Optimal Power Flow + Load Control

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Acknowledgment

Caltech

- Bose, Candy, Hassibi, Gan, Gayme, Li, Nicky, Topcu, Zhao

SCE

- Auld, Clarke, Montoya, Shah, Sherick

Motivations

Uncertainty in supply/demand

- Need for closing control loops, nonconvex physical flows

Uncertainty in data

- Unavailable, incomplete, inaccurate, dynamic and outdated

Control timescales and market mechanism

- How to co-design for efficiency & security?

Key messages

Uncertainty in supply/demand

- Exploit convexity structure, sparsity, locality

Uncertainty in data

- Exploit (not fight) system dynamics for robustness, scalability, simplicity

Control timescales and market mechanism

- How to co-design for efficiency & security?

AC OPF

Optimal power flow (OPF)

OPF underlies many applications

- Unit commitment, economic dispatch
- State estimation
- Contingency analysis
- Feeder reconfiguration, topology control
- Placement and sizing of capacitors, storage
- Volt/var control in distribution systems
- Demand response, load control
- Electric vehicle charging
- Market power analysis
- ...

ARPA-E GENI project

Goal: overcome nonconvexity of AC OPF

Status: new approach for AC OPF

- Theory
  - convex relaxations
- Algorithms
  - SDP, chordal relaxation, SOCP
- Simulations
  - IEEE test systems, Polish systems, SCE circuits

Seek: real-world applications

- Distributed volt/var control

Fast accurate AC OPF

Exploit convex relaxations of power flows

- Physical systems are nonconvex ...
- ... but have underlying convexity that should be exploited

Convexity is important for OPF

- Foundation of LMP, critical for efficient market theory
- Required to guarantee global optimality
- Required for real-time computation at scale

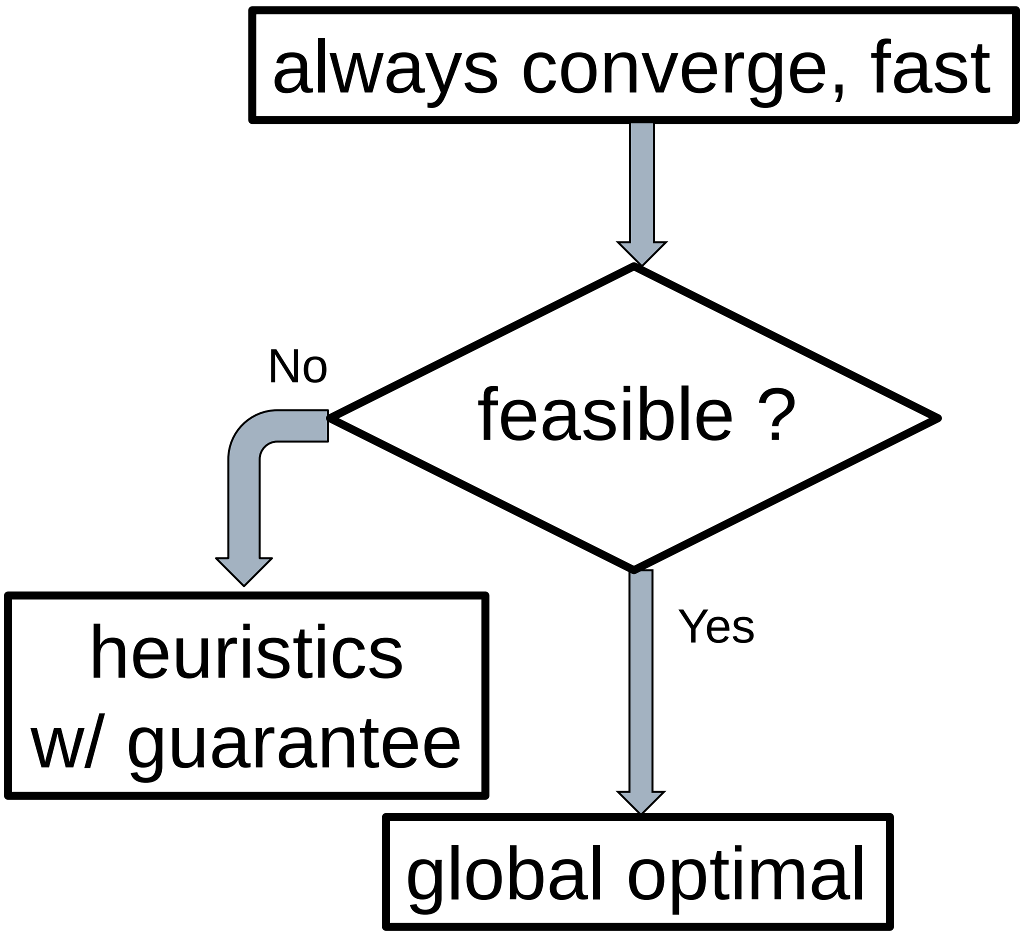
It's not just about accuracy and scalability

- New/enhanced applications where
  - AC power flow is a must (reactive, voltage, loss)
  - Guaranteed quality of solution is critical

Advantages of relaxations

Algorithms based on convex relaxation

Traditional algorithms



Nonlinear algorithms may not converge

DC OPF solution may not be feasible

No guarantee on solution quality

AC OPF: some details

$$\min \quad f(x)$$

$$\text{over} \quad x := (V, \theta, P^g, Q^g)$$

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$$\min \quad f(x)$$

$$\text{over} \quad x := (V, \theta, P^g, Q^g)$$

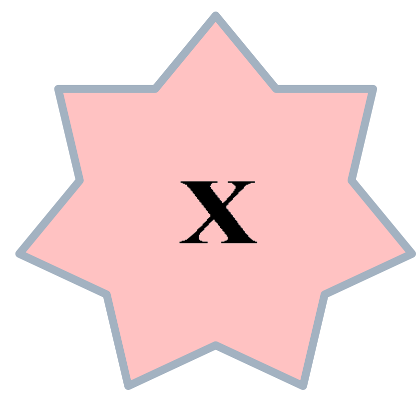
$$\text{s. t.} \quad \underline{P}_i^g \le P_i^g \le \overline{P}_i^g, \qquad \underline{Q}_i^g \le Q_i^g \le \overline{Q}_i^g$$

$$|\theta_i - \theta_j| \le \overline{\theta}_{ij}, \qquad \underline{V}_i \le V_i \le \overline{V}_i$$

simple resource & stability constraints

$$\boxed{x \in \mathbf{X}}$$

nonconvex physical law



simple model to focus on nonconvexity of power flow

AC OPF: some details

$$\min \quad f(x)$$

$$\text{over} \quad x := (V, \theta, P^g, Q^g)$$

$$\text{s. t.} \quad \underline{P}_i^g \le P_i^g \le \overline{P}_i^g, \qquad \underline{Q}_i^g \le Q_i^g \le \overline{Q}_i^g$$

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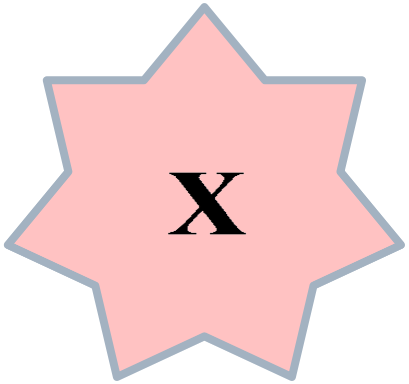
simple resource
& stability
constraints

$$\boxed{x \in \mathbf{X}}$$

nonconvex
physical law



how to deal with this nonconvexity ?

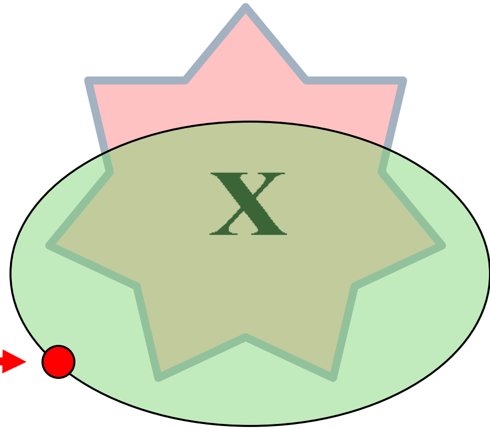
AC OPF: some details

Kirchhoff law:

$$S_j = \sum_{k:k\sim j} y_{jk}^* \left(\left|V_j\right|^2 - V_j V_k^* \right) \qquad \text{for all } j$$

X

AC OPF: some details



DC solution <u>infeasible</u>

$$\min \quad f(x)$$

$$\text{over} \quad x := (V, \theta, P^g, Q^g)$$

$$\text{s. t.} \quad \underline{P}_i^g \le P_i^g \le \overline{P}_i^g, \quad \cancel{\underline{Q}_i^g \le Q_i^g \le \overline{Q}_i^g}$$

$$|\theta_i - \theta_j| \le \overline{\theta}_{ij}, \quad \cancel{\underline{V}_i \le V_i \le \overline{V}_i}$$

DC OPF

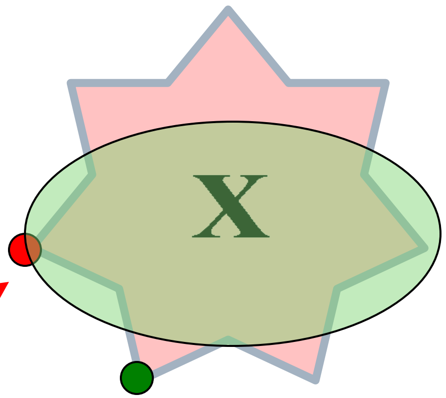
$$\boxed{P = MX^{-1}M^T\theta}$$

linear approximation

AC OPF: some details

$$
\begin{aligned}
\min \quad & f(x) \\
\text{over} \quad & x := (V, \theta, P^g, Q^g) \\
\text{s. t.} \quad & \underline{P}_i^g \le P_i^g \le \overline{P}_i^g, \quad \cancel{\underline{Q}_i^g \le Q_i^g \le \overline{Q}_i^g} \\
& |\theta_i - \theta_j| \le \overline{\theta}_{ij}, \quad \cancel{\underline{V}_i \le V_i \le \overline{V}_i}
\end{aligned}
$$

DC solution
local optimal

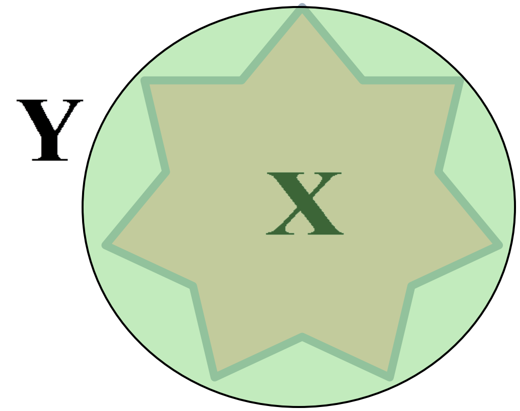


DC OPF

$$\boxed{P = M X^{-1} M^T \theta}$$

linear
approximation

AC OPF: some details



$$\begin{aligned}
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& |\theta_i - \theta_j| \le \overline{\theta}_{ij}, \qquad \underline{V}_i \le V_i \le \overline{V}_i
\end{aligned}$$

$$\boxed{x \in \mathbf{Y}}$$

convex relaxation

- $\mathbf{Y}$ is convex
- $\mathbf{Y}$ contains $\mathbf{X}$

AC OPF: some details

$$
\begin{aligned}
&\min \quad && f(x) \\
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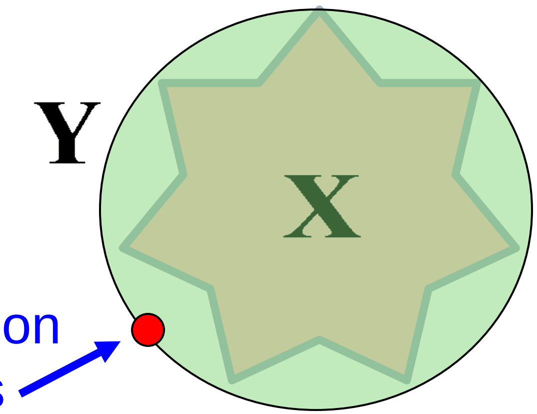
\end{aligned}
$$

relaxed solution
lower bounds

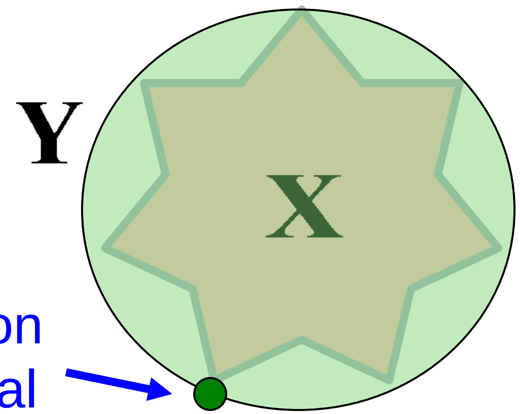


$$\boxed{x \in \mathbf{Y}}$$

convex relaxation

- always lower bounds

AC OPF: some details



relaxed solution
globally optimal

$$
\begin{aligned}
\min \quad & f(x) \\
\text{over} \quad & x := (V, \theta, P^g, Q^g) \\
\text{s. t.} \quad & \underline{P}_i^g \le P_i^g \le \overline{P}_i^g, \qquad \underline{Q}_i^g \le Q_i^g \le \overline{Q}_i^g \\
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\end{aligned}
$$

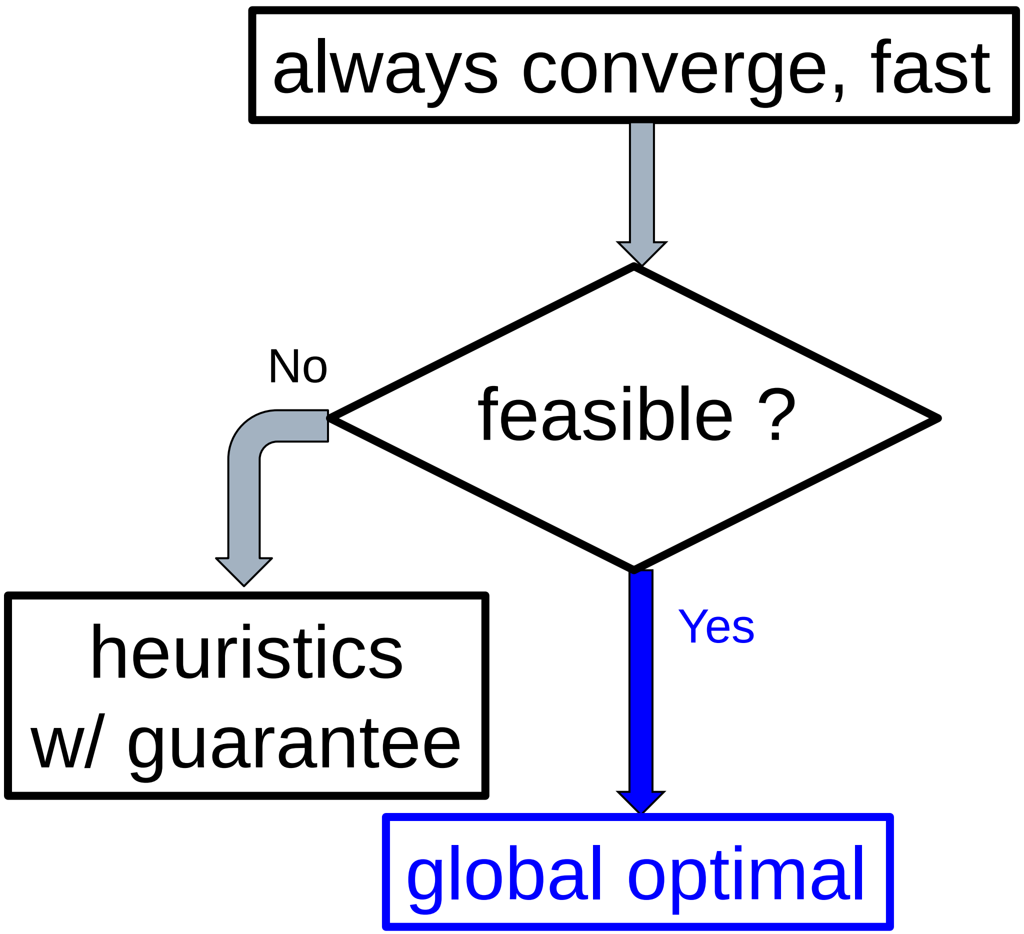
$\boxed{x \in \mathbf{Y}}$

convex relaxation

- always lower bounds
- often global optimal (checkable!)

Advantages of relaxations

Algorithms based on convex relaxation



Traditional algorithms

Nonlinear algorithms may not converge

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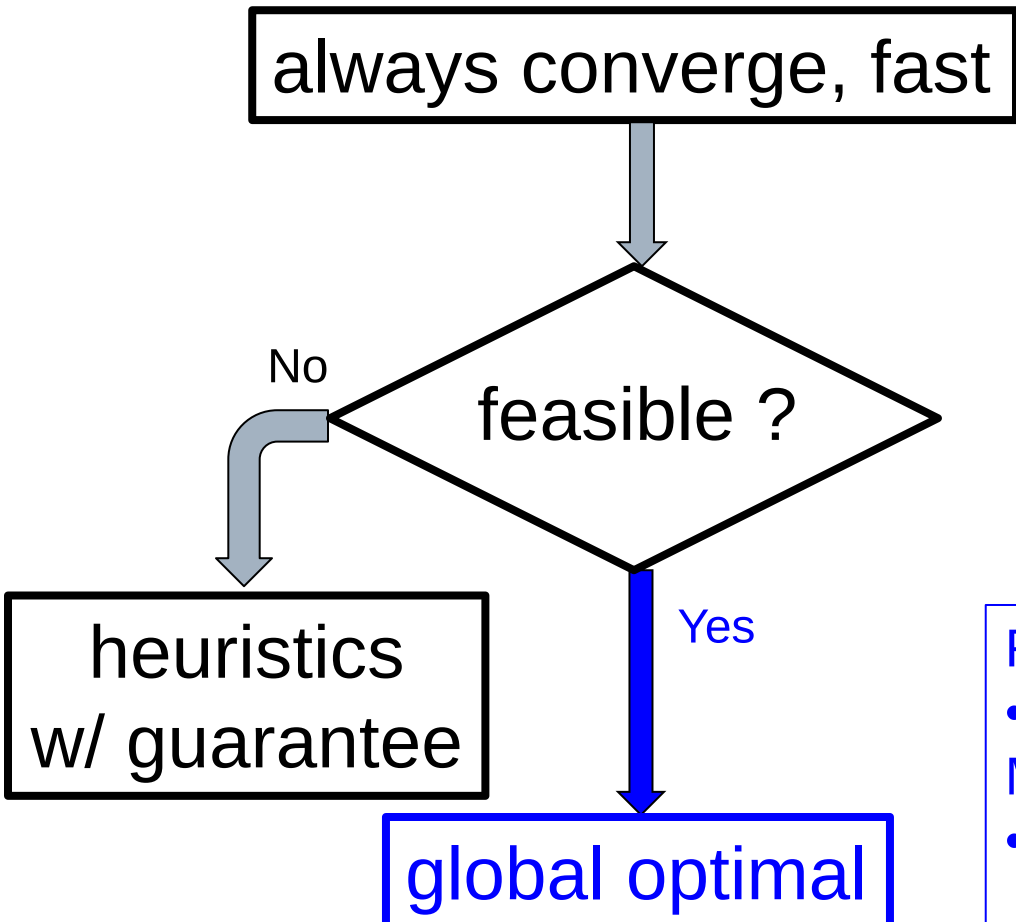
Advantages of relaxations

Algorithms based on convex relaxation



Radial networks

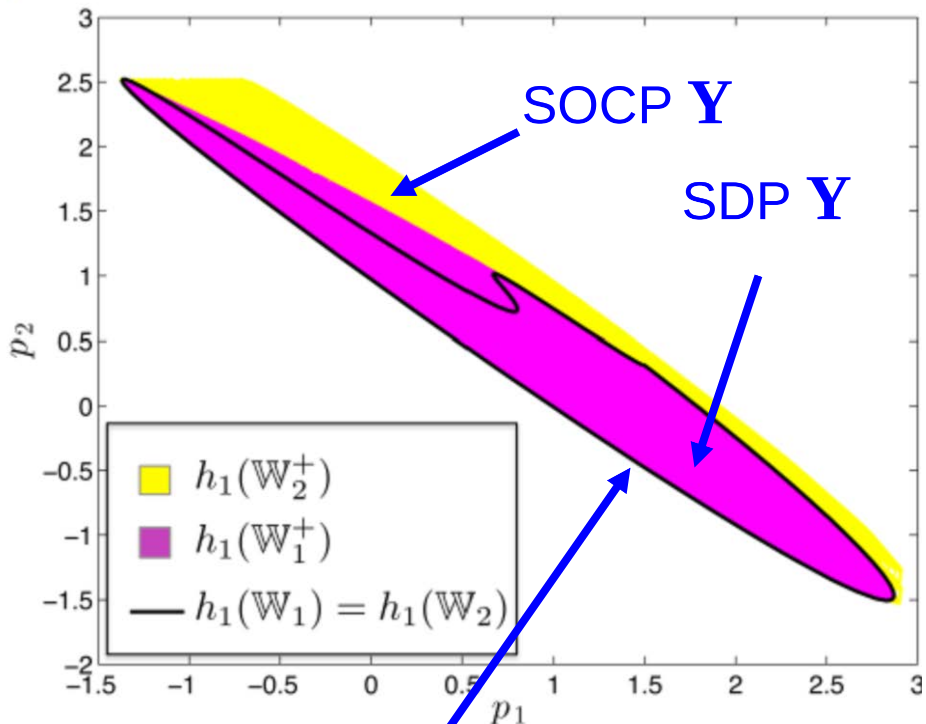
- Guaranteed to work (almost)

Mesh networks

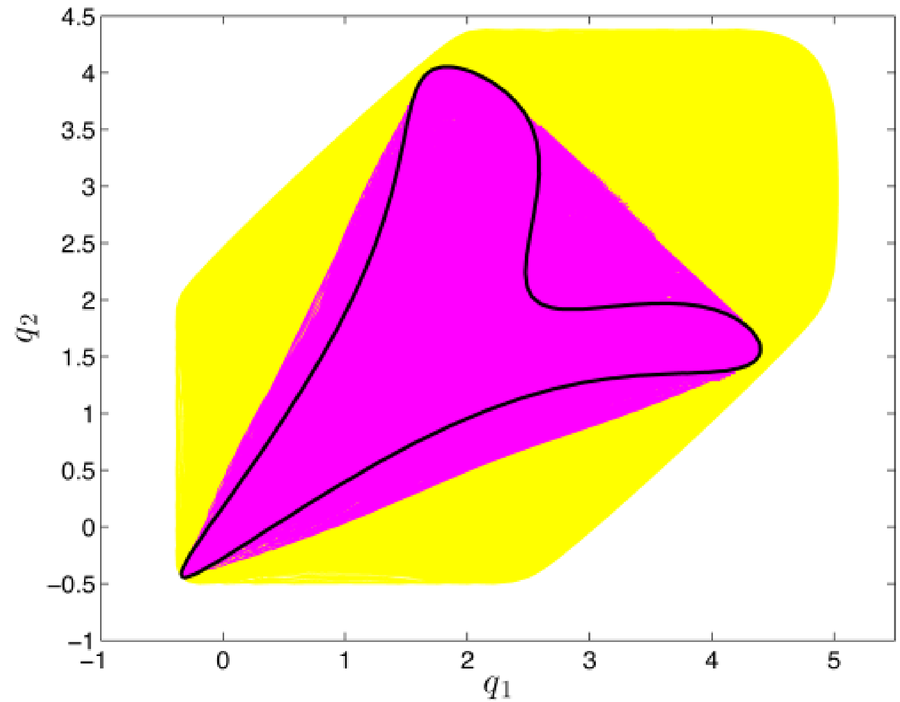
- Understand network structure needed for exact relaxation

Examples

Real Power



Reactive Power



power flow solution $\mathbf{X}$

- Relaxation is exact if $\mathbf{X}$ and $\mathbf{Y}$ have same Pareto front
- SOCP is faster but coarser than SDP

[Bose, et al 2013]

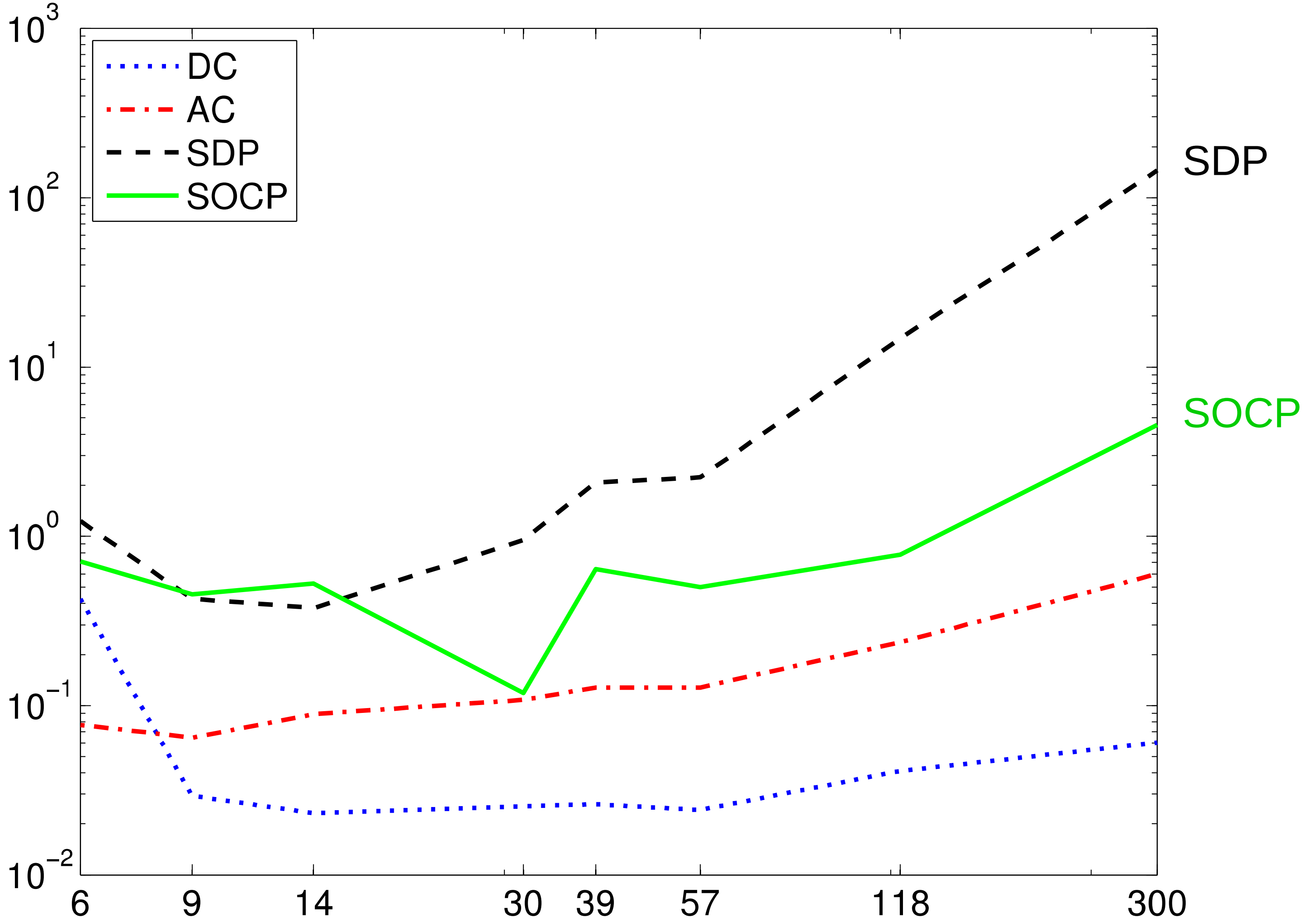
Examples

(secs)

optimal
solutions !

[Bose, et al 2013]

Examples



[Lingwen Gan, Caltech]

Examples

MatPower default nonlinear solver generally performs very well

- Fast, and computes global optimal (checked using convex relaxation method!)

Example difference: IEEE 39-bus

- 5% improvement in optimal network loss over MatPower default nonlinear solver

[Lingwen Gan, Caltech]

ARPA-E GENI project

Seek: real-world applications

- AC power flow is a must (reactive, voltage, loss)
- Guarantee on solution quality is a must

Key messages

Uncertainty in supply/demand

- Exploit convexity structure, sparsity, locality

Uncertainty in data

- Exploit (not fight) system dynamics for robustness, scalability, simplicity

Control timescales and market mechanism

- How to co-design for efficiency & security?

Load control
for freq regulation

Motivation

OPF applications determine operating point

- Economic efficiency through markets
- Setpoints for generators, taps, switches, ...
- Slow timescale: 5min – day

Fast timescale control tracks operating point

- Frequency regulations, AGC, PSS, ...
- Fast timescale: <1 sec – min
- Supplement with load-side control ?
- Market to incentivize huge number of small loads at fast timescale ?

Motivation

Synchronous network

- All buses synchronized to same nominal frequency (US: 60 Hz)
- Supply-demand imbalance ➔ frequency fluctuation

Frequency regulation

- Generator based
- Frequency sensitive (motor-type) loads

Freq-insensitive loads/generations

- Do not react to frequency deviation
- More & more: electronics
- Need active control – how?

Network model

$$\dot{\omega}_i = -\frac{1}{M_i}\left(\sum_{l \in i} d_l + D_i \omega_i - P_i^m + \sum_{i \to j} P_{ij} - \sum_{j \to i} P_{ji}\right)$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right) \qquad \forall\ i \to j$$

swing dynamics

small-signal (linear) model around setpoint

Suppose the system is in steady state, and suddenly …

$$\dot{\omega}_i = 0 \qquad \dot{P}_{ij} = 0$$

Optimal load control

Given: disturbance in gen/inelastic load P_i^m

How to control active load d_i

- Re-synchronize frequencies
- Re-balance supply and demand
- Minimize disutility in heterogeneous load reduction

Optimal load control (OLC)

$$\begin{array}{ll} \text{min} & \text{cost}(d_i, i \in N) \\ \text{over} & \text{active loads } d_i \in \left[\underline{d}_i, \bar{d}_i\right] \\ \text{s. t.} & \text{demand} = \text{supply across network/area} \end{array}$$

Punchline

Theorem

network dynamics

\+ frequency-based load control

= primal-dual algorithm that solves OLC

- Completely decentralized
- No need for explicit communication
- No need for accurate network data
- Exploit free global control signal

… reverse engineering swing dynamics

Punchline

network dynamics

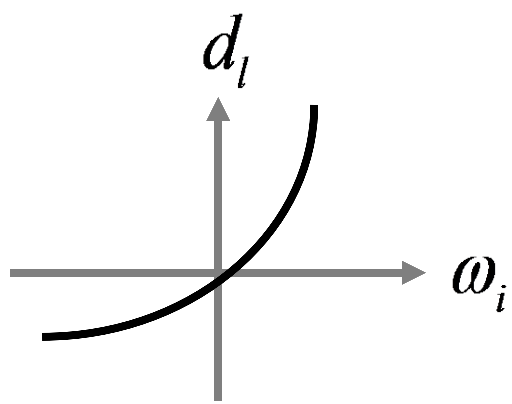
$$\dot{\omega}_i = -\frac{1}{M_i}\left(\sum_{l\in i} d_l(t) + D_i\omega_i(t) - P_i^m + \sum_{i\to j} P_{ij}(t) - \sum_{j\to i} P_{ji}(t)\right)$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i(t) - \omega_j(t)\right)$$

implicit

load control

$$d_l(t) := \left[c_l'^{-1}\left(\omega_i(t)\right)\right]_{\underline{d}_l}^{\overline{d}_l}$$



active control

freq deviations provide the right info,
but not the incentive (unlike prices) !

Punchline

Theorem

system trajectory $\left(d(t),\ \omega(t),\ P(t)\right)$
converges to $\left(d^*,\ \omega^*,\ P^*\right)$

- d^* : unique optimal load control
- ω^* : re-synchronized frequency
- P^* : re-balances gen-load

Zhao, Topcu, Li and Low, 2012. (http://netlab.caltech.edu)
Power system dynamics as primal-dual algorithm for optimal load control

Punchline

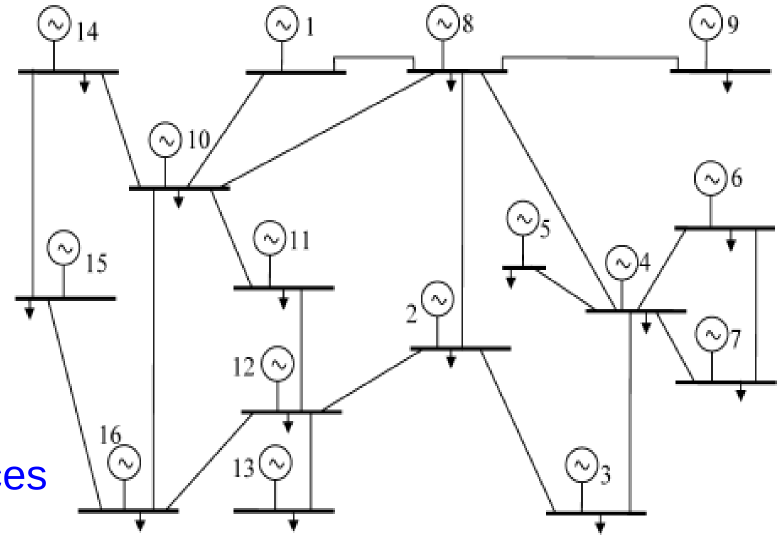
Key insights

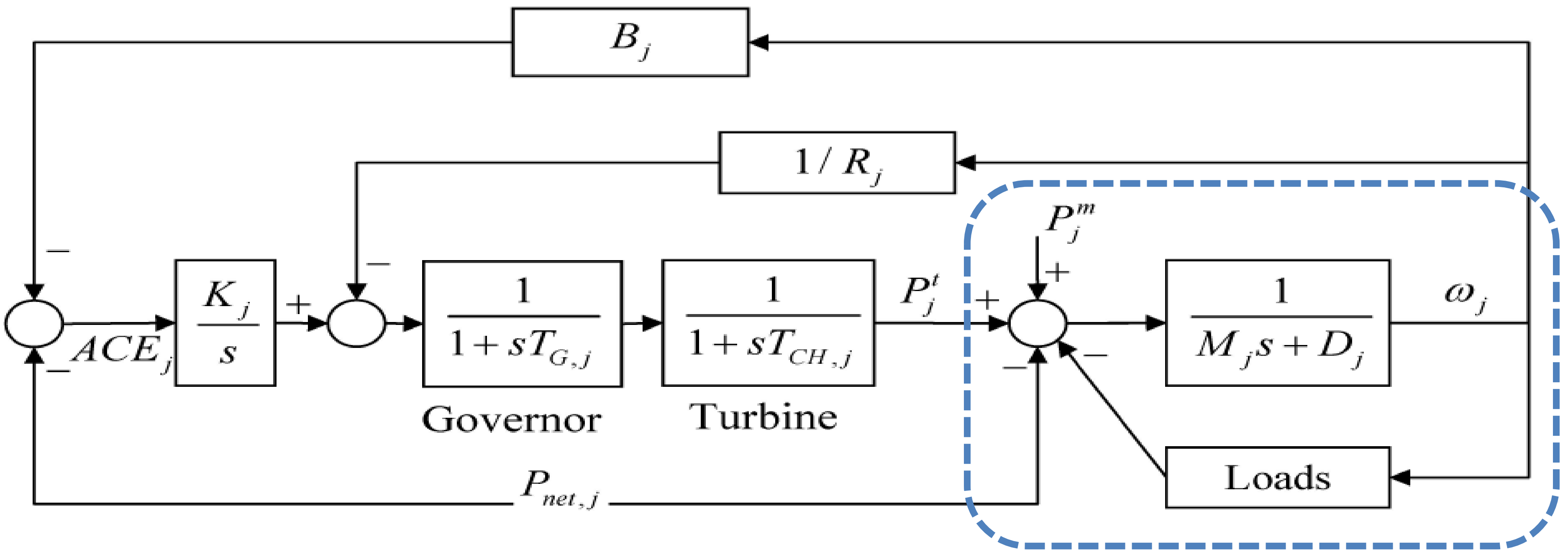
- freq deviations contain exactly right info on global power imbalance for local decisions
- natural system frequency should be exploited for robustness (e.g. to data), simplicity, scalability

Simulations

load control + generator control

16 buses
nonzero resistances





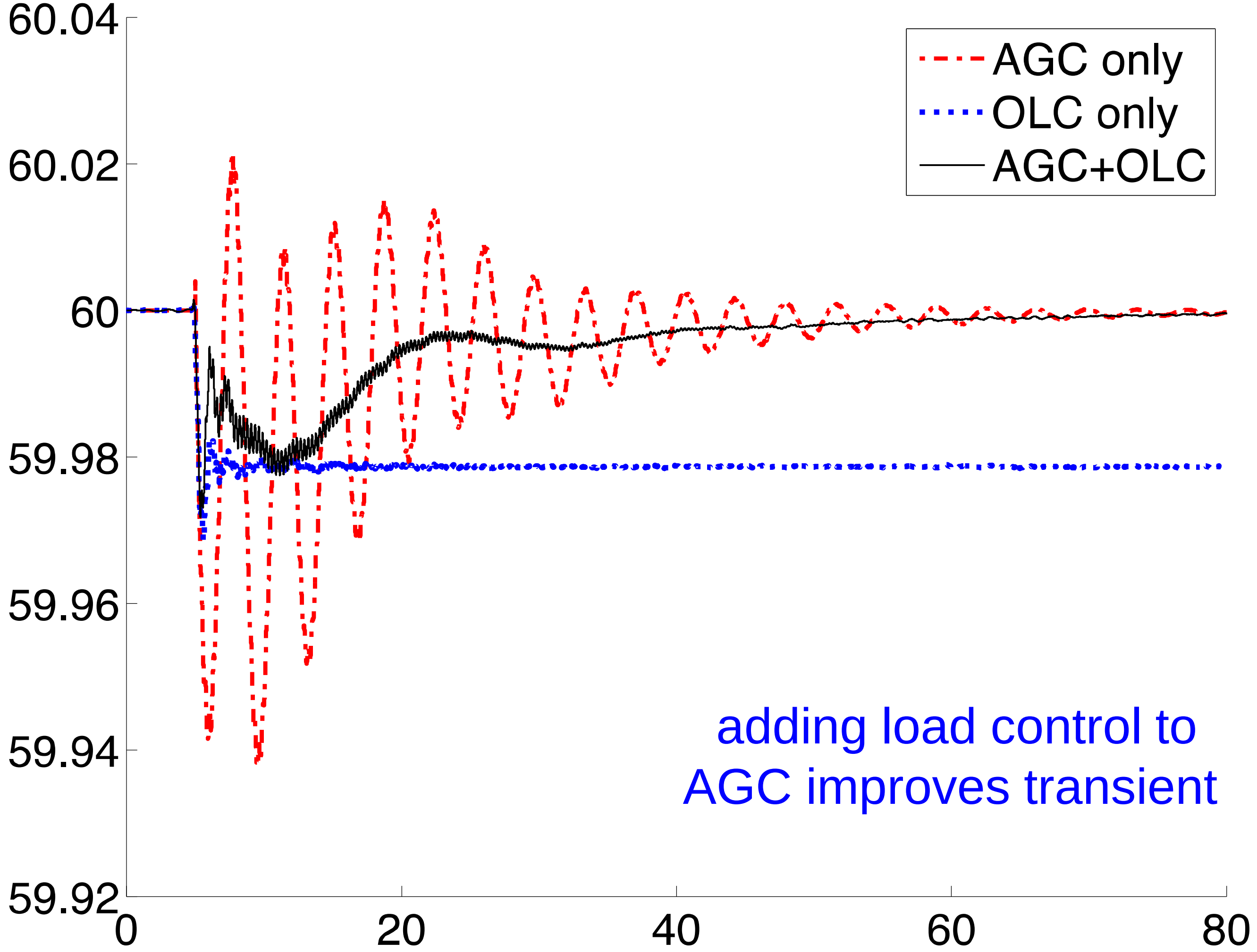
Automatic Generation Control (AGC)

optimal load control (this talk)

Simulations

total power imbalance

Simulations



bus 12

Key messages

Uncertainty in supply/demand

- Exploit convexity structure, sparsity, locality

Uncertainty in data

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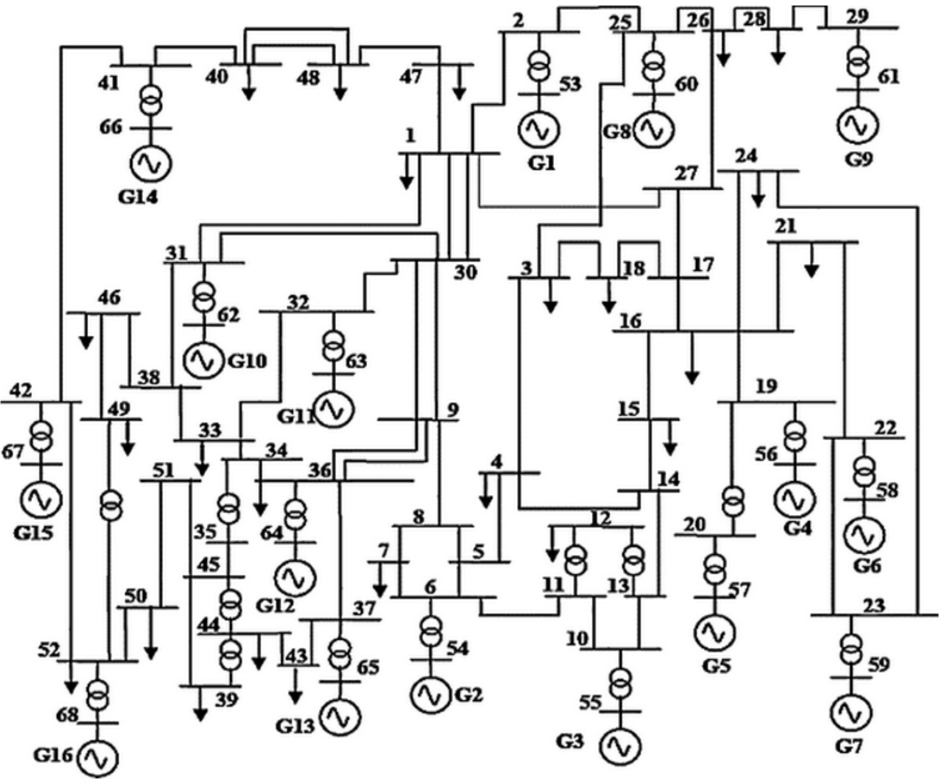
Control timescales and market mechanism

- Challenge: fast timescale, large small loads
- Market + standards ?

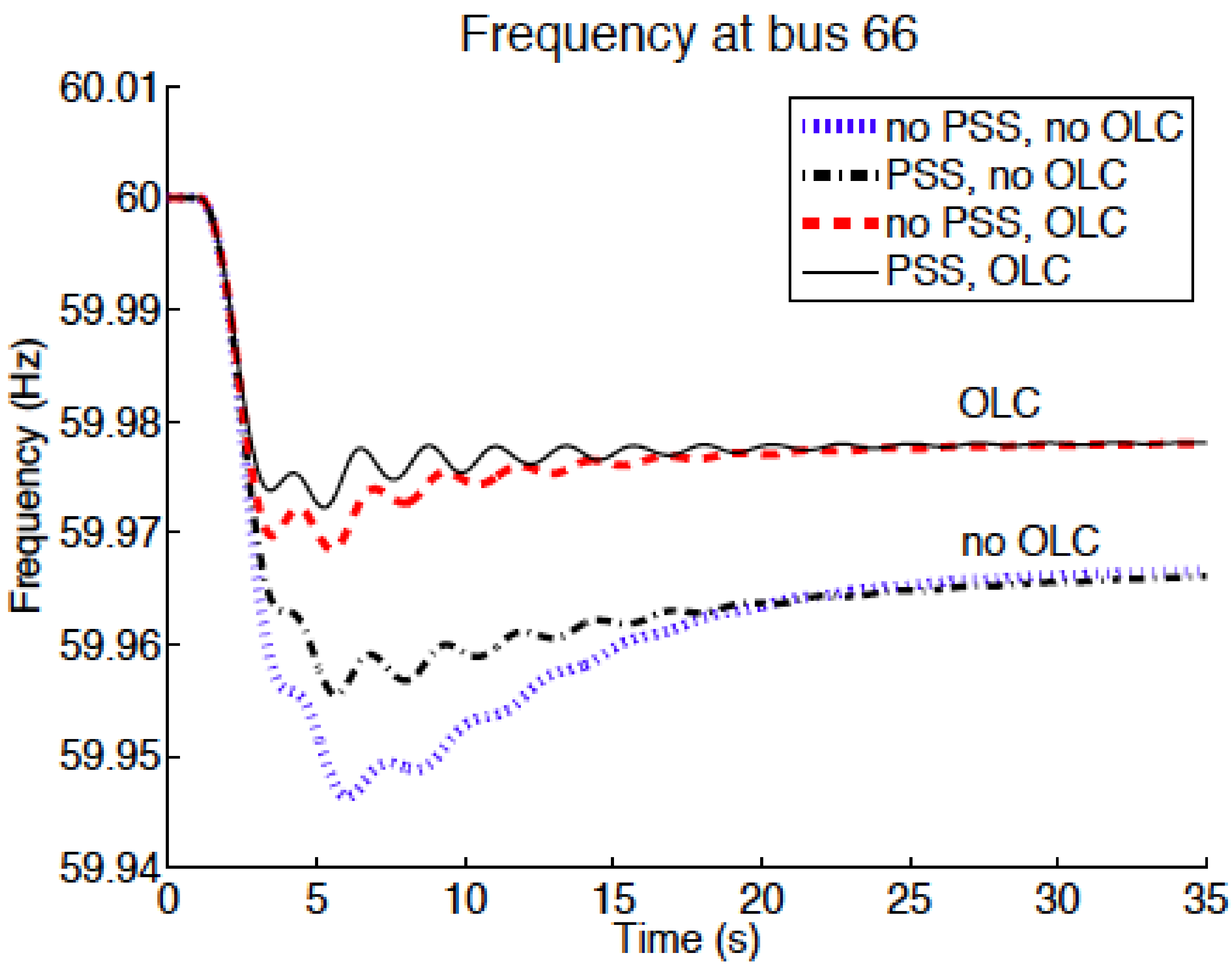
Simulations

Dynamic simulation of IEEE 68-bus system



- Power System Toolbox (Chow)
- Detailed generation model
- Exciter model, power system stabilizer model
- Nonzero resistance lines

Simulations



Simulations