Multi-Stage Robust Optimization in Electric Energy Systems

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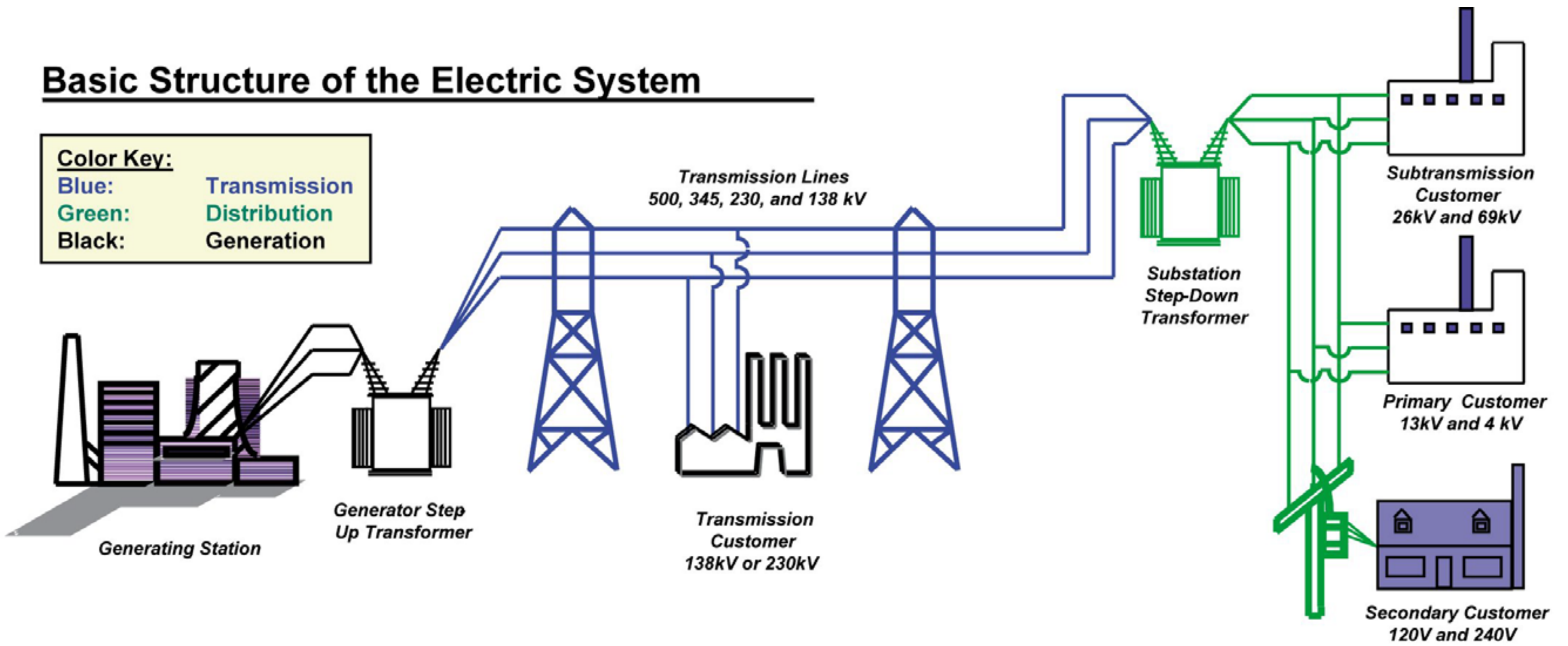
Outline

1. New Challenges of Power System Operations Under Uncertainty

2. Adaptive Robust Unit Commitment
   2.1 Two-stage Fully-Adaptive Robust UC
   2.2 Multi-stage Robust UC with Affine Policy

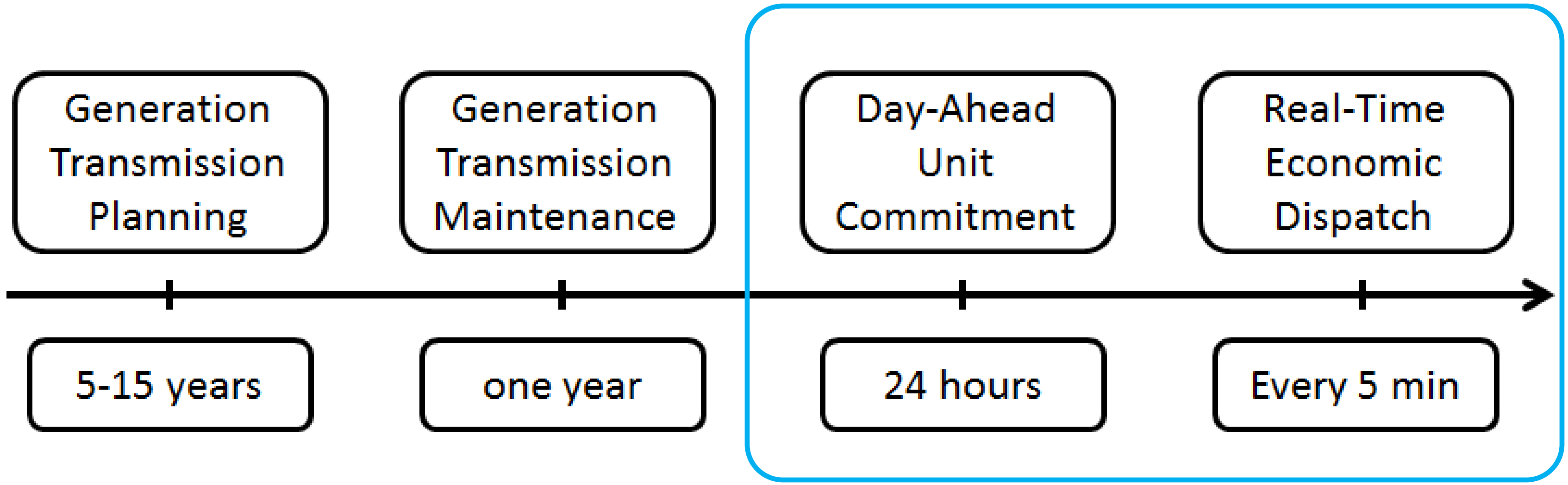
Electric Power Systems

- Electric power system is the backbone of modern society
- Electric power networks are among the world's most complex engineering system



Electric Power Systems Problems

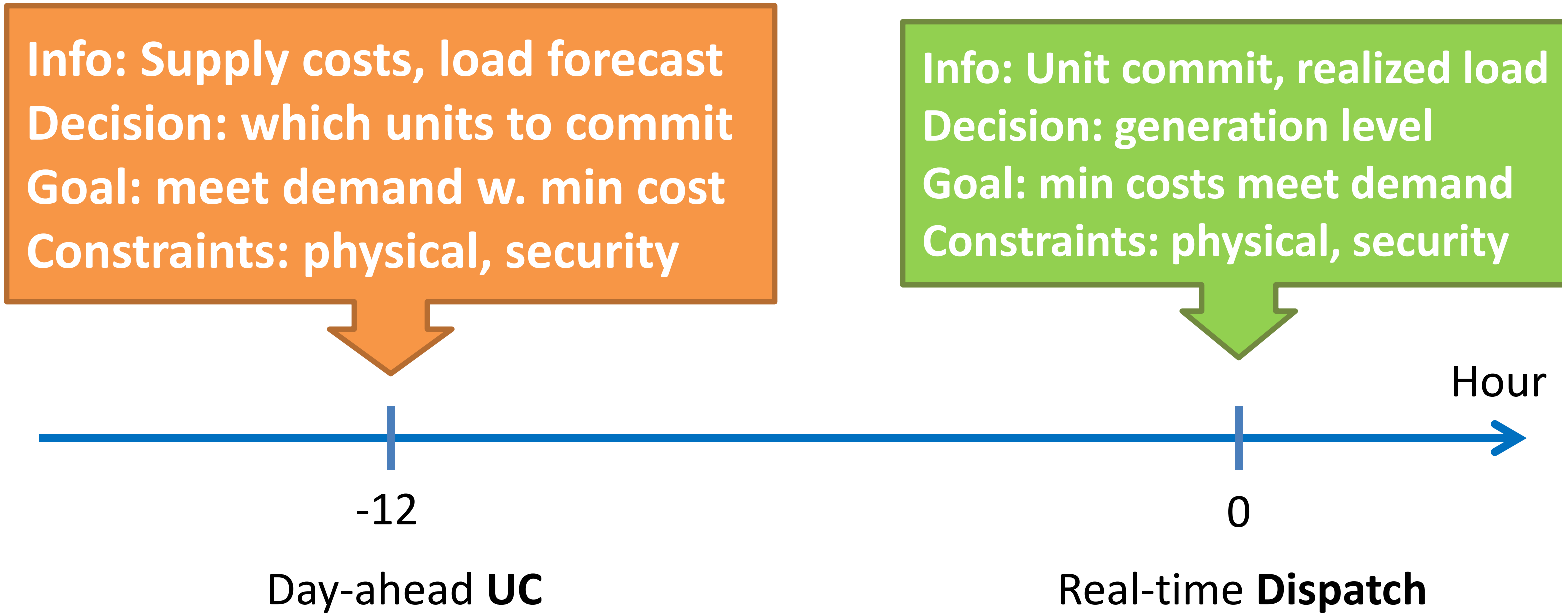
- Key problems:



  - Different time scales from min to decades
  - Multiple agents (GenCo, TransCo, DistCo, ISO, Utility…)
  - Significant uncertainties (load, generation, outages, construction, …)

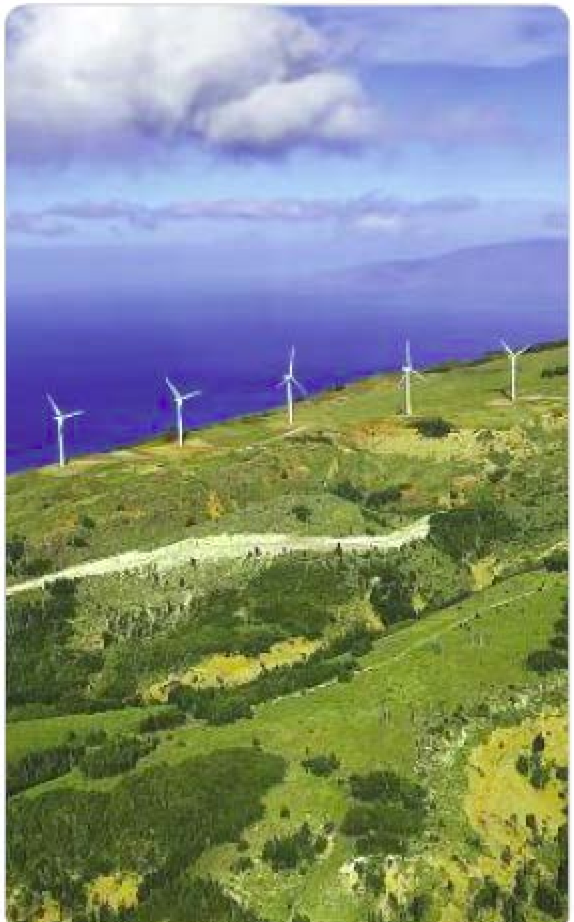
Daily Operation of Power System

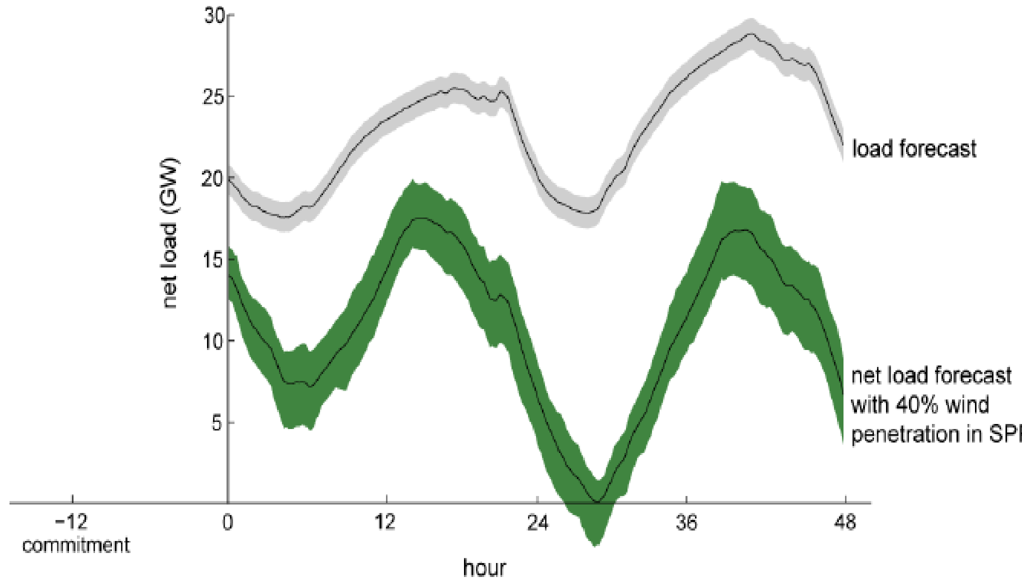
- Day-ahead unit commitment & Real-time economic dispatch



New Challenge: Growing Uncertainty

- New challenge





[Ruiz, Philbrick 10]

Supply Variation:
Wind Power Penetration
40% annual growth

Net Load Uncertainty
Can be Huge!

Robust Optimization for Short-Term Oper

- Two-stage RO for unit commitment
  - Adaptive robust SCUC models
    - [Jiang et. al. 2012], [Zhao, Zeng 2012], [Bertsimas et. al. 2013]
  - RO for security optimization
    - [Street et. al. 2011], [Wang et. al. 2013]
  - Unifying RO with Stochastic UC
    - [Wang et. al. 2013]
- Two-stage RO for economic dispatch
  - AGC control
    - [Zheng et. al. 2012]
  - Affine policy
    - [Jabr 2013]
  - Adaptive RO with dynamic uncertainty sets
    - [Lorca & S. 2014]
  - Three-stage for uncertain demand and demand-response
    - [Zhao et al 2013]

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Current Practice: Reserve Adjustment

- Deterministic Reserve adjustment approach

  Incorporating extra resources called reserve

  [Sen and Kothari 98] [Billinton and Fotuhi-Firuzabad 00]

Drawbacks:

1. Uncertainty not explicitly modeled
2. Both system and locational requirement are preset, heuristic, static

Existing Proposal: Stochastic Optim.

- Stochastic optimization approach

  Uncertainty modeled by distributions and scenarios

  [Takriti et. al. 96, 00] [Ozturk et. al. 05][Wong et. al. 07] [Wu et. al. 07]

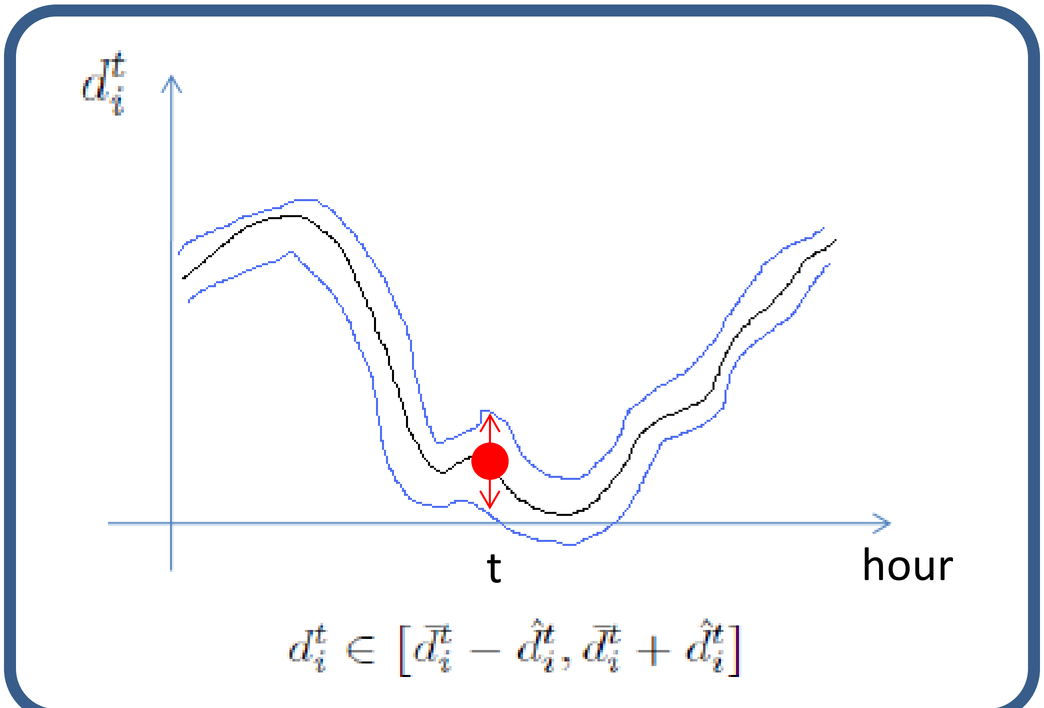
Weakness:

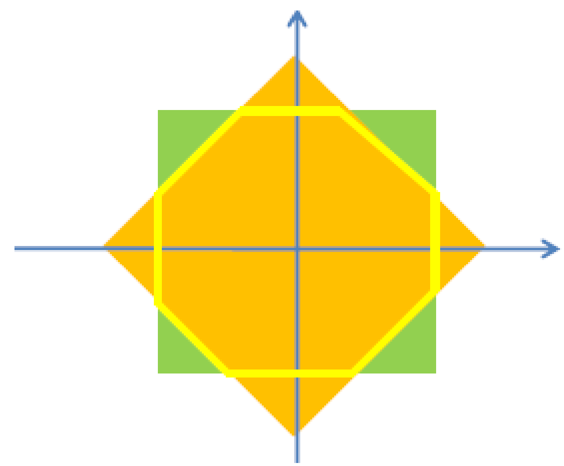
1. Hard to select “right” scenarios in large systems
2. Computational burden
3. Restricted by sample scenarios

Model of Uncertainty

- Uncertainty model of net load variation

$$\mathcal{D}^t(\bar{\mathbf{d}}^t, \hat{\mathbf{d}}^t, \Delta^t) := \Big\{ \mathbf{d}^t \in \mathbb{R}^{N_d} : \sum_{i \in N_d} \frac{|d_i^t - \bar{d}_i^t|}{\hat{d}_i^t} \le \Delta^t,$$
$$d_i^t \in [\bar{d}_i^t - \hat{d}_i^t, \bar{d}_i^t + \hat{d}_i^t], \forall i \in N_d \Big\}$$





Budgeted

Fully Adaptive Robust UC Problem

- The fully adaptive policy:
  - Objective: Fixed-Cost + Worst case Dispatch Cost

$$\min_{\mathbf{x},\mathbf{u},\mathbf{v}} \sum_t \sum_i F_i^t x_i^t + S_i^t u_i^t + G_i^t v_i^t + \boxed{\max_{\mathbf{d}\in\mathcal{D}} \boxed{\min_{\mathbf{p}\in\mathcal{W}(\mathbf{x},\mathbf{d})} \sum_t \sum_i C_i^t p_i^t}}$$

$$\text{s.t. } F(\mathbf{x},\mathbf{u},\mathbf{v}) \leq 0$$

$$\mathbf{x},\mathbf{u},\mathbf{v} \text{ binary.}$$

Constraints on commitment decision:
Startup/shutdown, Min-up/down…

Find worst case d for dispatch

For a fixed x, d minimize dispatch cost

Second-Stage Problem

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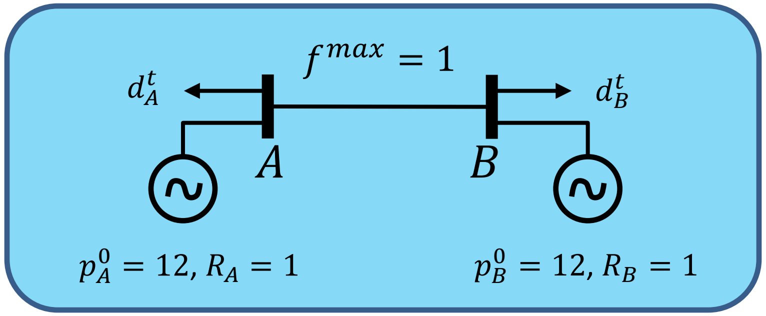
Issues with Two-Stage Robust UC

- A simple two-bus two-period example:



Demand uncertainty sets:
$D^1 = \{(12{,}12)\}$,
$D^2 = \left\{(d_A^2, d_B^2): d_A^2 + d_B^2 = 25, d_i^2 \in [10{,}15]\right\}$

- Claim: Two-stage robust UC is feasible
  - UC solution: $(x_A^t, x_B^t) = (1{,}1)$ for $t = 1{,}2$
  - Feasible dispatch solution:
    - $p_A^1(\boldsymbol{d}) = 12 + \frac{2}{5}(d_A^2 - 12.5), p_B^1(\boldsymbol{d}) = 12 - \frac{2}{5}(d_A^2 - 12.5)$
    - $p_A^2(\boldsymbol{d}) = 12.5 + \frac{3}{5}(d_A^2 - 12.5), p_B^2(\boldsymbol{d}) = 12.5 - \frac{3}{5}(d_A^2 - 12.5)$
  - Satisfy $p_A^t(\boldsymbol{d}) + p_B^t(\boldsymbol{d}) = d_A^t + d_B^t, f_{AB}(\boldsymbol{d}) \le f^{max}, \forall \boldsymbol{d} \in D$

Capture Multistage Nature is Critical

- Can we find a policy $p(\cdot)$ that does not look into the future? i.e. $\boldsymbol{p}^1(\boldsymbol{d}^1), \boldsymbol{p}^2(\boldsymbol{d}^1, \boldsymbol{d}^2)$?
  - Because real-time dispatch cannot depend on future

- No feasible **non-anticipative** policy exists!
  - No feasible $\boldsymbol{p}^1$ s.t. for any $\boldsymbol{d}^2 \in D^2$ there exists $\boldsymbol{p}^2$
  - If $p_A^1 \in [11,12]$: $p_A^2 \leq 13$, impossible to satisfy $\boldsymbol{d}^2 = (15,10)$
  - If $p_A^1 \in [12,13]$: $p_B^2 \leq 13$, impossible to satisfy $\boldsymbol{d}^2 = (10,15)$

- Bottleneck: ramping & transmission constraint

Multi-Stage Robust UC

$$\min_{\boldsymbol{x},\boldsymbol{u},\boldsymbol{v},\boldsymbol{p}(\cdot)} \left\{ \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}_g} (G_i x_i^t + S_i u_i^t) \; + \; \max_{\boldsymbol{d}\in\mathcal{D}} \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}_g} C_i\, p_i^t(\boldsymbol{d}^{[t]}) \right\}$$

s.t.

constraints for $\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{v}$

$$p_i^{min} x_i^t \le p_i^t(\boldsymbol{d}^{[t]}) \le p_i^{max} x_i^t \qquad \forall\, \boldsymbol{d} \in \mathcal{D},\; i \in \mathcal{N}_g,\; t \in \mathcal{T}$$

$$-RD_i x_i^t - SD_i v_i^t \le p_i^t(\boldsymbol{d}^{[t]}) - p_i^{t-1}(\boldsymbol{d}^{[t-1]}) \le RU_i x_i^{t-1} + SU_i u_i^t$$
$$\forall\, \boldsymbol{d} \in \mathcal{D},\; i \in \mathcal{N}_g,\; t \in \mathcal{T}$$

$$-f_l^{max} \le \alpha_l^\top \left(B^p\, \boldsymbol{p}^t(\boldsymbol{d}^{[t]}) - B^d\, \boldsymbol{d}^t \right) \le f_l^{max} \qquad \forall\, \boldsymbol{d} \in \mathcal{D},\; t \in \mathcal{T},\; l \in \mathcal{N}_l$$

$$\sum_{i\in\mathcal{N}_g} p_i^t(\boldsymbol{d}^{[t]}) = \sum_{j\in\mathcal{N}_d} d_j^t \qquad \forall\, \boldsymbol{d} \in \mathcal{D},\; t \in \mathcal{T}$$

Notation: $\boldsymbol{d}^{[t]} = (\boldsymbol{d}^1, \ldots, \boldsymbol{d}^t)$

Affine Multi-Stage Robust UC

- Tractable alternative for $p(\cdot)$:

$$p_i^t(\boldsymbol{d}^1, \ldots, \boldsymbol{d}^t) = w_i^t + \sum_{s\in\{1,\ldots,t\}} \sum_{j\in\mathcal{N}_d} W_{itjs}\, d_j^s$$

- Multi-stage robust UC with affine policy:

$$\min_{\boldsymbol{x},\boldsymbol{u},\boldsymbol{v},\boldsymbol{w},\boldsymbol{W}} \left\{ \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}_g} (G_i x_i^t + S_i u_i^t) + \max_{\boldsymbol{d}\in\mathcal{D}} \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}_g} C_i \left(w_i^t + \sum_{s\in\{1,\ldots,t\}} \sum_{j\in\mathcal{N}_d} W_{itjs} d_j^s \right) \right\}$$

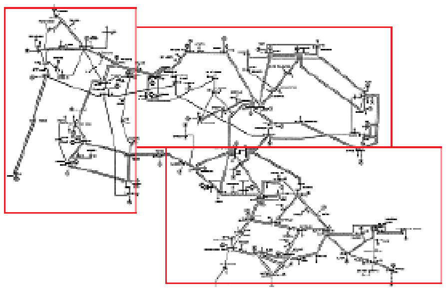
s.t.

constraints for $\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{v}$

$$p_i^{min} x_i^t \leq w_i^t + \sum_{s\in\{1,\ldots,t\}} \sum_{j\in\mathcal{N}_d} W_{itjs} d_j^s \leq p_i^{max} x_i^t \qquad \forall\, \boldsymbol{d} \in \mathcal{D},\ i \in \mathcal{N}_g,\ t \in \mathcal{T}$$

...

Simplified Affine Policies



Spatial Aggregation

Temporal Aggregation

General affine policy: $$p_i^t(\boldsymbol{d}^{[t]}) = w_i^t + \sum_{s\in\{1,\dots,t\}} \sum_{j\in\mathcal{N}_d} W_{itjs}\, d_j^s$$

Simpler information basis: $$p_i^t(\boldsymbol{d}^{[t]}) = w_i^t + \sum_{j\in\mathcal{N}_d} W_{itj}\, d_j^t$$

All loads aggregated: $$p_i^t(\boldsymbol{d}^{[t]}) = w_i^t + W_{it} \sum_{j\in\mathcal{N}_d} d_j^t$$

Loads and time periods aggregated: $$p_i^t(\boldsymbol{d}^{[t]}) = w_i^t + W_i \sum_{j\in\mathcal{N}_d} d_j^t$$

Solution Method

- **Dualization** approach does not work:
  - Traditionally, robust constraints are dualized
  - Resulting problem is too large for power systems

- **Constraint generation** makes sense:

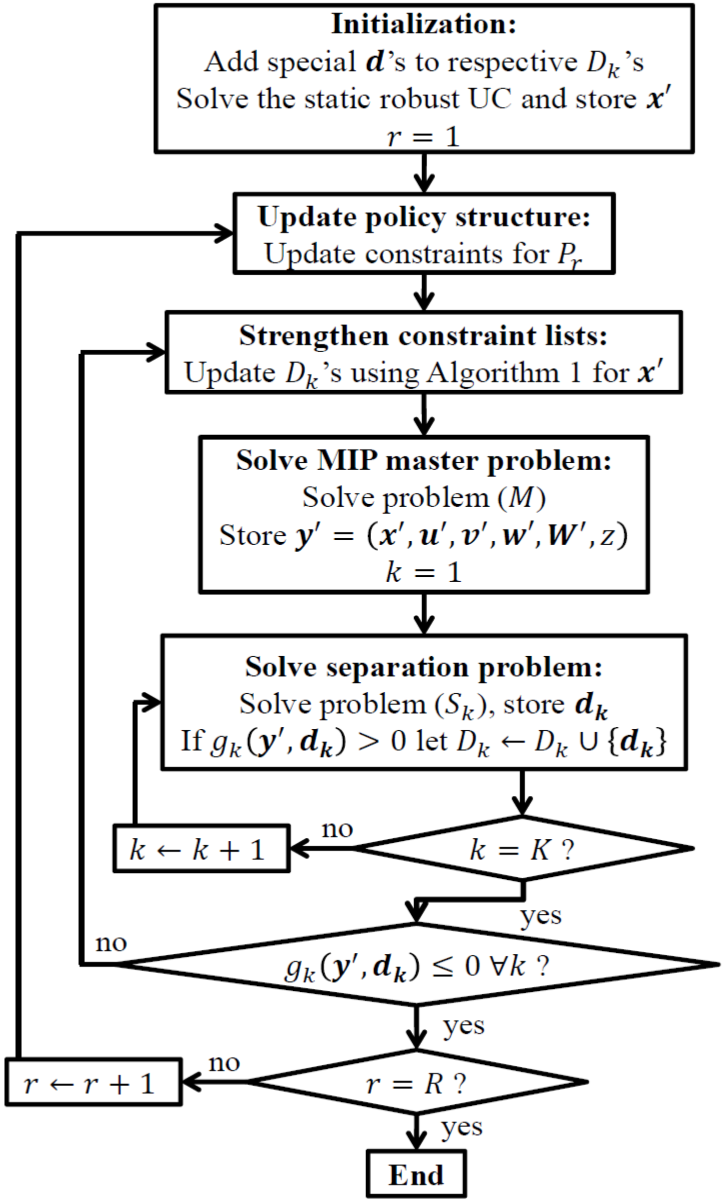
$$p_i^{min} x_i^t \leq w_i^t + W_{it} \sum_{j \in \mathcal{N}_d} d_j^t \leq p_i^{max} x_i^t \qquad \forall\, \boldsymbol{d} \in \mathcal{D},\, i \in \mathcal{N}_g,\, t \in \mathcal{T}$$

- However, naïve CG also does not work

Solution Method

- **Valid inequalities** for x and specific d's for ramping, generating limits, and line flow
- **Fixing** x and finding cuts by CG with an LP master
- **Iteratively improving** policy structure (e.g. $W_i \to W_{it}$) with approximate warm-start (not solving W_i fully)
- **Exploiting structure** of special policy form: e.g. pre-computing all needed constraints for ramping and generation limit constraints for W_{it}-policy.

Solution Method



Computational Study

- How good is the proposed algorithm?
  - Effectiveness of various algorithmic improvements
- How good is the simplified affine policy?
  - Compared to the “true” multi-stage robust UC
- Why should we use multi-stage formulation?
  - Worst case infeasibility of two-stage robust UC
  - Managing Ramping capability
- How good is affine UC “on average”?
  - Rolling-horizon Monte-Carlo simulation
  - Average performance in cost, std, reliability

How Good is the Algorithm?

Solution time (s) under "W_{it}" policy for IEEE-118 bus system (time limit = 15,000s)

Method	$\Gamma = 0.5$	$\Gamma = 1$	$\Gamma = 2$	$\Gamma = 4$
DBA	out of memory	out of memory	out of memory	out of memory
Basic CG	time lim.	time lim.	time lim.	time lim.
CG + DFX	8475	5639	3488	6965
CG + SE	80	961	1011	1227
CF + DFX + SE	67	77	78	218
CG + DFX + SE + CC	64	47	63	178

DBA: duality based approach
Basic CG: constraint generation
DFX: d's fixed x
SE: special structure of policy "W_{it}" exploited
CC: specific d for worst-case dispatch cost constraint

How Good is the Algorithm?

Solution time (s) for three test systems using W_{it} policy:

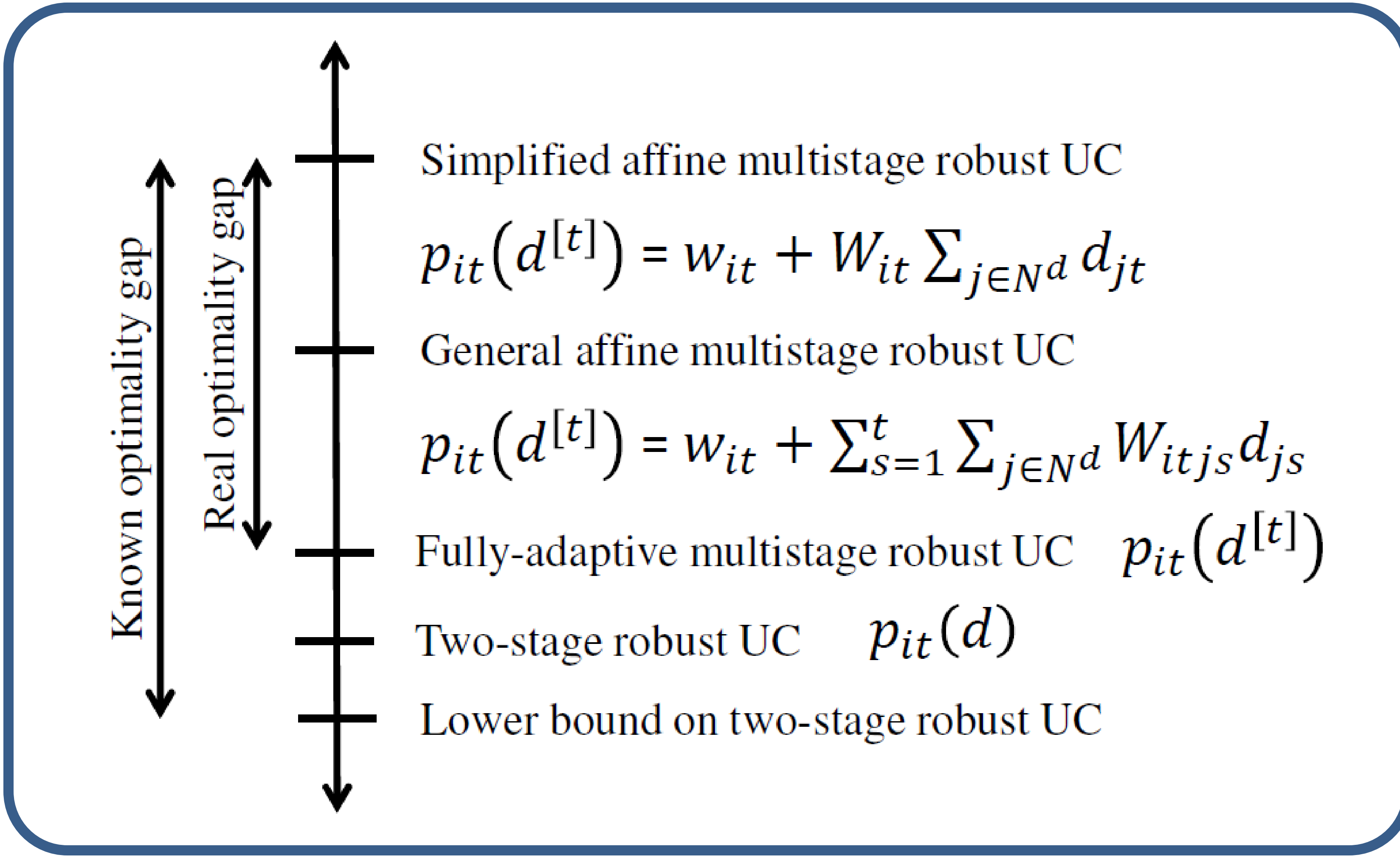
System	$\Gamma = 0.25$	$\Gamma = 0.5$	$\Gamma = 1$	$\Gamma = 2$	$\Gamma = 4$
30 bus	6s	3s	8s	6s	29s (inf)
118 bus	66s	64s	47s	63s	178s
2718 bus	3.6h	3.2h	2.3h	2.0h	0.4h (inf)

Note: “inf” indicates that the problem is infeasible

MIP optimality gap used for 30, 118, 2718 bus systems: 0.1%, 0.1%, 1%

How Good is the Simplified Affine Policy?

- How good is the simplified affine policy?



How Good is the Simplified Affine Policy?

Table : Opt. gap under different policy structures, for the 30 bus system.

$(n_g, n_\mathcal{T}, n_d, L)$	$\Gamma = 0.5$	$\Gamma = 1$	$\Gamma = 1.5$	$\Gamma = 2$
(3,1,1,0)	0.01%	0.20%	4.36%	8.73%
(6,1,1,0)	0.01%	0.20%	0.72%	2.78%
(6,4,1,0)	0.01%	0.20%	0.46%	1.08%
(6,24,1,0)	0.01%	0.18%	0.32%	0.61%
(6,24,4,0)	0.01%	0.18%	0.30%	0.53%
(6,24,1,1)	0.01%	0.10%	0.22%	0.49%
(6,24,4,1)	0.01%	0.10%	0.20%	0.45%

How Good is the Simplified Affine Policy?

Table : Opt. gap under different policy structures, for the 118 bus system.

$(n_g, n_{\mathcal{T}}, n_d, L)$	$\Gamma = 0.5$	$\Gamma = 1$	$\Gamma = 2$	$\Gamma = 4$
(10,1,1,0)	0.03%	0.06%	0.11%	0.95%
(21,1,1,0)	0.03%	0.05%	0.11%	0.77%
(31,1,1,0)	0.02%	0.04%	0.10%	0.74%
(54,1,1,0)	0.02%	0.04%	0.10%	0.67%
(54,4,1,0)	0.02%	0.03%	0.10%	0.52%
(54,24,1,0)	0.02%	0.03%	0.07%	0.35%

Table : Opt. gap for the 2718 bus system under the "W_{it}" policy.

$(n_g, n_{\mathcal{T}}, n_d, L)$	$\Gamma = 0.25$	$\Gamma = 0.5$	$\Gamma = 1$	$\Gamma = 1.5$	$\Gamma = 2$
(289,1,1,0)	0.09%	0.22%	0.42%	0.55%	1.05%
(289,24,1,0)	0.07%	0.11%	0.25%	0.35%	0.53%

Why Multi-Stage Robust UC?

Theorem: If the ramping constraint are not binding, the two-stage robust UC and the multi-stage robust UC are equivalent.

- Ramping capability is essential in dealing with high penetration of wind/solar
- Lately, ramping product has been discussed in ISOs, such as MISO and PJM
- Multi-stage robust UC provides a systematic way to manage system ramping

Why Multi-Stage Robust UC?

Table : Worst case cost (US$) comparison of commitment solutions found by the multistage approach and the two-stage approach, under the framework of policy structure with $(n_g, n_\mathcal{T}, n_d, L) = (6, 24, 1, 0)$, for the 30 bus case

	$\Gamma = 0.25$	$\Gamma = 0.5$	$\Gamma = 1$	$\Gamma = 2$
		Multistage		
Cost	87294.0	88516.2	91215.7	96655.3
Penalty	-	-	-	-
Opt. gap	0.04%	0.01%	0.18%	0.61%
		Two-stage		
Cost	87304.9	88516.2	103012.8	205382.6
Penalty	-	-	13372.0	114067.4
Opt. gap	0.05%	0.01%	13.13%	113.78%
		Cost lower bound		
	87259.8	88510.91	91056.38	96069.9

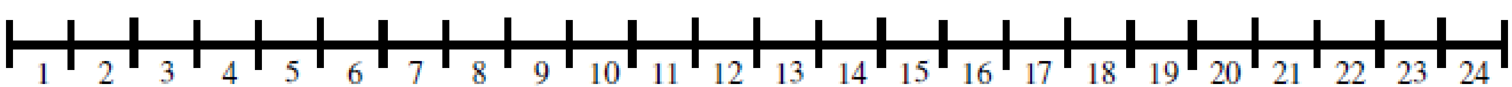
Why Multi-Stage Robust UC?

Table : Worst case cost (US$) comparison of commitment solutions found by the multistage approach and the two-stage approach, under the framework of policy structure with $(n_g, n_\mathcal{T}, n_d, L) = (54, 24, 1, 0)$, for the 118 bus case

	$\Gamma = 0.5$	$\Gamma = 1$	$\Gamma = 2$	$\Gamma = 4$
Multistage				
Cost	1696304.2	1725470.4	1784542.7	1909954.9
Penalty	-	-	-	-
Opt. gap	0.02%	0.03%	0.07%	0.35%
Two-stage				
Cost	1696455.7	1749765.6	1897212.2	4374468.6
Penalty	-	52500.8	196100.5	2632574.2
Opt. gap	0.03%	1.44%	6.39%	129.84%
Cost lower bound				
	1696021.6	1724970.9	1783208.9	1903234.9

How Good is Affine UC on Average?

- Solve two robust UC models:
  - Multi-stage affine robust UC
  - Two-stage fully-adaptive robust UC
- Economic dispatch:
  - Affine policy dispatch using multi-stage affine UC
  - Look-ahead ED using two-stage robust UC
- Rolling-horizon simulation for N=1000 trajectories:



How Good is Affine UC on Average?

- Multi-stage robust UC with policy enforcement for IEEE-118.

Γ	0.25	0.5	1.5	2	4
Cost Avg (M$)	1.8099	1.7025	1.6710	1.6712	1.6788
Cost Std (M$)	0.1913	0.0774	0.0058	0.0058	0.0056
Penalty Avg (M$)	0.1417	0.0337	0.0000	0.0000	0.0000
Penalty Freq Avg	4.06%	1.78%	0.00%	0.00%	0.00%

- Two-stage robust UC with look-ahead ED for IEEE-118.

Γ	0.25	0.5	1.5	2	4
Cost Avg (M$)	1.9100	1.8473	1.7380	1.7372	1.7387
Cost Std (M$)	0.2564	0.2100	0.0976	0.0971	0.0968
Penalty Avg (M$)	0.2424	0.1794	0.0687	0.0674	0.0656
Penalty Freq Avg	5.60%	4.77%	2.53%	2.27%	2.06%

- Avg cost saving = $\frac{1.7372-1.6710}{1.7372} = 3.81\%$, std reduced more than 15 times

How Good is Affine UC on Average?

- Multi-stage robust UC with policy enforcement for 2718-bus.

Γ	0.25	0.5	1	1.5	2	3
Cost Avg ($)	9397528	9319396	9342754	9360359	9379464	9442858.486
Cost Std ($)	113724.53	15969.69	12828.10	12509.33	12362.86	12091.84
Penalty Avg ($)	93552	3497	727	61	5	0
Penalty Freq Avg	10.00%	1.47%	0.40%	0.01%	0.00%	0.00%

- Two-stage robust UC with look-ahead ED for 2718-bus.

Γ	0.25	0.5	1	1.5	2	3
Cost Avg ($)	9398109	9456599	9408732	9383569	9407290	9362379
Cost Std ($)	93470	195774	173884	144698	162469	45584
Penalty Avg ($)	80127	152637	98113	66801	82864	6103
Penalty Freq Avg	9.93%	12.26%	7.80%	5.11%	5.57%	0.37%

- Avg cost saving = $\frac{9.3624-9.3194}{9.3624} = 0.46\%$, std reduced about 2.8 times

How Good is Affine UC on Average?

- Multi-stage robust UC with policy enforcement for 2718-bus.

Γ	0.25	0.5	1	1.5	2	3
Cost Avg ($)	9397528	9319396	9342754	9360359	9379464	9442858.486
Cost Std ($)	113724.53	15969.69	12828.10	12509.33	12362.86	12091.84
Penalty Avg ($)	93552	3497	727	61	5	0
Penalty Freq Avg	10.00%	1.47%	0.40%	0.01%	0.00%	0.00%

- Det UC with reserve and look-ahead ED for 2718-bus.

Reserve factor	0%	5%	10%	15%	20%
Cost Avg ($)	9525885	9696879	9878738	9734593	9719669
Cost Std ($)	200708	282636	130167	125845	71023
Penalty Avg ($)	223406	379704	520708	276776	60192
Penalty Freq Avg	16.72%	17.52%	22.81%	17.27%	11.53%

- Avg cost saving = $\frac{9.5259-9.3194}{9.5259} = 2.17\%$, std reduced about 12.6 times

Summary

- Progress on large-scale power systems operations under uncertainty:
  - Two-stage robust UC models significantly improve over deterministic approach
  - Multistage modeling is important for ramping constrained systems
  - Multistage robust UC with affine policy
    - Significantly improves over two-stage robust UC
    - Computationally tractable for large systems
- Promising directions

THANK YOU!

Questions?

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