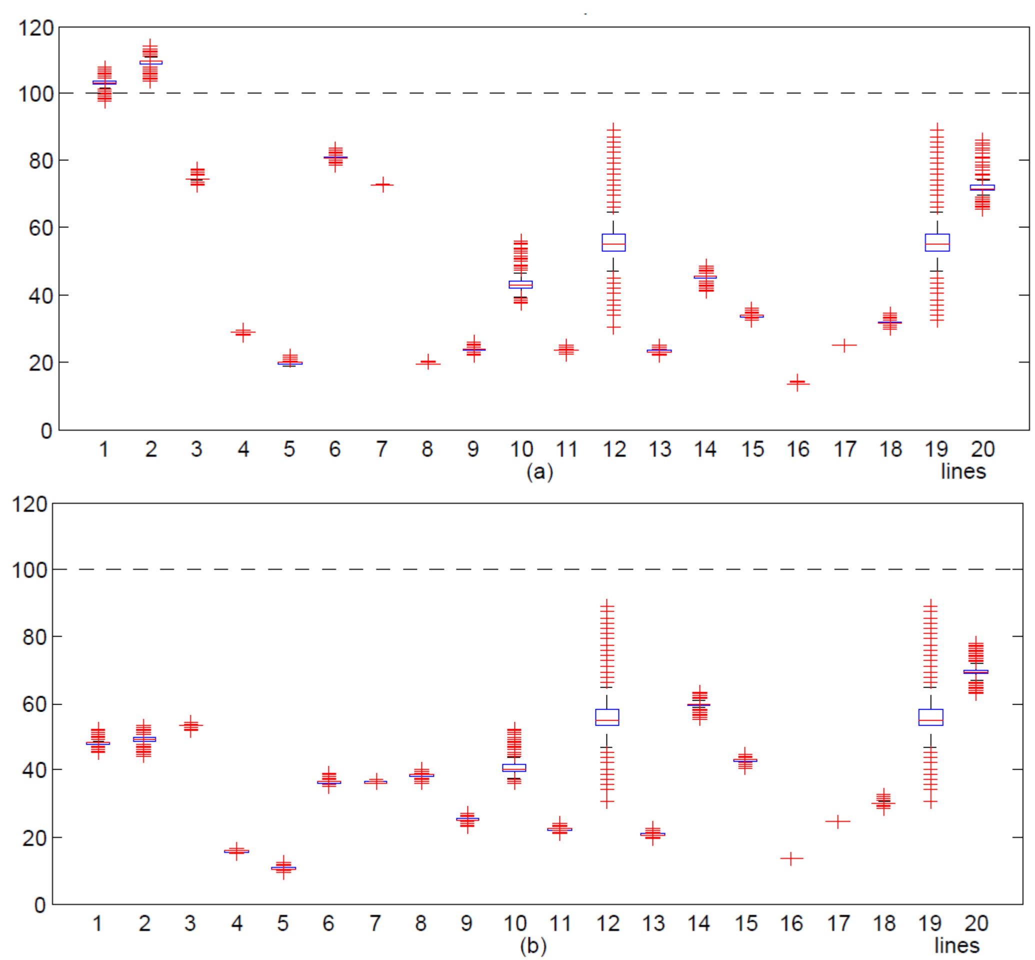
# 2. Probabilistic AC based SC-OPF

## d) Case study

**Line loadings for N-1 cases and 10000 wind power scenarios**

**DC set-up**

**AC set-up**



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3. **Exploiting component controllability**

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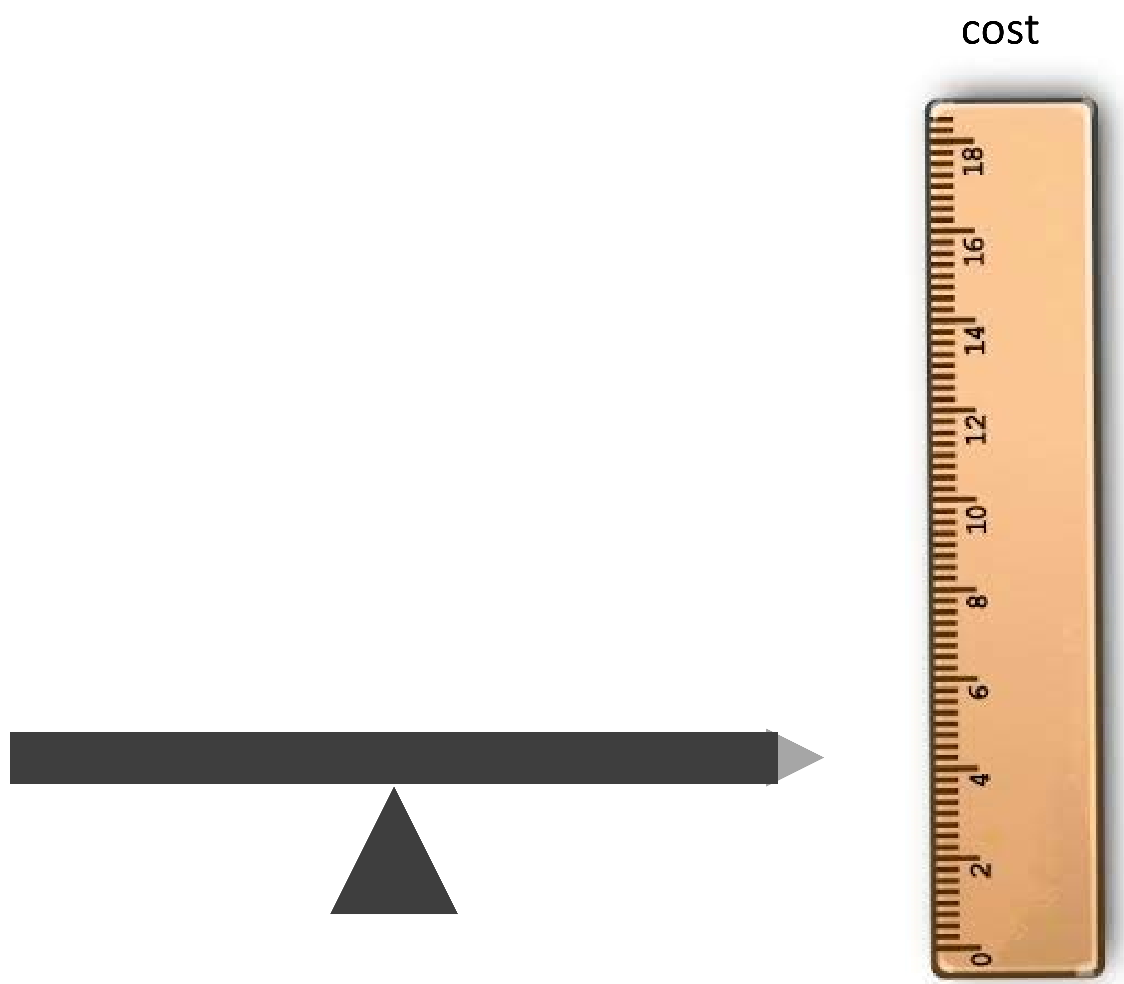
# 3. Exploiting component controllability

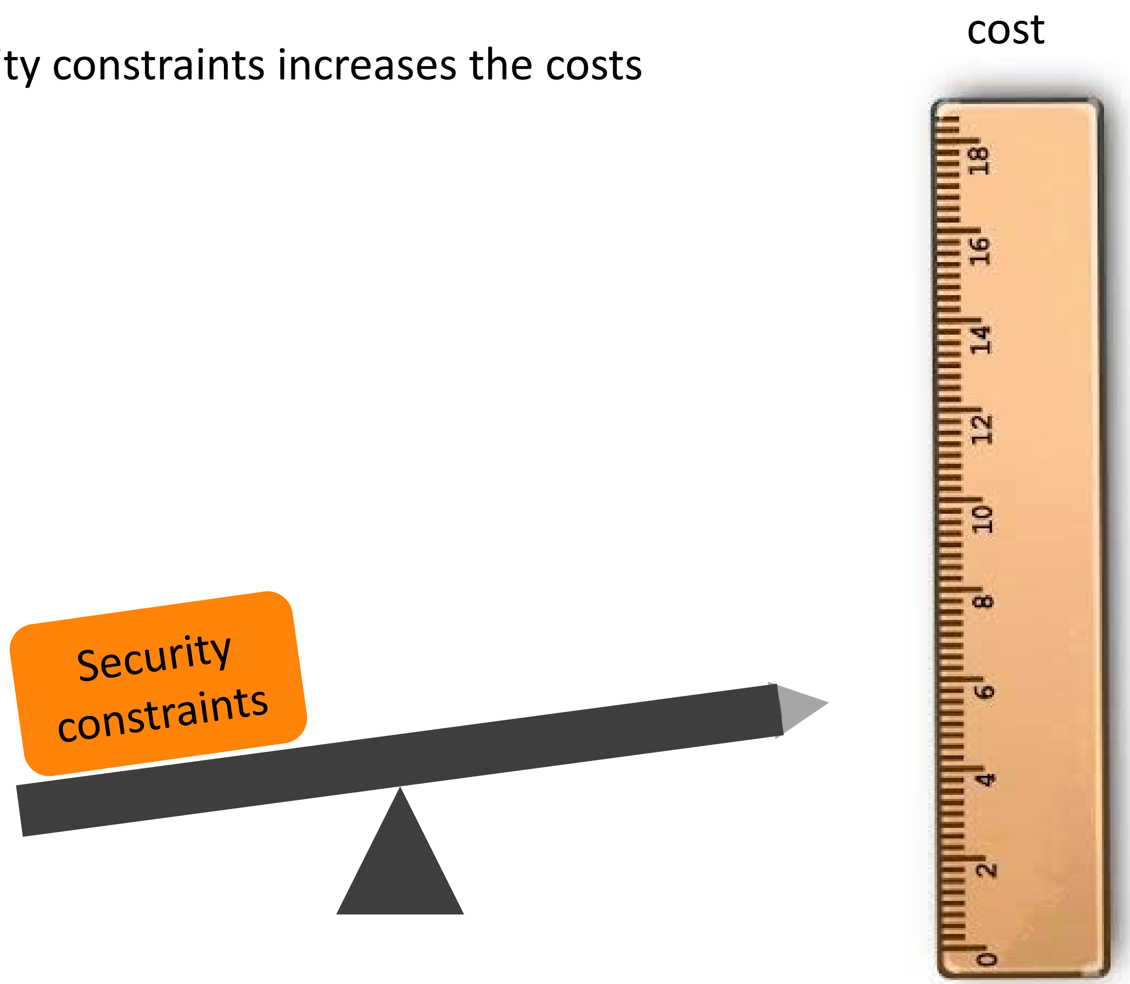
## a) Operational costs



# 3. Exploiting component controllability

## a) Operational costs

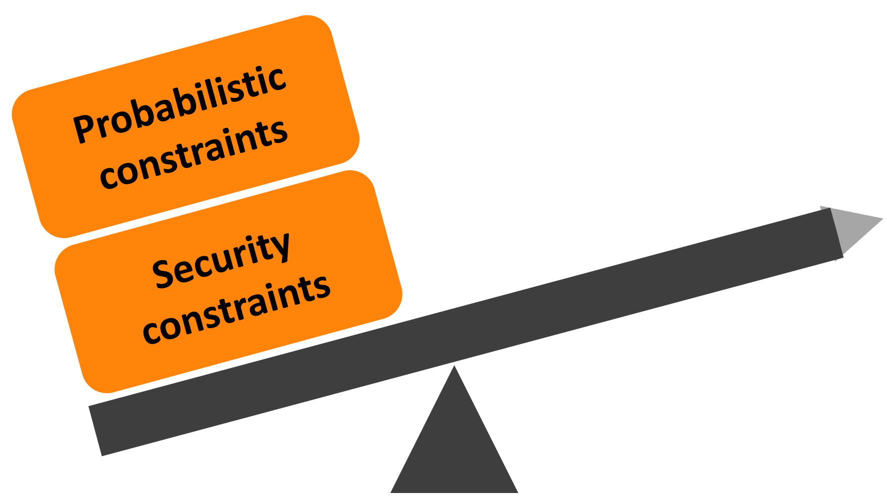
- Including security constraints increases the costs



# 3. Exploiting component controllability

## a) Operational costs

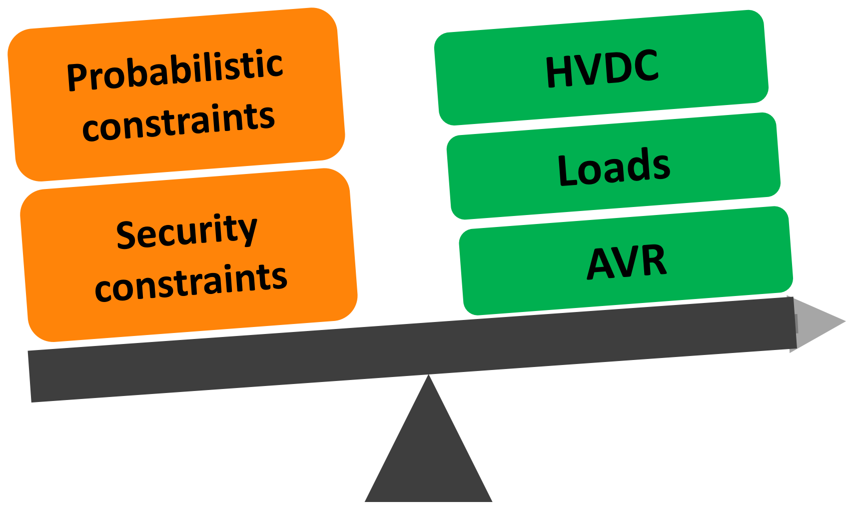
- Including security constraints increases the costs
- Probabilistic robustness increases the cost as well




# 3. Exploiting component controllability

## a) Operational costs - OPF

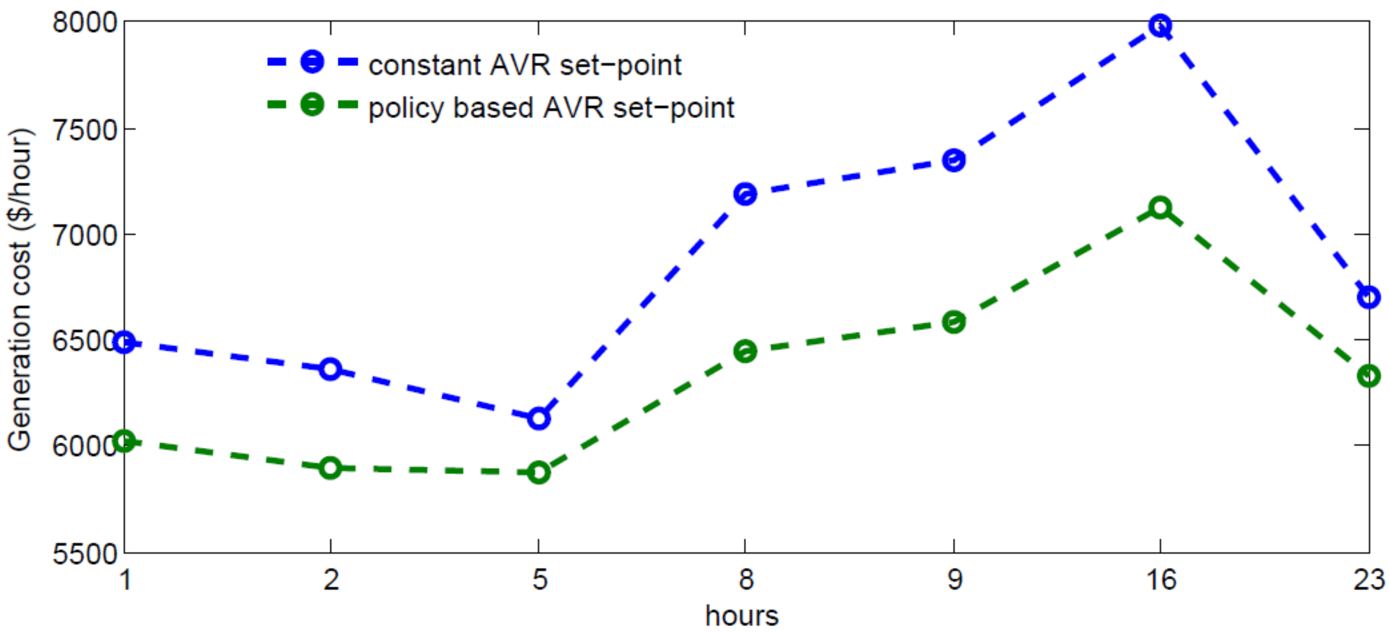
- Decrease cost by exploiting controllability of certain componets (e.g. AVR, HVDC, Loads)
- Model their post-disturbance set-point as a function of the uncertainty (e.g. affine policies)




cost

# 3. Exploiting component controllability

## a) Case study

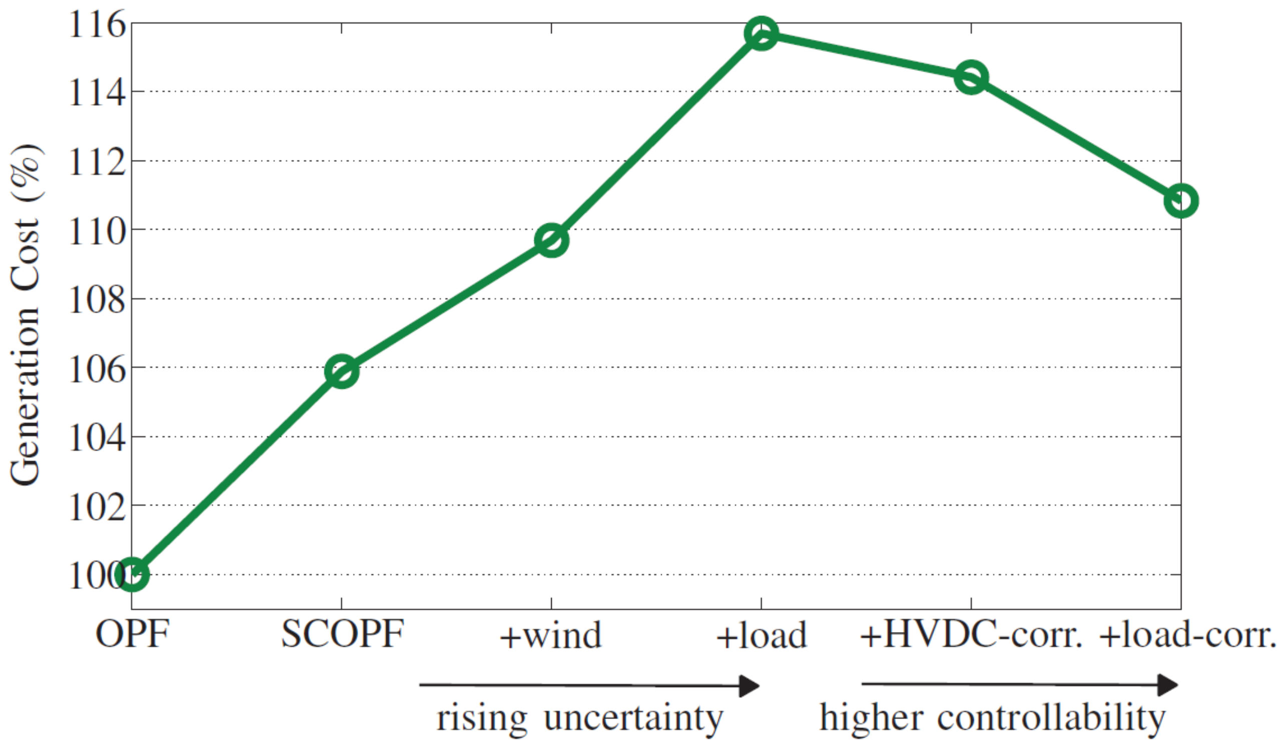



**Corrective control action of the AVR setpoint**

- Reduce the operational cost
- Enables higher wind penetration

# 3. Exploiting component controllability

## a) Case studies




**Using HVDC + load controllability reduced the costs due to uncertainty to 50%**

[Vrakopoulou, Chatzivasileiadis, Iggland, Imhof, Krause, Mäkelä, Mathieu, Roald, Wiget, Andersson, IREP 2013]

# Outline

1. Probabilistic DC based SC-OPF
2. Probabilistic AC based SC-OPF
3. Exploiting component controllability
4. **Co-optimization of energy and reserves**

# 4. Co-optimization of energy and reserves

## a) Reserve representation (Secondary frequency control)

- Up to now…

$$P_{G,post} = P_G - d \cdot P_{mismatch}$$

$d$: considered to be constant

# 4. Co-optimization of energy and reserves

## a) Reserve representation (Secondary frequency control)

- Up to now…

$$P_{G,post} = P_G - d \cdot P_{mismatch}$$

" Power correction term $R$ "

$d$: considered to be constant

# 4. Co-optimization of energy and reserves

## a) Reserve representation (Secondary frequency control)

- Up to now…

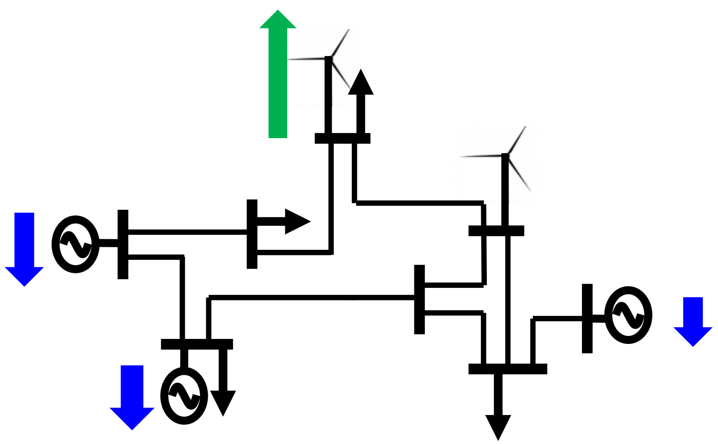
$$P_{G,post} = P_G - d \cdot P_{mismatch}$$ “ Power correction term $R$ ”

$d$: considered to be constant

- Move to optimal mismatch distribution

$$R = \begin{cases} d_{dn} P_{mismatch} \\ -d_{up} P_{mismatch} \end{cases}$$

$d_{dn}, d_{up}$: Treat them as decision variables



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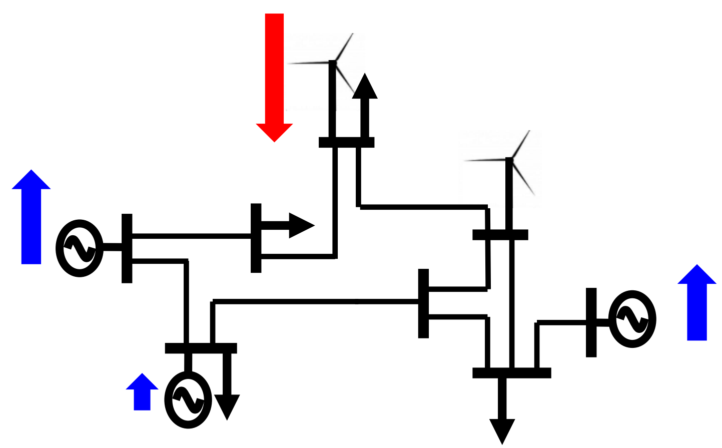

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$$R = \begin{cases} d_{dn} \, P_{mismatch} \\ -d_{up} \, P_{mismatch} \end{cases}$$

- **Reserves strategy for real time deployment**
  - Piece-wise function of the uncertainty with coefficient $d_{dn}, d_{up}$

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  - Worst-case (probabilistically) values of $R$ incorporating

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$$-R_{dn} \leq R^i \leq R_{up}$$

- **What happens in case of outages ?**

## 4. Co-optimization of energy and reserves

### a) Reserve representation (Secondary frequency control)

$$R^i = \begin{cases} d_{dn}^i \, P_{mismatch} \\ -d_{up}^i \, P_{mismatch} \end{cases}$$

- **Reserves strategy for real time deployment**
  - Piece-wise function of the uncertainty with coefficient $d_{dn}, d_{up}$
- **Amount of reserves to be purchased**
  - Worst-case (probabilistically) values of $R$ incorporating

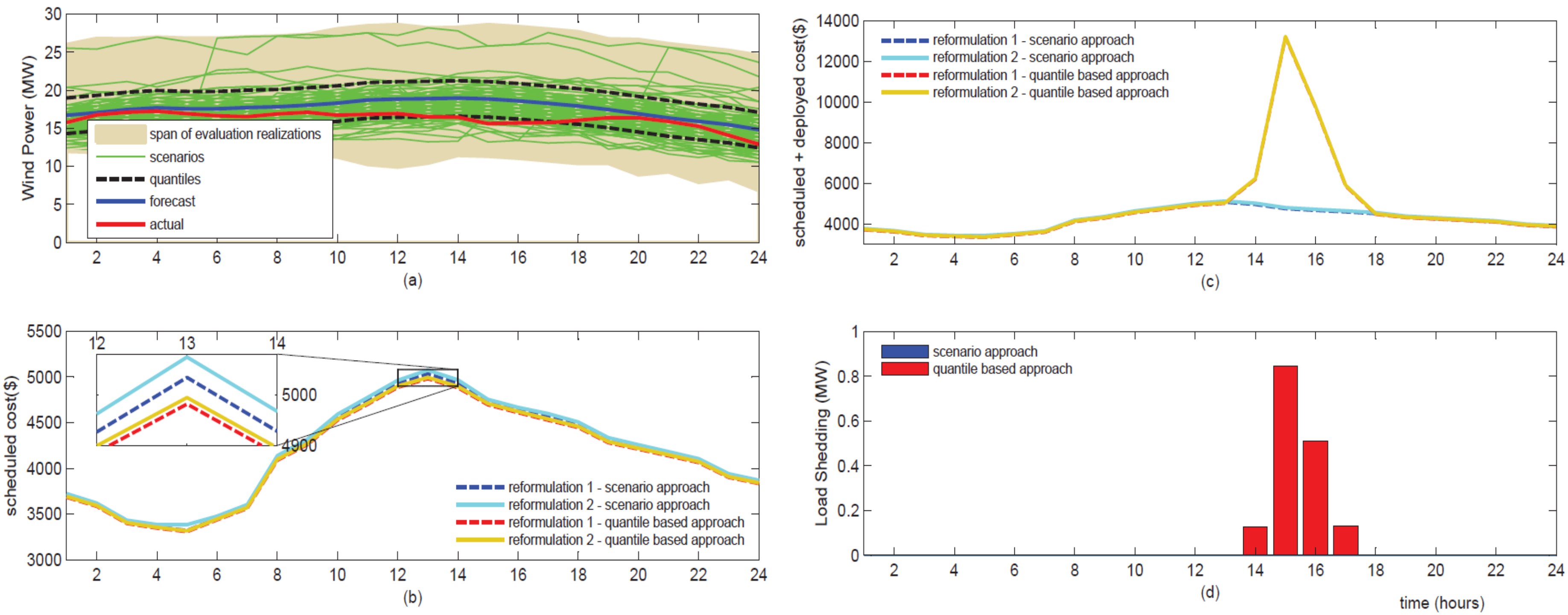
$$-R_{dn} \leq R^i \leq R_{up}$$

- **What happens in case of outages ?**
  - Introduce different distribution vector per outage
  - The mismatch depends on $P_G$ so bilinear terms appear!
  - We proposed two solutions → Heuristic iterative algorithm → Convex reformulation

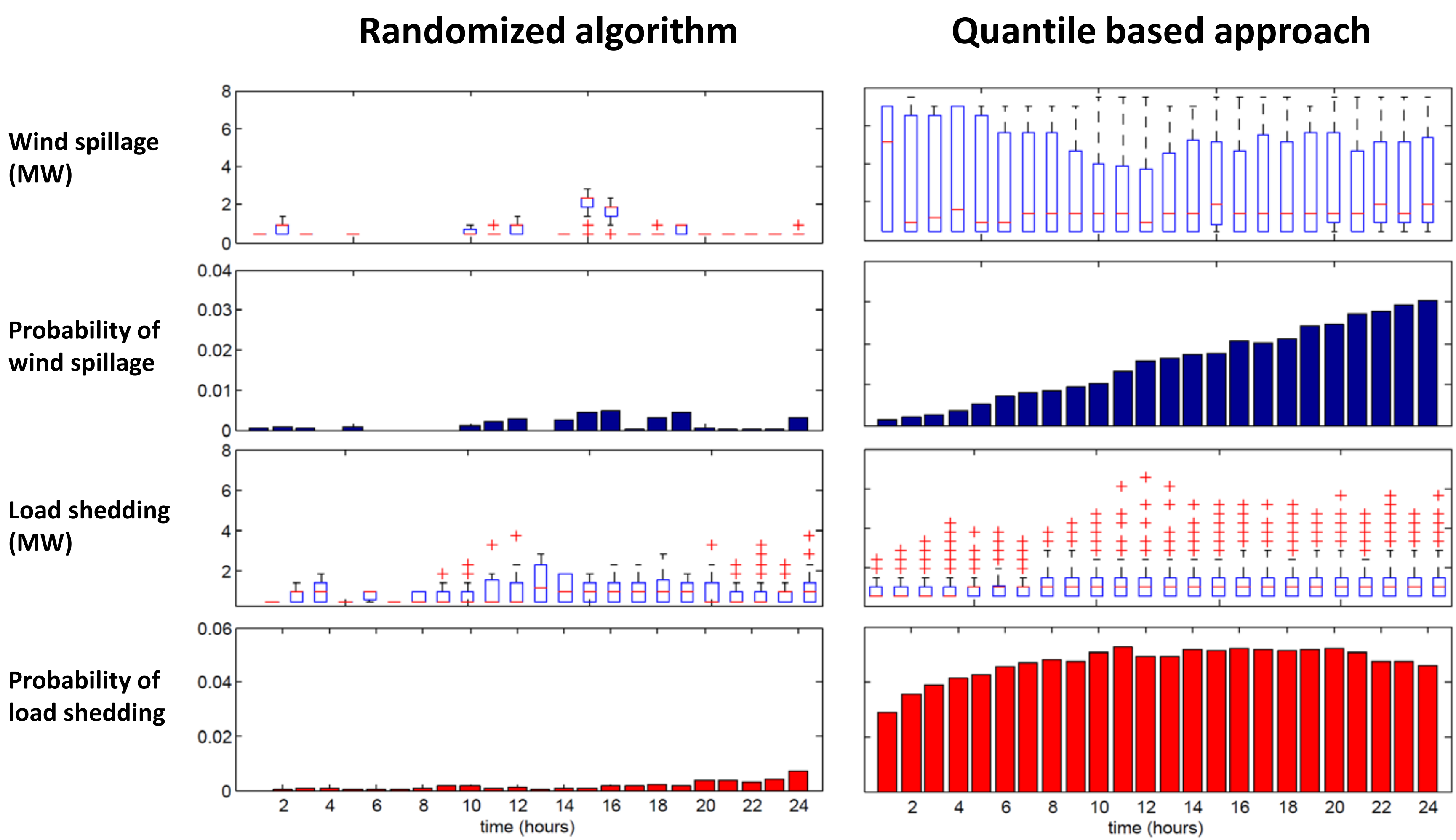
# 4. Co-optimization of energy and reserves

## b) Case study

- Results for 1 day
- Evaluation over the "actual" wind power realization




- No N-1 insecure instance was encountered




**Mean reduction per hour for Randomized algorithm**

in probability of load shedding: **94%**

in probability of wind spillage: **92%**

in wind spillage: **88%**

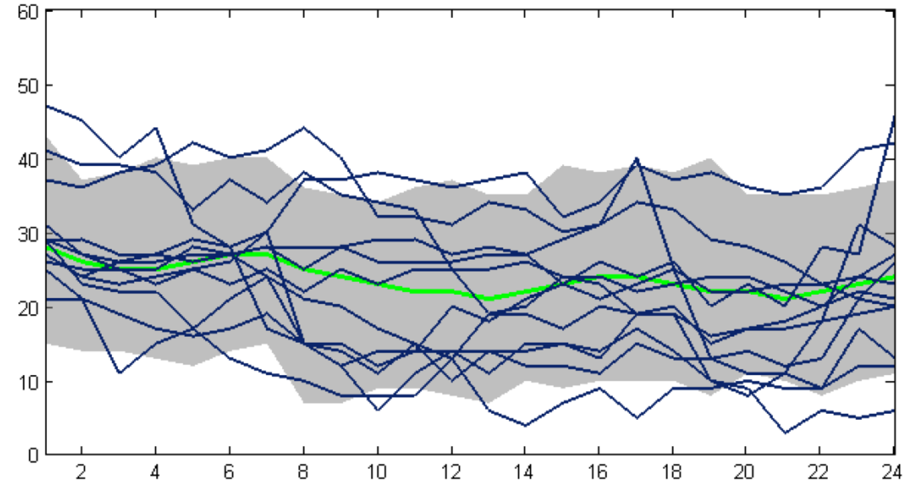
in load shedding: **36%**

**With maximum 1% increase in scheduling costs**

**Randomized algorithm** (grey lines)
**Criterion-based method for reduced number of scenarios** (dark blue lines)

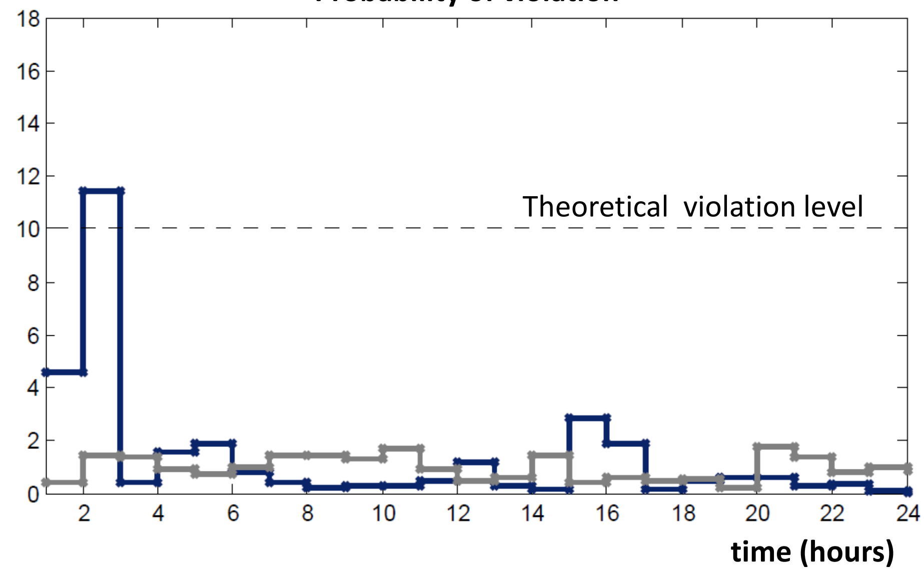
The randomized algorithm results in **9%** lower total scheduling costs than the criterion-based method

**Wind power (MW) scenarios**
(grey shadow: span of the scenarios for the randomized algorithm)




The criterion based method , though more expensive, fails to satisfy the violation level at the second hour

**Probability of violation**




[Panagou, ETH Zurich, Master thesis 2013]

# Summary

**Developed a probabilistic framework for optimal decision making in the presence of uncertainty**

- A-priori guaranteed performance
- Exploiting the trade-off between robust and economical operation

**Probabilistic DC and AC power flow set-up**

**Exploit component controllability**

- Determined a strategy for their post-disturbance operating point as functions of the uncertainty
- Achieve more economic performance, allow more wind integration

**Constructed a mechanism for reserve provision**

- Determined the minimum cost amount of reserves
- Determined a reserve strategy for real time deployment

# Extensions

This presentation mostly based on:

- N-1 security
- Reserve strategy

[Vrakopoulou, Margellos, Lygeros, Andersson, PMAPS 2012, EnergyCon 2012
Transcacions of Power Systems 2013]

- N-1 security
- AC OPF

[Vrakopoulou, Katsampani, Margellos, Lygeros, Andersson, PowerTech 2013]

Some extensions up to now:

- Unit commitment
- Reserve strategy

[Margellos, Rostampour, Vrakopoulou, Prandini, Andersson, Lygeros, ECC 2013]

- Reserve strategy
- Reserves from Demand Response

[Vrakopoulou, Mathieu, Andersson, HICSS 2014]

- N-1 security
- Risk based constraints

[Roald, Vrakopoulou, Oldewurtel, Andersson, PSCC 2014]

- Unit commitment
- Reserve strategy
- N-1 security
- Non-spinning reserves

[Hreinsson, Vrakopoulou, Andersson, PSCC 2014]

# Thank you for your attention!

contact: mariavr@umich.edu

# Implementation