Stochastic Optimal Power Flow with Uncertain Reserves from Flexible Loads

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Joint work with Dr. Maria Vrakopoulou (Michigan, formerly ETH Zurich), Prof. Göran Andersson (ETH Zurich), and Prof. Siqian Shen (Michigan)
How can loads provide reserves?
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→ your air conditioner is already flexible

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Thousands of coordinated thermal loads can track signals

➔ Air conditioners, heat pumps, space heaters, electric water heaters, refrigerators

[Mathieu, Koch, and Callaway IEEE Transactions on Power Systems 2013]
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Direct load control for fast, reliable power system service provision
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→ Distributed!
→ Accurate?
→ Cheap?
→ Green?
“But load control will never be reliable enough to provide trustworthy reserves!”

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• Why?
  – Too much uncertainty: People! Weather! etc.

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“But load control will never be reliable enough to provide trustworthy reserves!”

- Why?
  - Too much uncertainty: People! Weather! etc.
- Two options:
  - Be conservative in how much reserve you schedule
  - Explicitly consider reserve uncertainty in the planning algorithm
Outline

• Modeling reserves from loads
• Understanding uncertainty
• Stochastic optimal power flow
• Preliminary results
• New directions
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Time-varying “thermal battery” model

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Mathieu, Kamgarpour, Lygeros, & Callaway ECC 2013
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Time-varying “thermal battery” model

Baseline power

Charging and discharging power

Mathieu, Kamgarpour, Lygeros, & Callaway ECC 2013

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Time-varying “thermal battery” model

\[ S(k + 1) = S(k) + (P(k) - P_{\text{baseline}}(k)) \Delta T \]
Time-varying “thermal battery” model

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Mean power over an interval

\[ P_{\text{min}}(k) \leq P(k) \leq P_{\text{max}}(k) \]

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\[ P_{\text{min}}(k) \leq P(k) \leq P_{\text{max}}(k) \]

State of charge

\[ S_{\text{min}}(k) \leq S(k) \leq S_{\text{max}}(k) \]
What are the power and energy capacity intuitively?

\[ P_{\text{max}} \Delta T_{\text{max}} = S_{\text{max}} \]

[Graph showing power over time with lines at different power levels and a formula relating maximum power to maximum temperature change and capacity.]
What are the power and energy capacity intuitively?

\[ P_{\text{max}} \Delta T_{\text{max}} = S_{\text{max}} \]
Time-varying power & energy capacities

1000 electric space heaters

- Energy Capacity, $S$
  - $S_{\text{max}}(k)$
  - $S_{\text{min}}(k) = 0$

- Power Capacity, $P_C$
  - $P_{\text{max}}(k)$
  - $P_{\text{min}}(k) = 0$

Outdoor air temperature (°C), $T$

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Time-varying power & energy capacities
1000 electric space heaters

In planning problems, we have temperature forecast uncertainty
Time-varying power & energy capacities
1000 electric space heaters

In planning problems, we have temperature forecast uncertainty
What if we try to hold a fixed trajectory?
What if we try to hold a fixed trajectory?

→ The power capacity is actually a function of external forcing!
Handling reserve uncertainty within a dc optimal power flow (DC-OPF) formulation

- **Decision variables:**
  - Generation set points & reserve levels
  - Load set points & reserve levels given that load flexibility is uncertain!

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• **Objective:** Minimize cost of energy generation and reserves subject to generator, load, and power system constraints
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- Uncertainty:
  - Wind power
  - Outdoor temperature
    → Load uncertainty
    → DR uncertainty
    → Reserve uncertainty
Reserve Modeling

• Secondary frequency control (AGC) provided by loads and generators
  – Assumes loads are cheaper!
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• Re-dispatch (15-minute market, Tertiary control) provided by generators
  – Covers power mismatch between expected and actual generation (as it does today)
  – Provides energy to return loads to their scheduled energy state (like CAISO’s Regulation Energy Management)!

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Solution Approach

• Chance-constrained OPF
• Solved with probabilistically robust design [Margellos, Goulart, and Lygeros 2012], inspired by a scenario-based technique [Calafiore and Campi 2006]
• Probabilistic guarantees
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- More details this afternoon...
  “Quantifying the Trade-off Between Secure and Economic Operation of Power Systems Under Uncertainty” – Maria Vrakopoulou
How much does outdoor temperature uncertainty really matter?
How much does outdoor temperature uncertainty really matter?
Operational costs

- no load uncertainty, no control
Operational costs

- dotted line: no load uncertainty, no control
- dashed line: uncertain load, no control

Stochastic formulation

Graph shows the total cost over time (hours) for two scenarios: one with no load uncertainty and no control, and another with uncertain load but no control. The dotted line indicates a lower total cost compared to the dashed line, suggesting a more efficient operation in the absence of load uncertainty.
Operational costs

- no load uncertainty, no control
- uncertain load, no control
- uncertain load, control

Stochastic formulation
DR

Total cost vs. time (hours)
How well do the probabilistic guarantees work?
Key Takeaway

Reserves from loads may not be of the same “quality” as those provided by generators, BUT we can *plan* for load uncertainty by explicitly considering it in our problem formulation.

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New directions...

• How do uncertainty and reserve costs interact?

• How can we handle the full complexity of load control uncertainty?
  – Multi-dimensional
  – Exogenous and endogenous
  – Non-stationary
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  – Insufficient data!

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Scenario-based approaches vs. Robust formulations vs. Analytical reformulation

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THANK YOU!  QUESTIONS?

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