A Synergistic Combination of Surrogate Lagrangian Relaxation and Branch-and-Cut for MIP Problems in Power Systems

Peter B. Luh, Mikhail A. Bragin, Yaowen Yu
University of Connecticut

Joseph H. Yan, Gary A. Stern, Nanpeng Yu
Southern California Edison

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Motivation

- A problem from power systems: Simultaneous auctions in ISO day-ahead markets are used to commit units to satisfy bidding demand and reserve requirements

- Such auctions and many other practical problems are modeled as mixed-integer programming problems which are computationally intensive
  - Unit commitment is complicated by both system-wide and individual unit constraints
  - Unit commitment is further complicated when extended to incorporate uncertainties introduced by renewable resources
  - Payment cost minimization is very complicated since the objective function involves cross-products of decision variables and constraints defining prices are global

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- To solve these problems, Lagrangian relaxation and branch-and-cut have been used

Standard Lagrangian Relaxation

- Lagrangian relaxation has been powerful for solving separable mixed-integer programming problems by introducing Lagrange multipliers
  - Relaxed problem can be decomposed into subproblems
  - Multipliers are updated based on levels of constraint violations
- The subgradient method, the most widely used method to update multipliers, can suffer from slow convergence
  - The relaxed problem is difficult to fully optimize (NP-hard, non-separable)
  - The method can suffer from zigzagging of multipliers
  - Convergence does not require the optimal dual value q^*
- How to solve problems efficiently and guarantee convergence without requiring q^*? ~ an open issue

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Standard Branch-and-Cut

- Branch-and-cut has been a powerful method to solve linear mixed-integer programming problems when
  - Strong cuts are used
  - Branch-and-cut tree is reasonably small

- The method can suffer from slow convergence because
  - Strong globally valid cuts can be problem-dependent
  - Other cuts (e.g., Gomory cuts) can suffer from infinite convergence
  - When strong cuts are not available, branching can lead to a large number of branching operations

- Difficulty: Complicated global and local constraints can pose challenges when strong cuts are unavailable

In This Talk

- Development of novel surrogate Lagrangian relaxation to guarantee convergence without fully optimizing the relaxed problem and without requiring q^*

- Innovative synergy of surrogate Lagrangian relaxation and branch-and-cut

- Applications for solving mixed-integer programming problems with complicated constraints prevalent in ISO day-ahead markets
  - Multi-stage combined cycle units
  - Payment cost minimization
  - Stochastic unit commitment

- Conclusions

Surrogate Subgradient Method

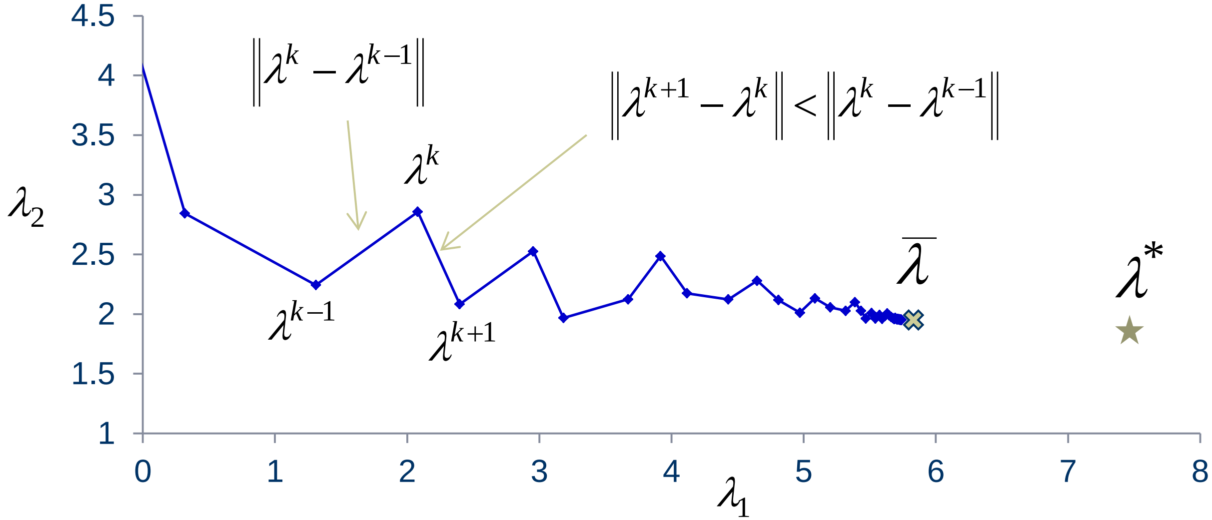
- To overcome difficulties of the subgradient method, the Lagrangian relaxation and surrogate subgradient method (Zhao et al, 1999) has been introduced
- To reduce computational effort, the relaxed problem is approximately optimized subject to the surrogate optimality condition
- Surrogate subgradient directions form small acute angles toward λ^*, thereby alleviating zigzagging and reducing the number of iterations required for convergence
- Convergence was proved by using q^*

Reference: Zhao, X., Luh, P. B., and Wang, J.: Surrogate Gradient Algorithm for Lagrangian Relaxation. Journal of Optimization Theory and Applications 100 (3), 699-712 (1999)

Innovative Surrogate Lagrangian Relaxation

- Main Contribution: Develop a new method, prove and guarantee convergence
  - Without fully optimizing the relaxed problem
  - Without requiring q^*

- Main Idea 1: Decrease distances between multipliers at successive iterations ($\|\lambda^{k+1} - \lambda^k\|$ decreases)



- $\|\lambda^{k+1} - \lambda^k\|$ decreases $\Rightarrow$ fixed-point mapping $\Rightarrow \lambda^k \to \bar{\lambda}$

Innovative Surrogate Lagrangian Relaxation

- $$\left.\begin{array}{c} \lambda^{k+1} = \lambda^k + c^k \tilde{g}\left(x^k\right) \\ \left\|\lambda^{k+1} - \lambda^k\right\| = \alpha_k \left\|\lambda^k - \lambda^{k-1}\right\|,\ 0 < \alpha_k < 1, \\ \left\|c^k \tilde{g}\left(x^k\right)\right\| = \alpha_k \left\|c^{k-1} \tilde{g}\left(x^{k-1}\right)\right\| \end{array}\right\} \Rightarrow \boxed{c^k = \alpha_k \frac{c^{k-1}\left\|\tilde{g}\left(x^{k-1}\right)\right\|}{\left\|\tilde{g}\left(x^k\right)\right\|}} \qquad (1)$$

- Parameters α_k should satisfy $c^k \sim \boxed{\prod_{i=1}^{k} \alpha_i \to 0}$

- If α_k are small, $c^k \to 0$ too fast $\Rightarrow$ premature convergence

- Main Idea 2:
  - To avoid premature convergence, stepsizes ($c^k \to 0$) should be kept sufficiently large
  - To ensure that, stepsize setting parameters α_k should converge to 1 faster than $c^k \to 0$

$$\boxed{\lim_{k \to \infty} \frac{1 - \alpha_k}{c^k} = 0}$$

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Main Theorem

- Multipliers converge to the optimum λ^* without requiring q^* provided α_k satisfy:

1) $c^k \sim \prod_{i=1}^{k} \alpha_i \to 0$ (Main idea 1)

2) $\lim_{k\to\infty} \frac{1-\alpha_k}{c^k} = 0$ (Main idea 2)

Without requiring q^*!

- One possible example of α_k that satisfy conditions 1) and 2): $\alpha_k = 1 - \frac{1}{M \cdot k^p},\ 0 < p < 1,\ M > 1,\ k = 1, 2, \ldots$

- At convergence, the surrogate dual value approaches the (optimal) dual value q^* ~ valid lower bound on the feasible cost

Synergistic Combination with Branch-and-cut

- To overcome difficulties of existing methods, surrogate Lagrangian relaxation and branch-and-cut can be synergistically combined by approximately optimizing the relaxed problem using branch-and-cut subject to the surrogate optimality condition

- The synergistic combination is efficient and implementable in practice since it makes the best use of problem structures
  - Surrogate Lagrangian relaxation exploits separability and local characteristics by relaxing global constraints
  - Branch-and-cut exploits linearity

and convergence of the surrogate Lagrangian relaxation is guaranteed without requiring q^*

Synergistic Combination with Branch-and-cut

- The relaxed problem can be efficiently optimized approximately by using branch-and-cut
  - If decomposable, solving one subproblem approximately is sufficient to satisfy the surrogate optimality condition
  - If not decomposable, optimizing with respect to selected decision variables while keeping other decision variables fixed is sufficient to satisfy the surrogate optimality condition
- To further improve efficiency, a warm start uses solutions from previous iterations to set the basis and
  - Eliminate portions of the search space thereby leading to smaller branch-and-cut trees
  - Allow heuristics to improve the incumbent solution

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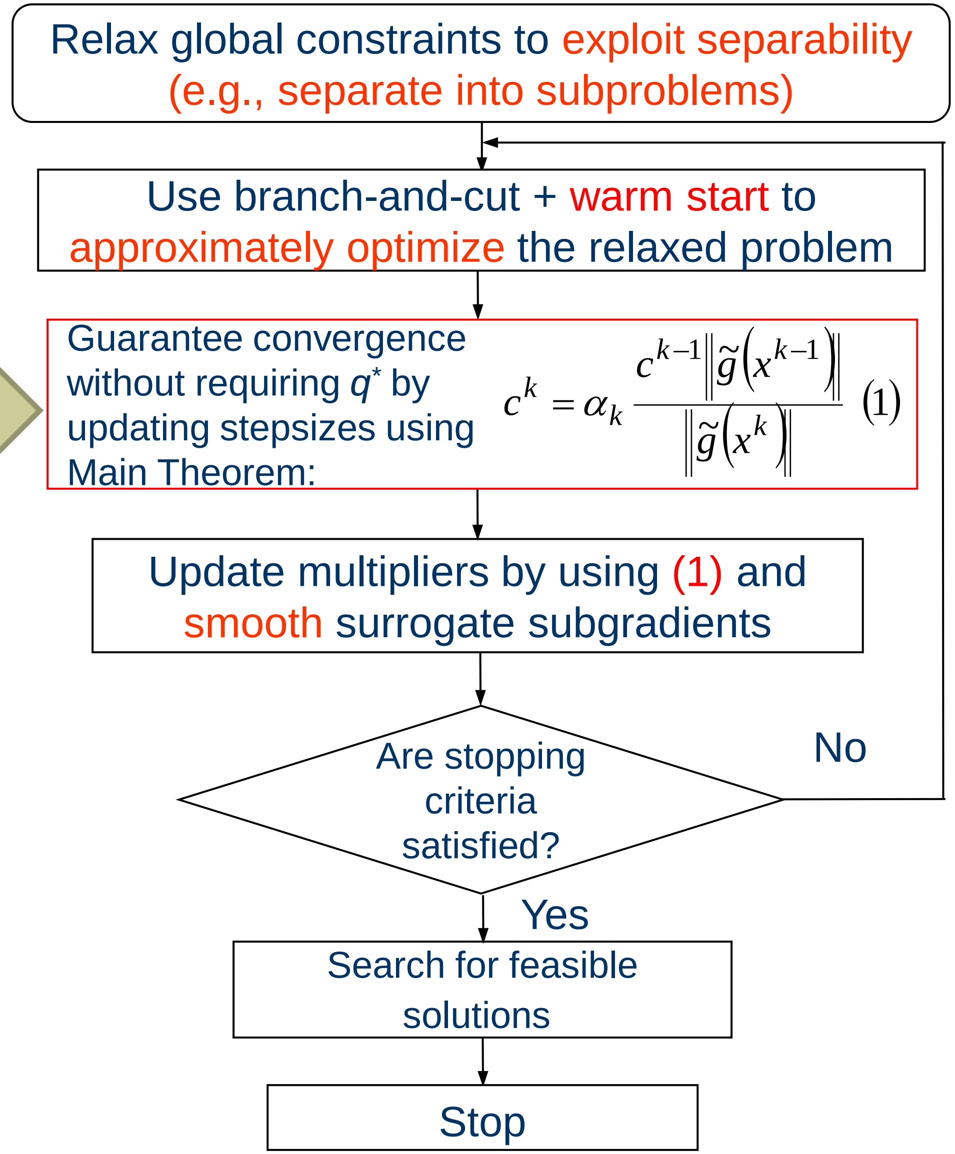
Flow-Chart of the Synergistic Approach

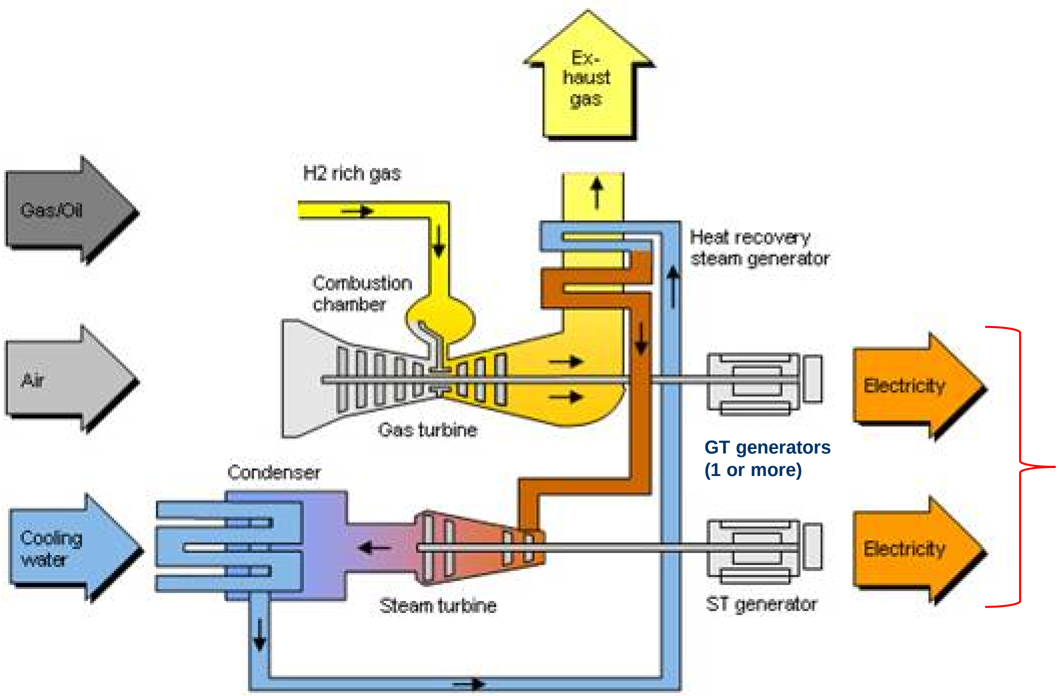
Convergence without requiring q^* enables the synergistic combination



Multi-Stage Combined Cycle Units

- Problem: Commit units to satisfy bidding demand subject to transitions between configurations of a combined cycle plant and conventional units constraints

- Importance: Problems involving combined cycles have been used in power markets but are not well-handled



Generators are coupled:

1) Status change of one generator can lead to status changes of other generators
2) A large number of possible transitions between states complicate the problem

Multi-Stage Combined Cycle Units

- Difficulty: Constraints modeling transitions between configurations of generators are logical and complex
- While such constraints can be linearized, because of the complex nature of transitions, strong cuts are not available within branch-and-cut to handle combined cycle constraints efficiently globally
- In the novel approach, branch-and-cut can handle local constraints associated with each subproblem locally thereby efficiently optimizing the relaxed problem approximately with respect to one combined cycle subproblem

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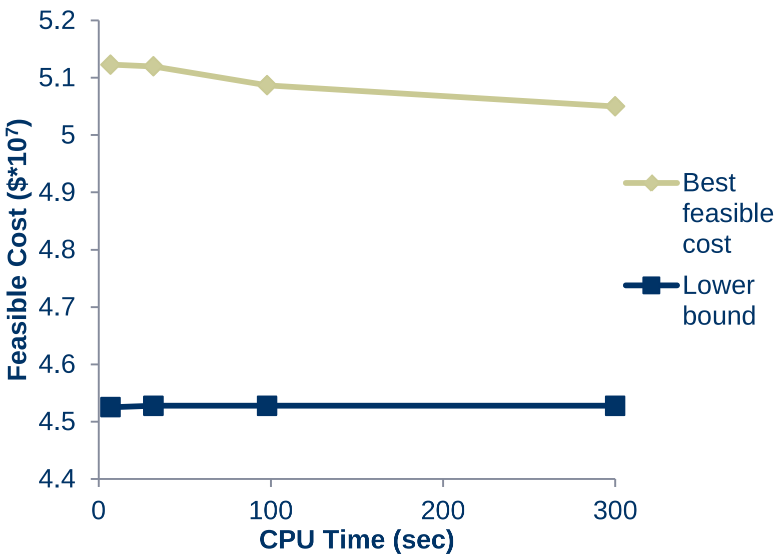
Multi-Stage Combined Cycle Units

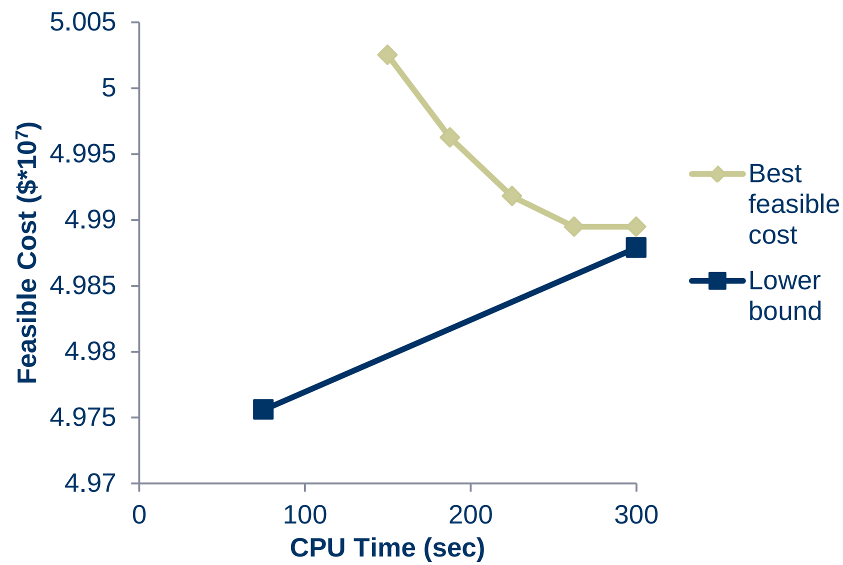
- To demonstrate the efficiency of surrogate Lagrangian relaxation, a problem with 10 CC plants and 300 conventional units is considered

Method	Feasible Cost	Lower Bound	Gap (%)	CPU Time (min)
Branch-and-cut	50,260,500	45,305,200	9.859	30
Novel method	49,894,806	49,879,027	0.032	5

Branch-and-cut



Surrogate Lagrangian relaxation



Payment Cost Minimization

- Problem: Payment Cost Minimization (PCM) commits units to minimize the total payment cost to satisfy bidding demand P^D, and individual unit constraints

- Simplified formulation w/o considering transmission:

$$\min_{\{MCP(t),\, p_i(t),\, x_i(t)\}} \sum_{t=1}^{T} \left(MCP(t) P^D(t) + \sum_{i=1}^{I} S_i(t) \right)$$

Energy cost — Start-up cost

- Simplified price definition constraints:

$$MCP(t) \geq c_i(t) x_i(t)$$ ~ global constraint

Bid *i*'s price — Status of a bid *i*

- Importance: To assess advantages and disadvantages of PCM and facilitate a comparative analysis with other auction mechanisms, an efficient solution methodology is required

Payment Cost Minimization

- While the general problem formulation of the problem is nonlinear, the simplified formulation can be efficiently linearized
- Difficulties: Because of the complicated role of prices
  - Strong cuts are not available within pure branch-and-cut to handle global price definition constraints
  - When demand constraints are relaxed, the problem is not decomposable because of price definition constraints
- To solve the problem efficiently, the relaxed problem is approximately optimized with respect to one (or several) bid at a time by using branch-and-cut and efficiency is further improved by using warm start

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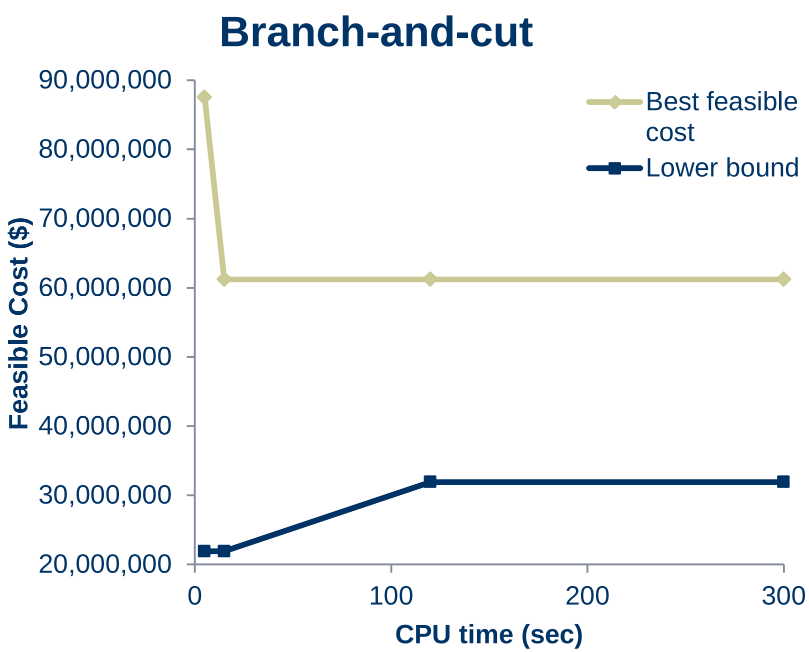
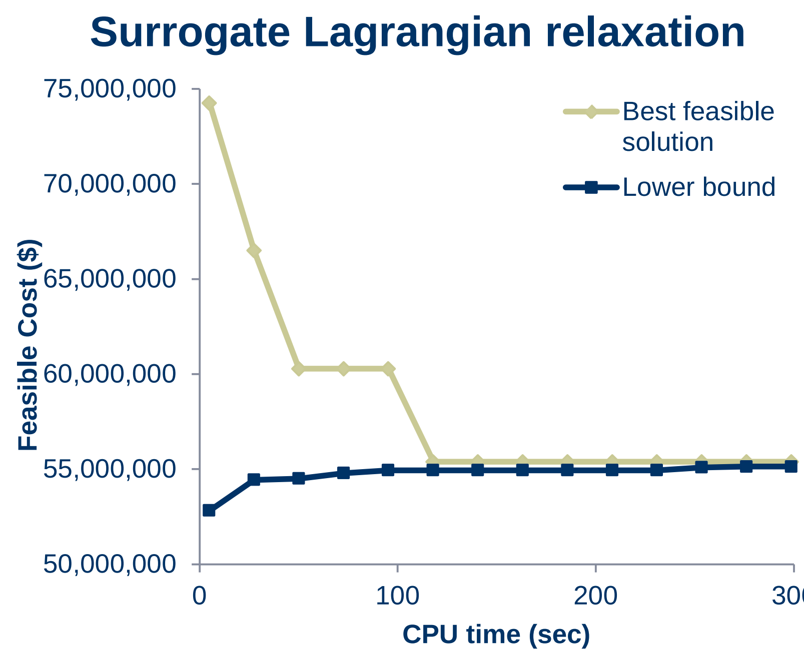
Payment Cost Minimization

- To demonstrate the efficiency of surrogate Lagrangian relaxation, a problem 300 conventional units is considered

Method	Feasible Cost	Lower Bound	Gap (%)	CPU Time (min)
Branch-and-cut	61,271,446	31,903,846	47.91	5
Novel method	55,553,996	55,240,853	0.56	5





Conclusion

- Major novel theoretical result: Within the surrogate Lagrangian relaxation framework, multipliers converge to the optimum without requiring q^*
- Surrogate Lagrangian relaxation has been synergistically combined with branch-and-cut to solve mixed-integer programming problems efficiently
- Numerical results demonstrate that the innovative approach is powerful and efficient for solving mixed-integer programming problems
- Broad Impact: The novel methodology opens new directions to efficiently solve mixed-integer programming problems such as Stochastic Unit Commitment

Thank You!