An AC-Feasible Linear Approximation Approach to Finding the Optimal Power Flow

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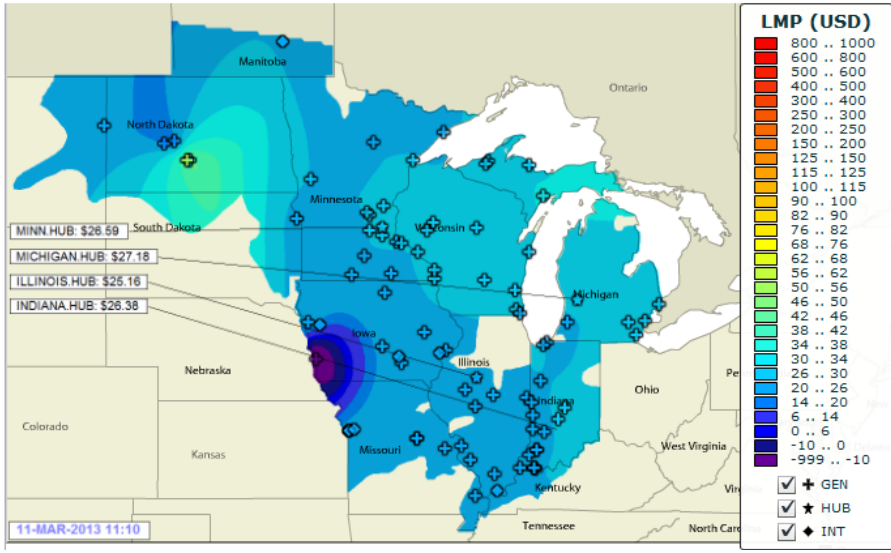
Optimal Power Flow: Decisions

Power Production Schedules

Locational Marginal Prices



Optimal Power Flow: Requirements

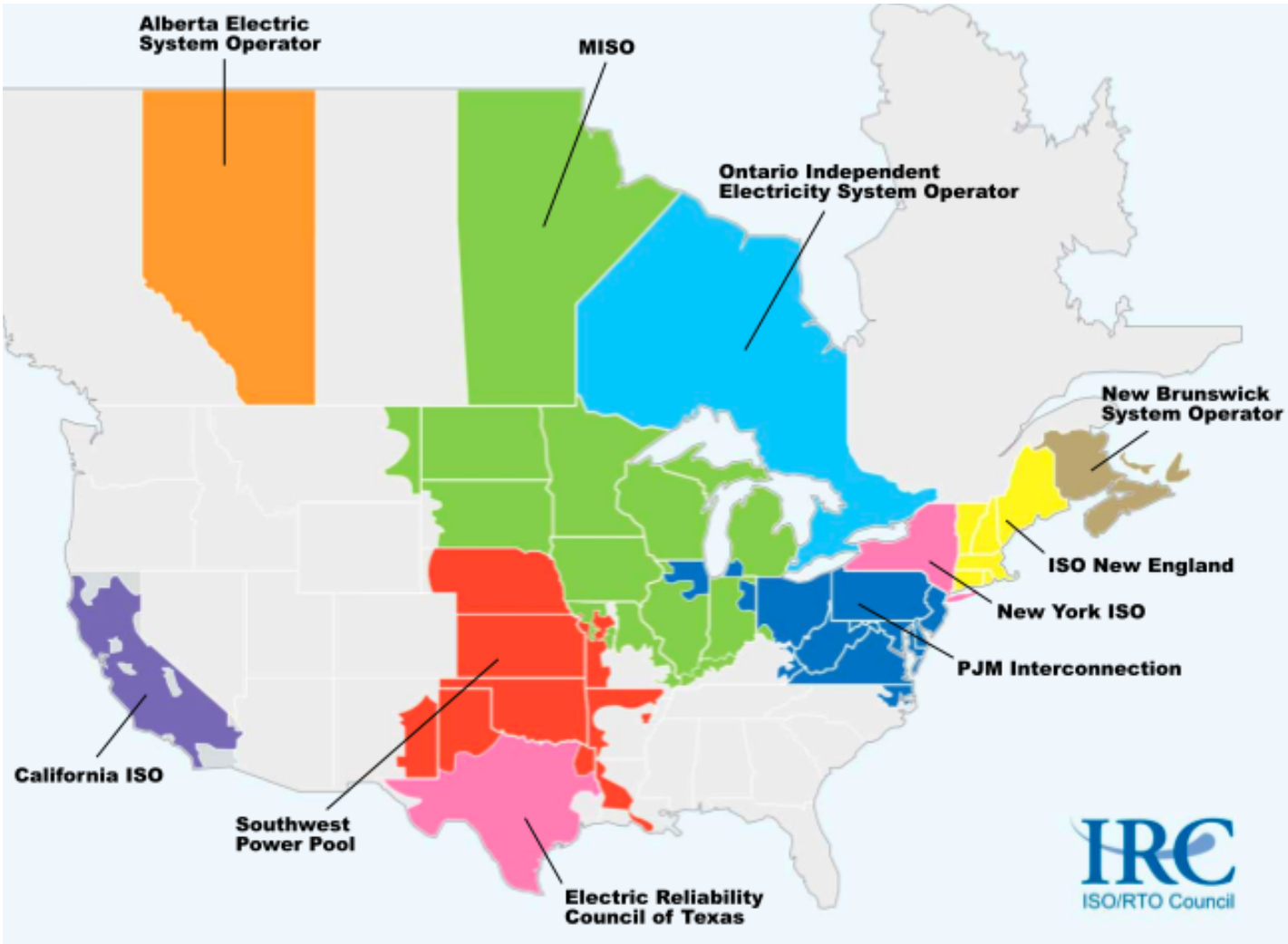
Thermal Line Limits

Power Demand

Generator Production Limits

Optimal Power Flow: Goal

ISO/RTO Perspective: Minimize total power production cost



Optimal Power Flow
Mathematical Formulation

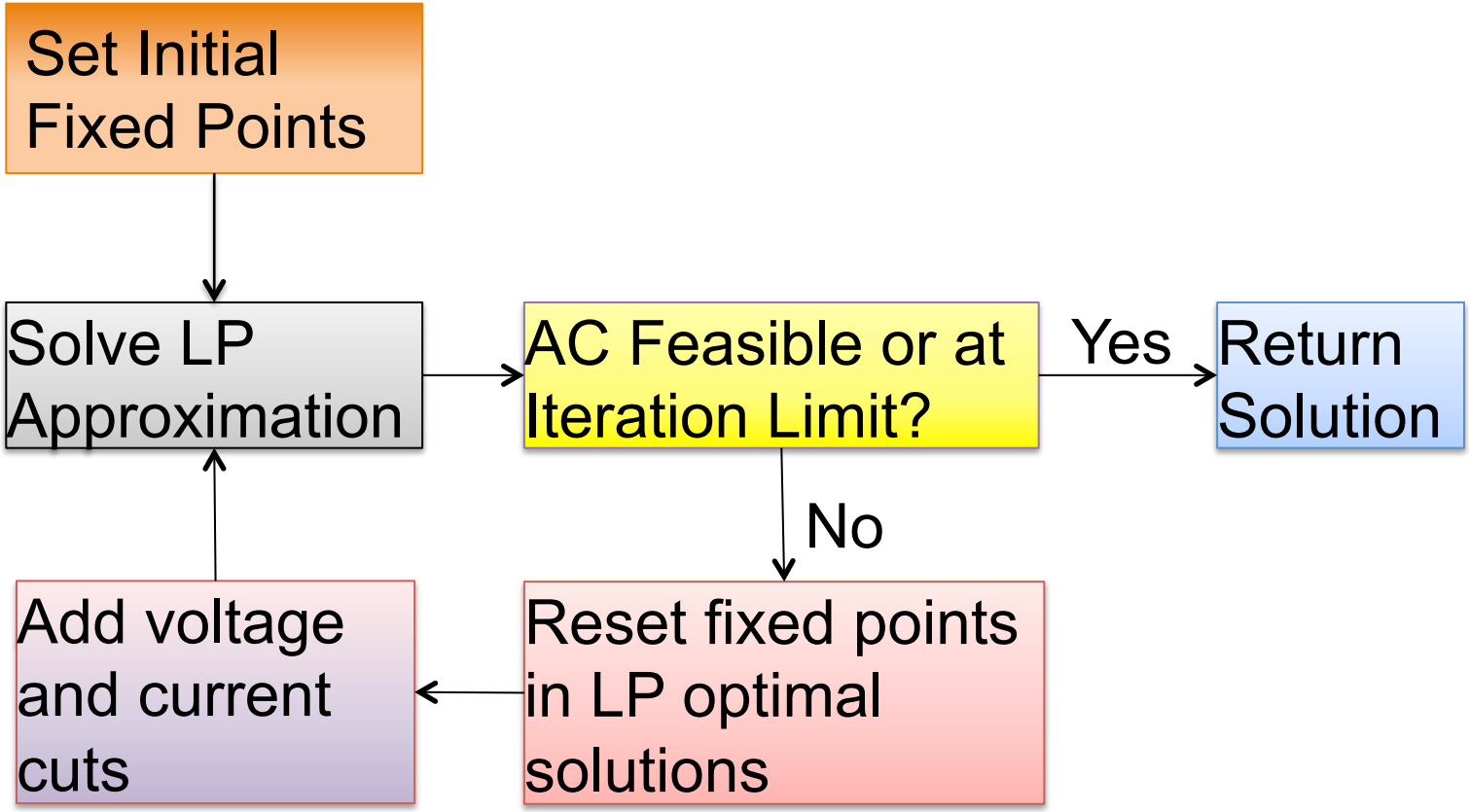
- Objective: Minimize Power Production Cost
- Constraints:
  - Min/Max Voltage Limits
  - Max thermal Line Limits
  - Generator Min/Max Limits
  - Meet Real and Reactive Power Demand

Nonlinear and Nonconvex
Large Problems

Approaches to Finding the Optimal Power Flow

- ISOs & RTOs: Decoupled AC/DC
- Alternatives:
  - Exact Methods (i.e. Newton-Raphson)
  - Semidefinite Representation
  - Conic Relaxation
  - AC Linear Approximation
    - Power-Voltage Model: Coffrin & Van Hentenryck
    - Current-Voltage Model: This presentation

LP-ACOPF Overview



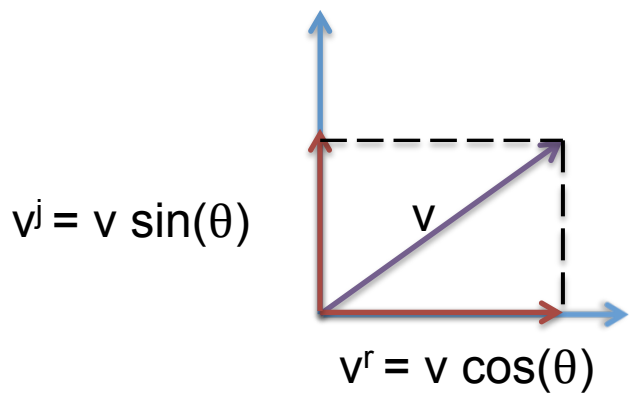
Nomenclature

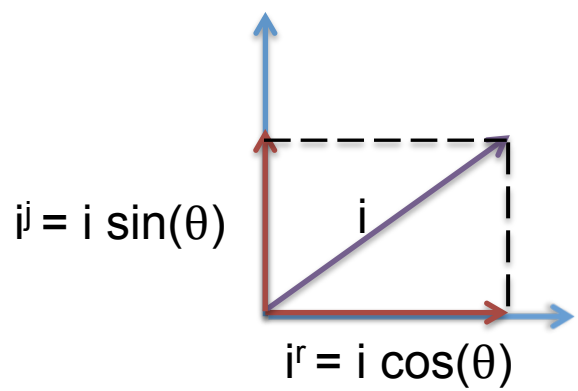
- Variables
  - v = voltage
  - i = current
  - p = Real power
  - q = Reactive Power
- Indexing: n, m = buses
- Parameters
  - g = conductance
  - b = susceptance
  - c^p = real power cost
  - c^q = reactive power cost
  - c^{pen} = penalty cost of violating constraints

Nomenclature

Fixed Points:

- $\underline{v}^r$ = value of Re{v} from prior iteration
- $\underline{v}^j$ = value of Im{v} from prior iteration
- $\underline{i}^r$ = value of Re{i} from prior iteration
- $\underline{i}^j$ = value of Im{i} from prior iteration

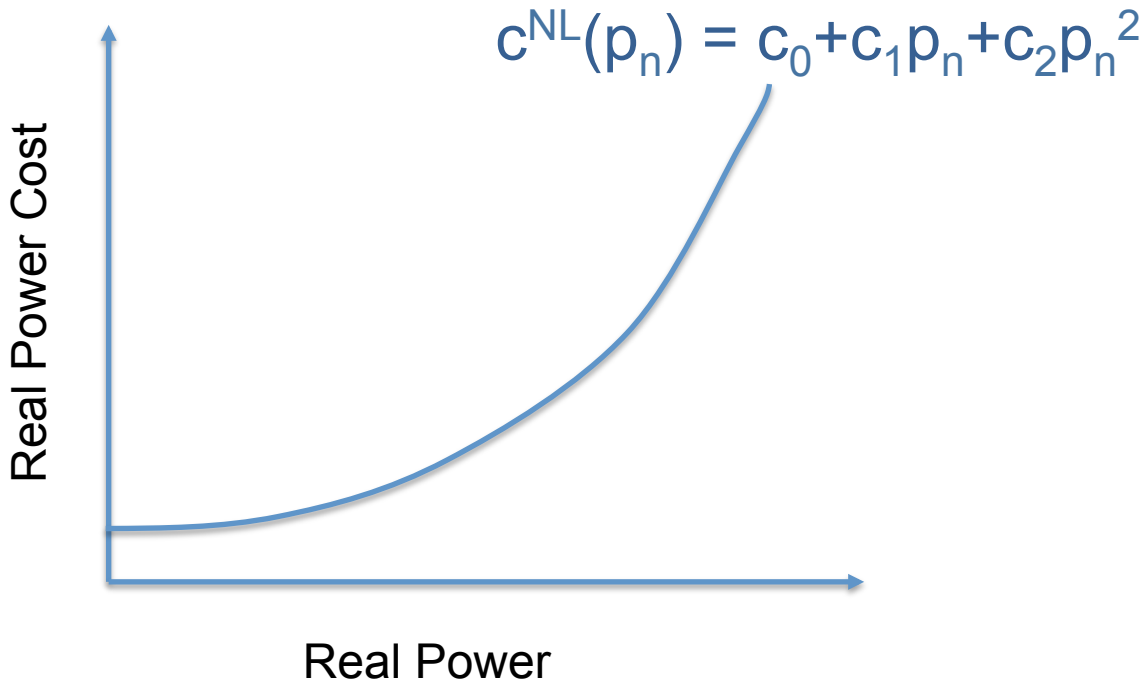




ACOPF I-V Formulation: Cost Function

Cost = Real Power Cost + Reactive Power Cost + Penalty Cost

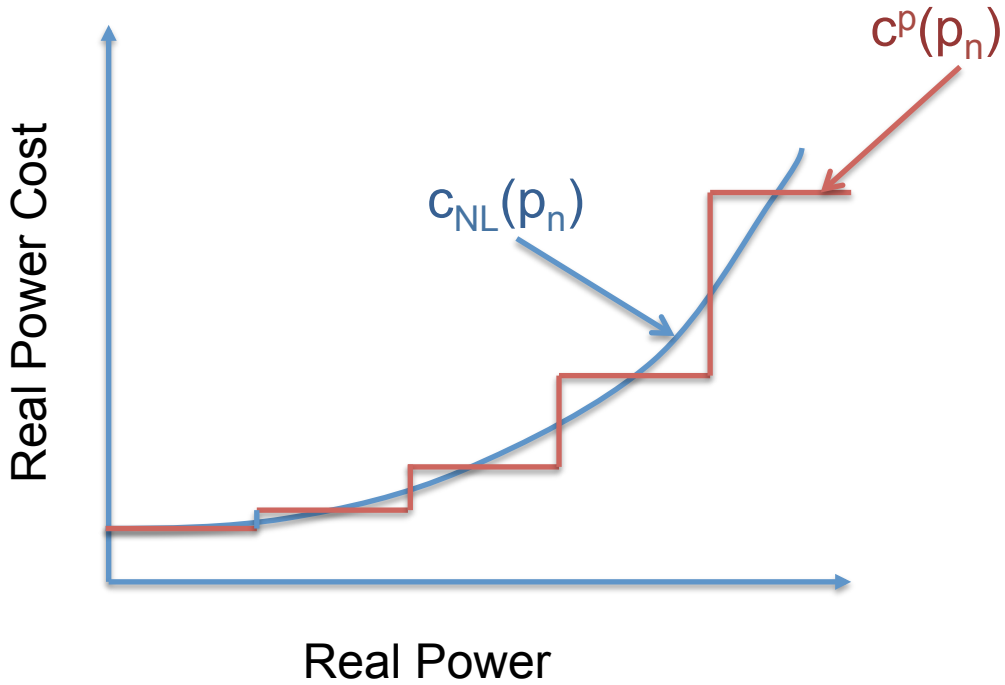
Real Power Cost



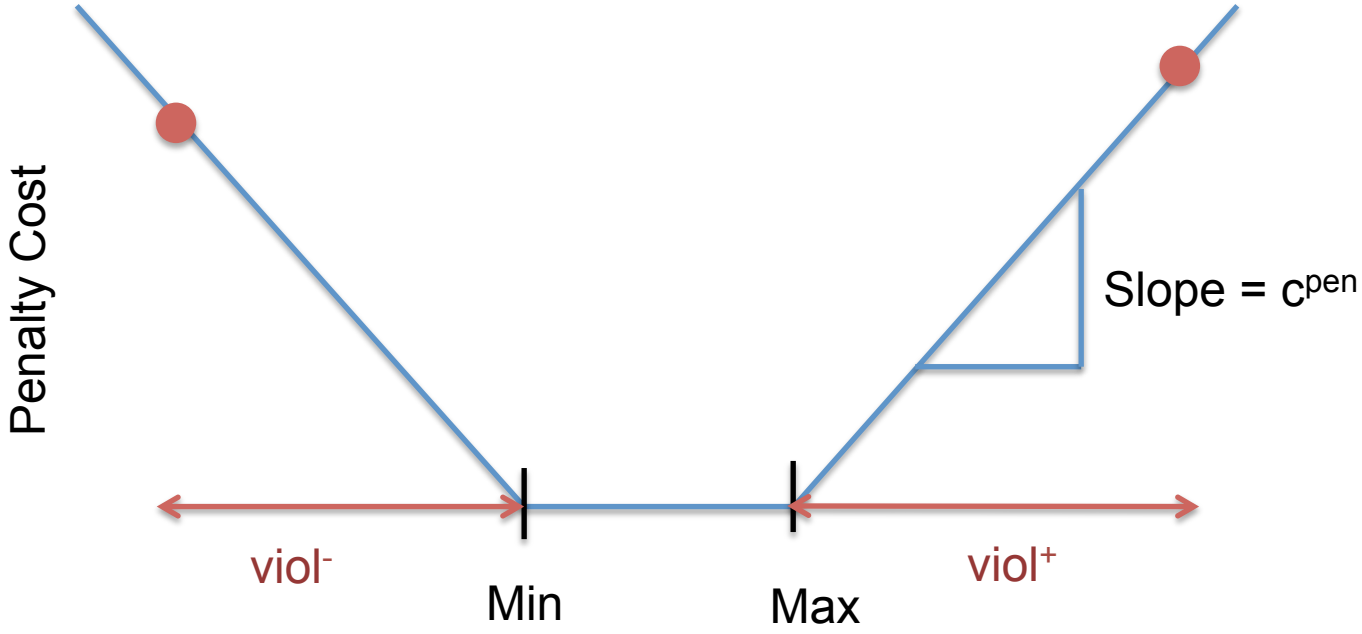
ACOPF I-V Formulation: Linear Real Power Step Function

Cost = Real Power Cost + Reactive Power Cost + Penalty Cost

Real Power Cost



ACOPF I-V Formulation: Penalty Cost



$viol = viol^- + viol^+$

Penalty cost = $c^{pen}(v^{viol}+p^{viol}+q^{viol})$

ACOPF I-V Formulation: Linear Constraints

Minimize $\sum_n c^p_n (p_n) + c^q_n (q_n) + c^{pen}(v_{viol} + p_{viol} + q_{viol})$

Subject to

Current-Voltage Relationship of Lines:

$$i^r_{nm} = g_{nm}(v^r_n - v^r_m) - b_{nm}(v^j_n - v^j_m) \text{ "real" part}$$

$$i^j_{nm} = b_{nm}(v^r_n - v^r_m) + g_{nm}(v^j_n - v^j_m) \text{ "imaginary" part}$$

Definition of current at a bus:

$$i^r_n = \sum_m i^r_{nm} \qquad \text{"real" part}$$

$$i^j_n = \sum_m i^j_{nm} \qquad \text{"imaginary" part}$$

Linear Constraints

ACOPF I-V Formulation

Minimize $\sum_n c_n^p(p_n) + c_n^q(q_n) + c^{pen}(v_{n,viol} + p_{n,viol} + q_{n,viol})$

Subject to:

$i_{nm}^r = g_{nm}(v_n^r - v_m^r) - b_{nm}(v_n^j - v_m^j)$	"real" line current definition
$i_{nm}^j = b_{nm}(v_n^r - v_m^r) + g_{nm}(v_n^j - v_m^j)$	"imag" line current definition
$i_n^r = \sum_m i_{nm}^r$	"real" bus current definition
$i_n^j = \sum_m i_{nm}^j$	"imag" line current definition

Linear Constraints

$(v_n^{\min})^2 - v_{n,viol}^- \le (v_n^r)^2 + (v_n^j)^2 \le (v_n^{\max})^2 + v_{n,viol}^+$	min/max voltage limits
$(i_{nm}^r)^2 + (i_{nm}^j)^2 \le (i_{nm}^{\max})^2$	max current limits
$p_n^{gen} - p_n^{dem} = v_n^r i_n^r + v_n^j i_n^j$	real power definition
$q_n^{gen} - q_n^{dem} = v_n^j i_n^r - v_n^r i_n^j$	reactive power definition
$p_n^{\min} - p_{n,viol}^- \le p_n^{gen} - p_n^{dem} \le p_n^{\max} + p_{n,viol}^+$	min/max real power limits
$q_n^{\min} - q_{n,viol}^- \le q_n^{gen} - q_n^{dem} \le q_n^{\max} + q_{n,viol}^+$	min/max reactive power limits

Nonlinear Constraints

ACOPF I-V Formulation: Nonlinear Constraints

<u>Convex Constraints</u>:

$$\left(v_n^r\right)^2 + \left(v_n^j\right)^2 \le \left(v_n^{\max}\right)^2 + v_{n,viol}^+$$

$$\left(i_{nm}^r\right)^2 + \left(i_{nm}^j\right)^2 \le \left(i_{nm}^{\max}\right)^2$$

<u>Nonconvex Constraints</u>:

$$\left(v_n^{\min}\right)^2 - v_{n,viol}^- \le \left(v_n^r\right)^2 + \left(v_n^j\right)^2$$

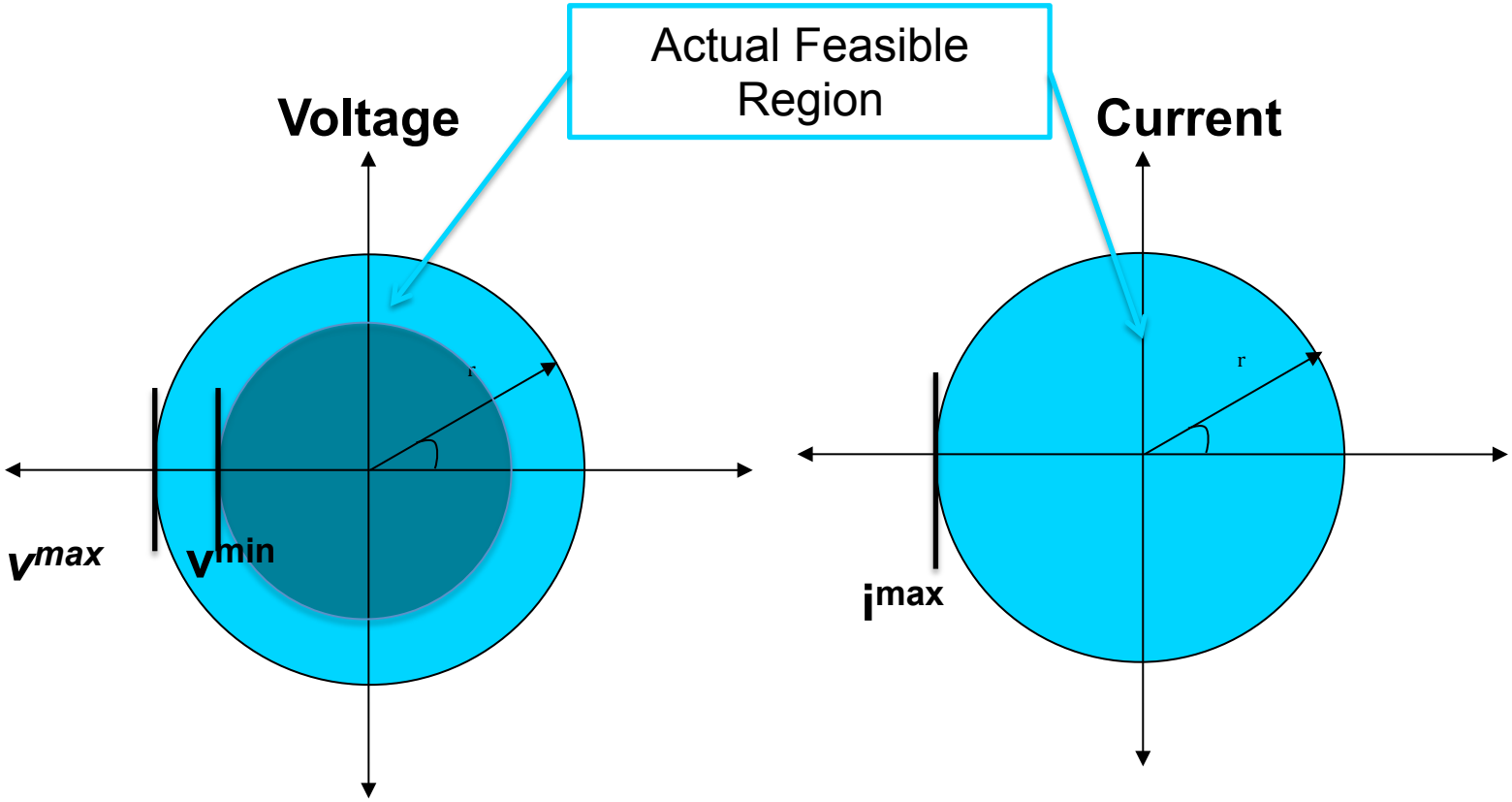
$$p_n^{gen} - p_n^{dem} = v_n^r i_n^r + v_n^j i_n^j$$

$$q_n^{gen} - q_n^{dem} = v_n^j i_n^r - v_n^r i_n^j$$

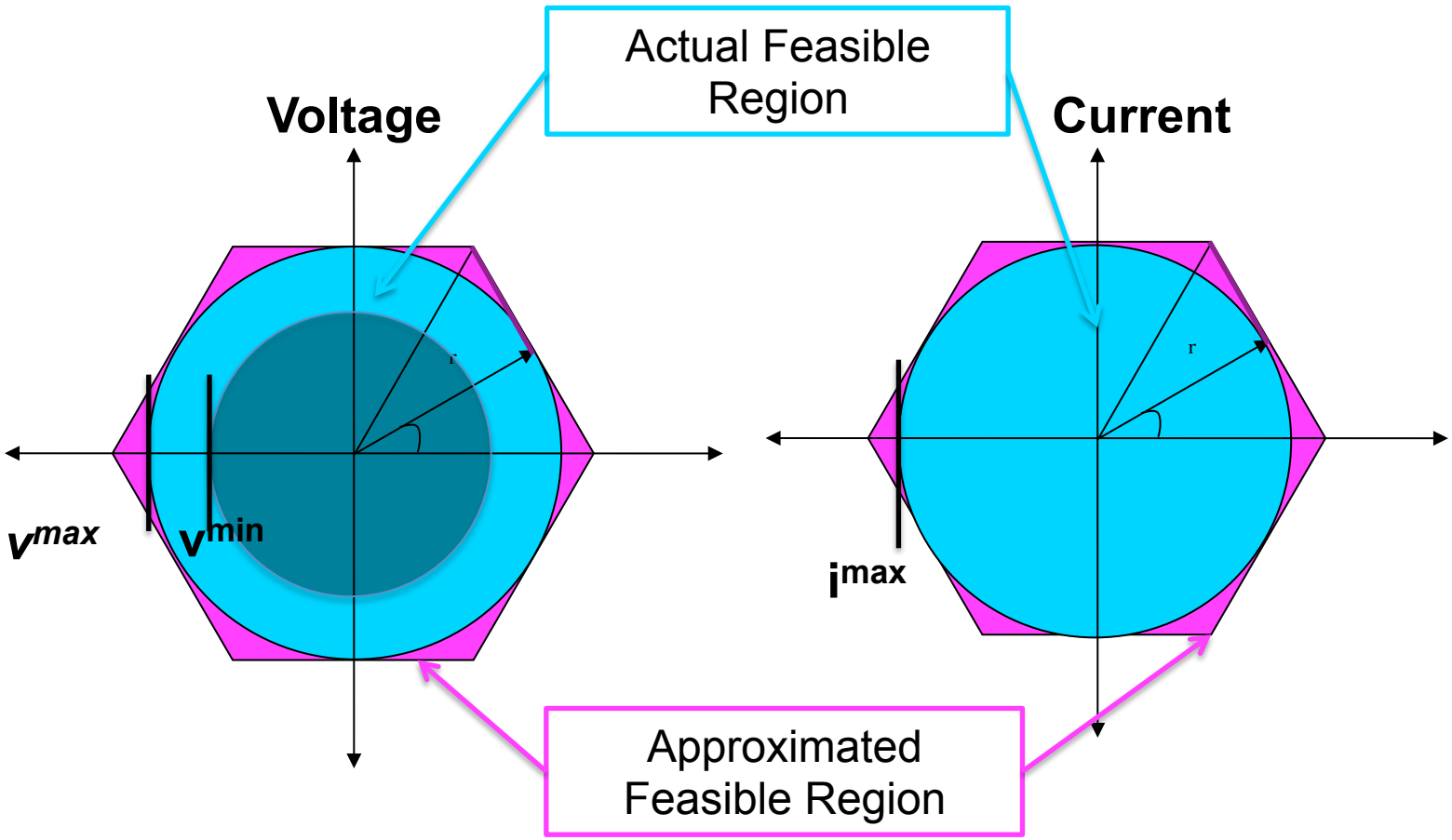
Voltage & Current Constraints

$$\left(v_n^{\min}\right)^2 \le \left(v_n^r\right)^2 + \left(v_n^j\right)^2 \le \left(v_n^{\max}\right)^2$$

$$\left(i_{nm}^r\right)^2 + \left(i_{nm}^j\right)^2 \le \left(i_{nm}^{\max}\right)^2$$

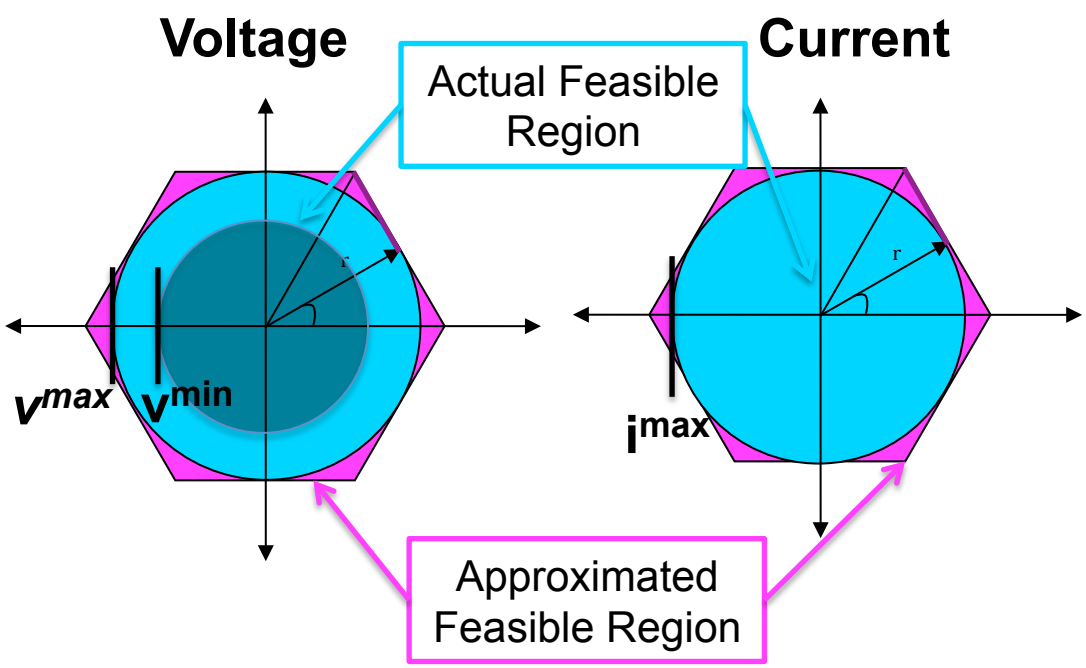


Linearizing Voltage & Current Constraints: Initial Approximations

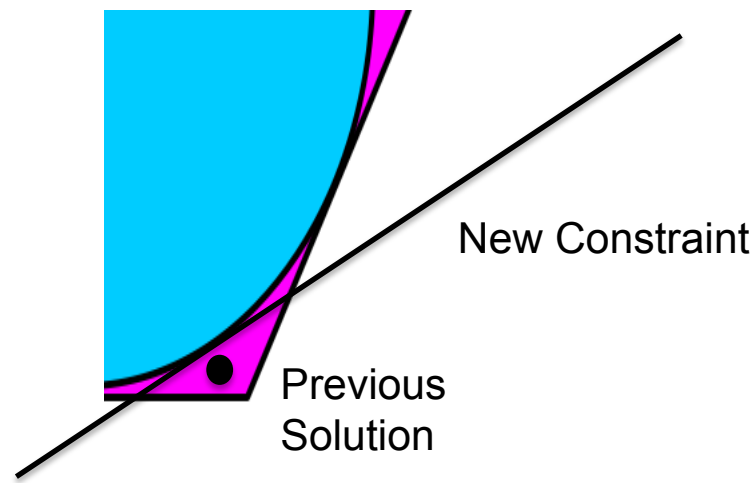


Linearizing Voltage & Current Constraints

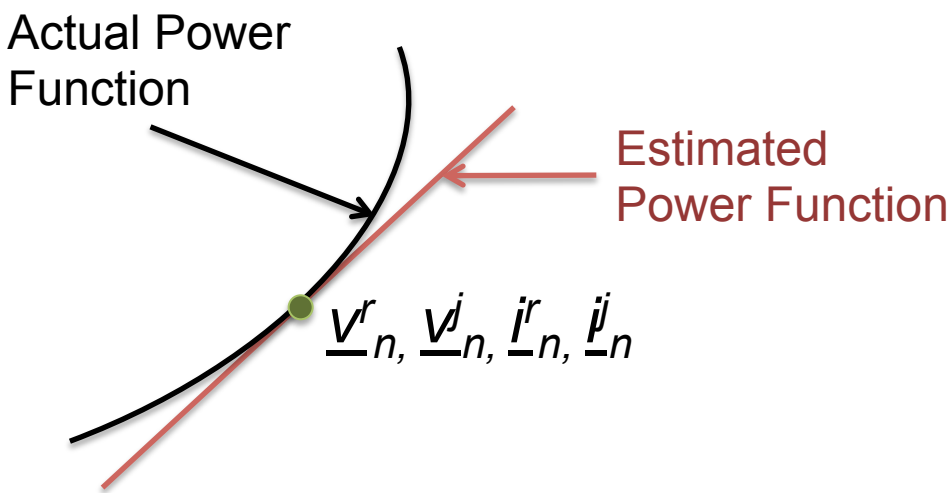
Initial Approximation



Iterative Cuts



Power Approximation:
1st Order Taylor Series Approximation



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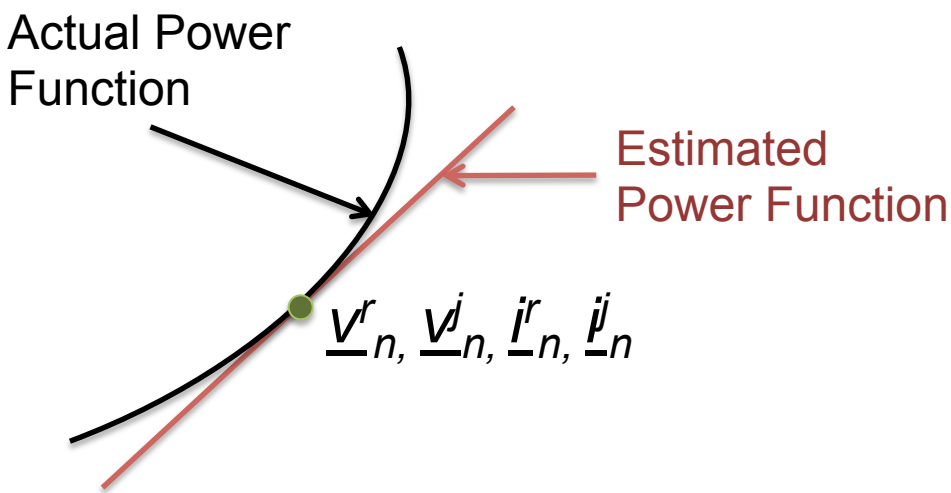
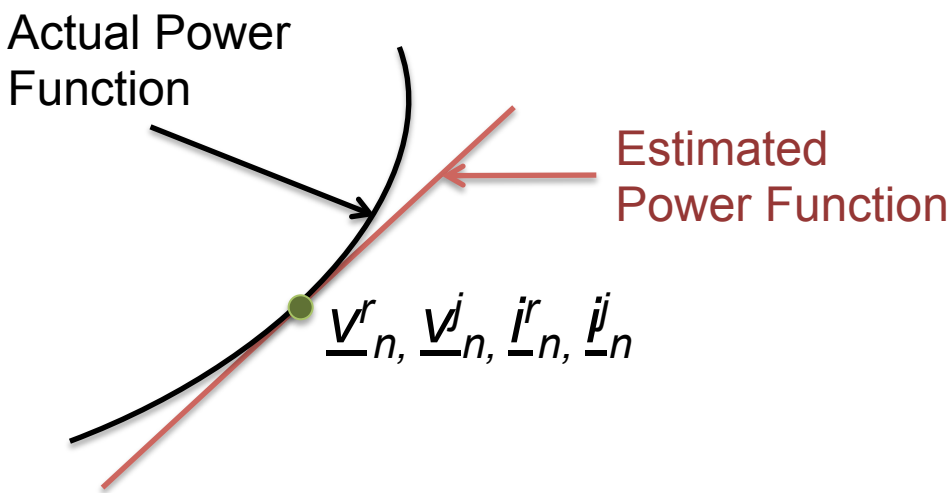


$$p_n^{gen} - p_n^{dem} = \underline{v}_n^r i_n^r + v_n^j \underline{i}_n^j + v_n^r \underline{i}_n^r + \underline{v}_n^j i_n^j - \left(\underline{v}_n^r \underline{i}_n^r + \underline{v}_n^j \underline{i}_n^j\right)$$

$$q_n^{gen} - q_n^{dem} = \underline{v}_n^j i_n^r - v_n^r \underline{i}_n^j - v_n^r \underline{i}_n^j + \underline{v}_n^j i_n^r + \left(\underline{v}_n^j \underline{i}_n^r - \underline{v}_n^r \underline{i}_n^j\right)$$

Power Approximation:
1st Order Taylor Series Approximation



$$p_n^{gen} - p_n^{dem} = \underline{v_n^r}\, i_n^r + v_n^j\, \underline{i_n^j} + v_n^r\, \underline{i_n^r} + \underline{v_n^j}\, i_n^j - \left(\underline{v_n^r}\, \underline{i_n^r} + \underline{v_n^j}\, \underline{i_n^j}\right)$$

$$q_n^{gen} - q_n^{dem} = \underline{v_n^j}\, i_n^r - v_n^r\, \underline{i_n^j} - v_n^r\, \underline{i_n^j} + \underline{v_n^j}\, i_n^r + \left(\underline{v_n^j}\, \underline{i_n^r} - \underline{v_n^r}\, \underline{i_n^j}\right)$$

$$p_n^{act} - p_n^{est} = \left(v_n^r - \underline{v_n^r}\right)\left(i_n^r - \underline{i_n^r}\right) + \left(v_n^j - \underline{v_n^j}\right)\left(i_n^j - \underline{i_n^j}\right)$$

$$q_n^{act} - q_n^{est} = \left(v_n^j - \underline{v_n^j}\right)\left(i_n^r - \underline{i_n^r}\right) + \left(v_n^r - \underline{v_n^r}\right)\left(i_n^j - \underline{i_n^j}\right)$$

Linearized ACOPF Formulation

<u>Linearized Real and Reactive Power Constraints:</u>

$$p_n^{gen} - p_n^{dem} = \underline{v_n^r} i_n^r + v_n^j \underline{i_n^j} + v_n^r \underline{i_n^r} + \underline{v_n^j} i_n^j - \left(\underline{v_n^r} \underline{i_n^r} + \underline{v_n^j} \underline{i_n^j} \right)$$

$$q_n^{gen} - q_n^{dem} = \underline{v_n^j} i_n^r - v_n^r \underline{i_n^j} - v_n^r \underline{i_n^j} + \underline{v_n^j} i_n^r + \left(\underline{v_n^j} \underline{i_n^r} - \underline{v_n^r} \underline{i_n^j} \right)$$

Linearized ACOPF Formulation

Linearized Real and Reactive Power Constraints:

$$p_n^{gen} - p_n^{dem} = \underline{v_n^r} i_n^r + v_n^j \underline{i_n^j} + v_n^r \underline{i_n^r} + \underline{v_n^j} i_n^j - \left(\underline{v_n^r} \underline{i_n^r} + \underline{v_n^j} \underline{i_n^j} \right)$$

$$q_n^{gen} - q_n^{dem} = \underline{v_n^j} i_n^r - v_n^r \underline{i_n^j} - v_n^r \underline{i_n^j} + \underline{v_n^j} i_n^r + \left(\underline{v_n^j} \underline{i_n^r} - \underline{v_n^r} \underline{i_n^j} \right)$$

Initial Voltage & Current Constraints:

$$v_n^r \cos\left(2\pi a / L \right) + v_n^j \sin\left(2\pi a / L \right) \le v_n^{\max}$$

$$i_{nm}^r \cos\left(2\pi a / L \right) + i_{nm}^j \sin\left(2\pi a / L \right) \le i_{nm}^{\max}$$

L = # of cuts;
a= 1, 2, 3, …, L

Linearized ACOPF Formulation

Linearized Real and Reactive Power Constraints:

$$p_n^{gen} - p_n^{dem} = \underline{v_n^r} i_n^r + v_n^j \underline{i_n^j} + v_n^r \underline{i_n^r} + \underline{v_n^j} i_n^j - \left(\underline{v_n^r}\,\underline{i_n^r} + \underline{v_n^j}\,\underline{i_n^j} \right)$$

$$q_n^{gen} - q_n^{dem} = \underline{v_n^j} i_n^r - v_n^r \underline{i_n^j} - v_n^r \underline{i_n^j} + \underline{v_n^j} i_n^r + \left(\underline{v_n^j}\,\underline{i_n^r} - \underline{v_n^r}\,\underline{i_n^j} \right)$$

Initial Voltage & Current Constraints:

$$v_n^r \cos\left(2\pi a / L \right) + v_n^j \sin\left(2\pi a / L \right) \le v_n^{\max}$$

$$i_{nm}^r \cos\left(2\pi a / L \right) + i_{nm}^j \sin\left(2\pi a / L \right) \le i_{nm}^{\max}$$

L = # of cuts;
a= 1, 2, 3, …, L

Iterative Voltage and Current Cuts:

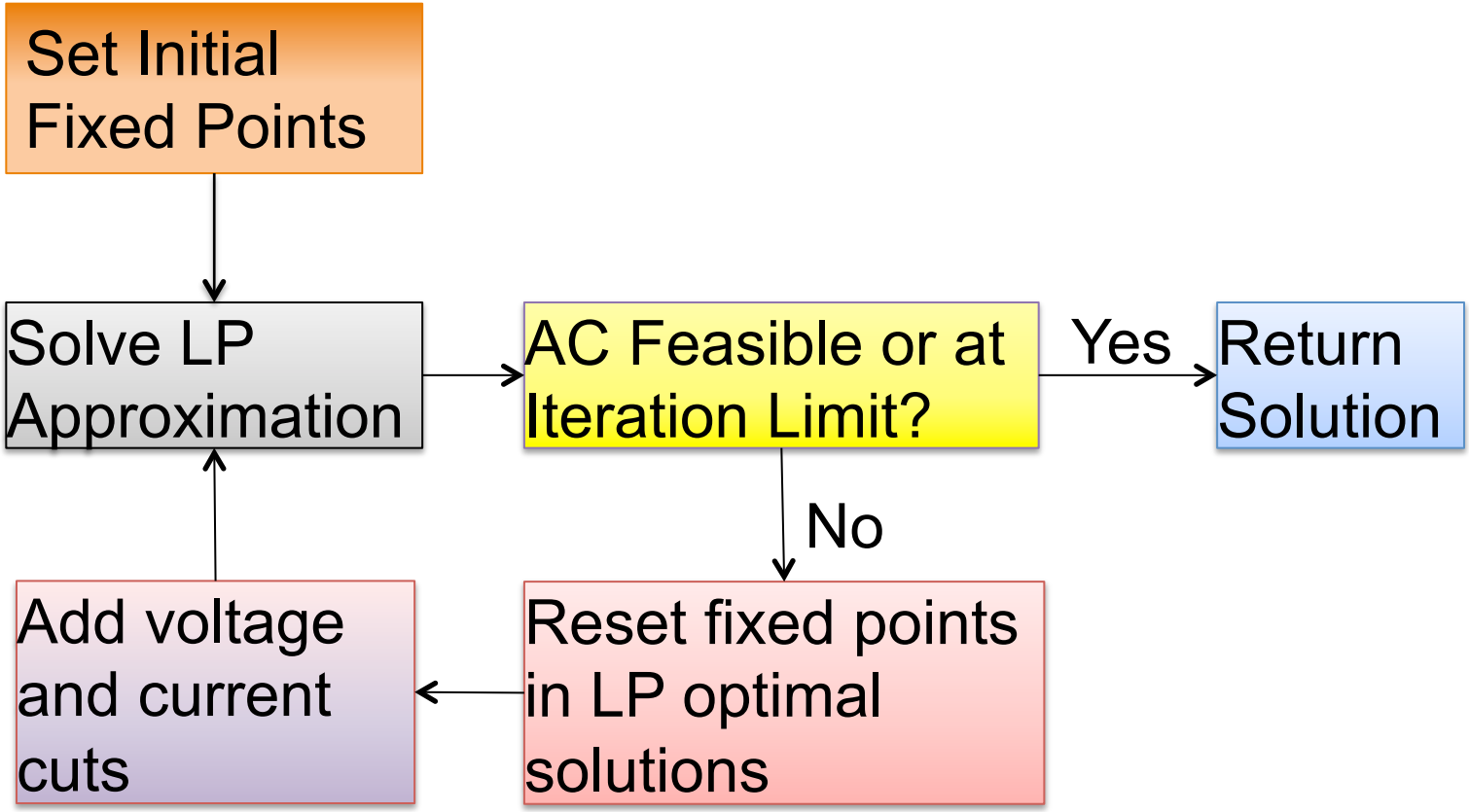
$$v_n^r \left(\underline{v_n^{rt}} \Big/ \left| \underline{v_n} \right| \right) + v_n^j \left(\underline{v_n^{jt}} \Big/ \left| \underline{v_n} \right| \right) \le v_n^{\max}$$

$$i_{nm}^r \left(\underline{i_{nm}^{rt}} \Big/ \left| \underline{i_{nm}} \right| \right) + v_{nm}^j \left(\underline{i_{nm}^{jt}} \Big/ \left| \underline{i_{nm}} \right| \right) \le i_{nm}^{\max}$$

t = 1, 2, .., # of iterations

$\underline{v_n}^{rt}$, $\underline{v_n}^{jt}$ is the fixed point at iteration *t*

LP-ACOPF Procedure



Testing Data

Data Sets: IEEE Test Systems

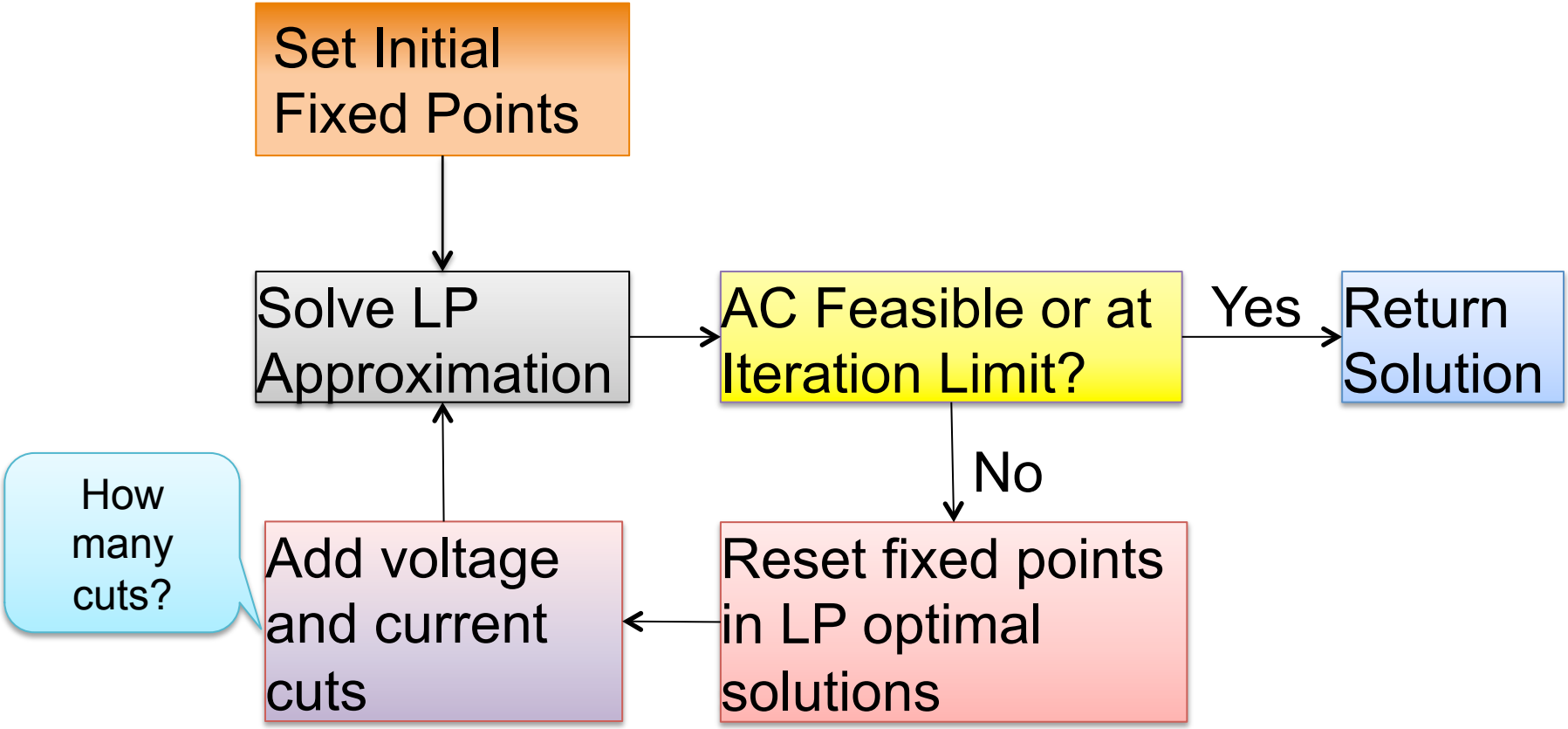
Buses	Lines	Generators		Demand	V^{max}	V^{min}
		Number	Total Capacity			
14	20	5	7.72	2.59	1.06	0.94
30	41	6	326.80	42.42	1.10	0.94
57	80	7	326.78	235.26	1.06	0.94
118	186	54	99.66	42.42	1.06	0.94

Current Constraints: Created to be binding on some lines

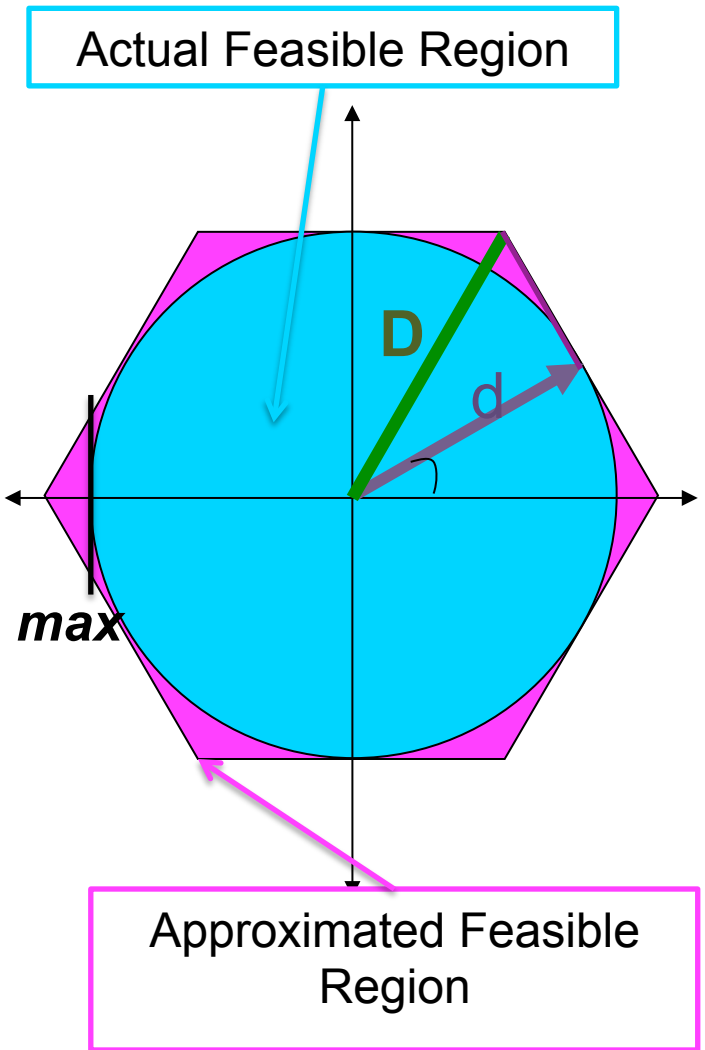
# Buses	14	30	57	118
Tightly Constrained Max Current	0.2264	0.3092	1.4027	0.9294
Loosely Constrained Max Current	0.6246	0.3162	1.4168	2.7536

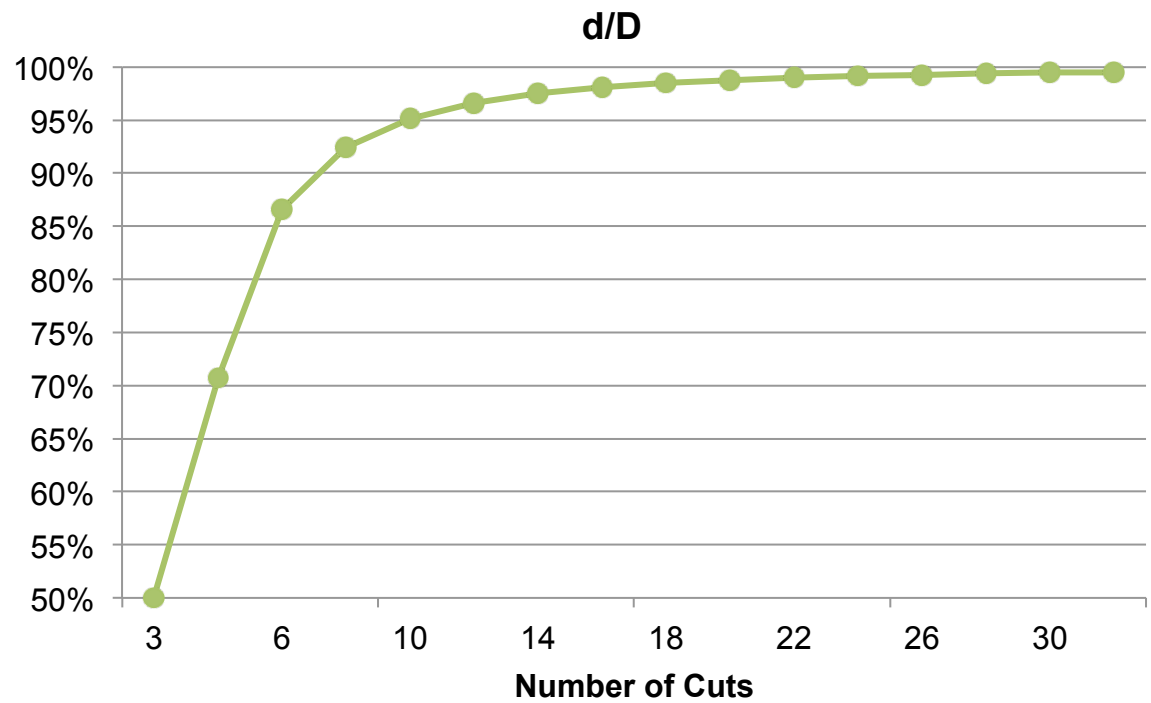
Linear Solver: Gurobi 4.0
Nonlinear Solver: IPOPT 3.8

LP-ACOPF Procedure

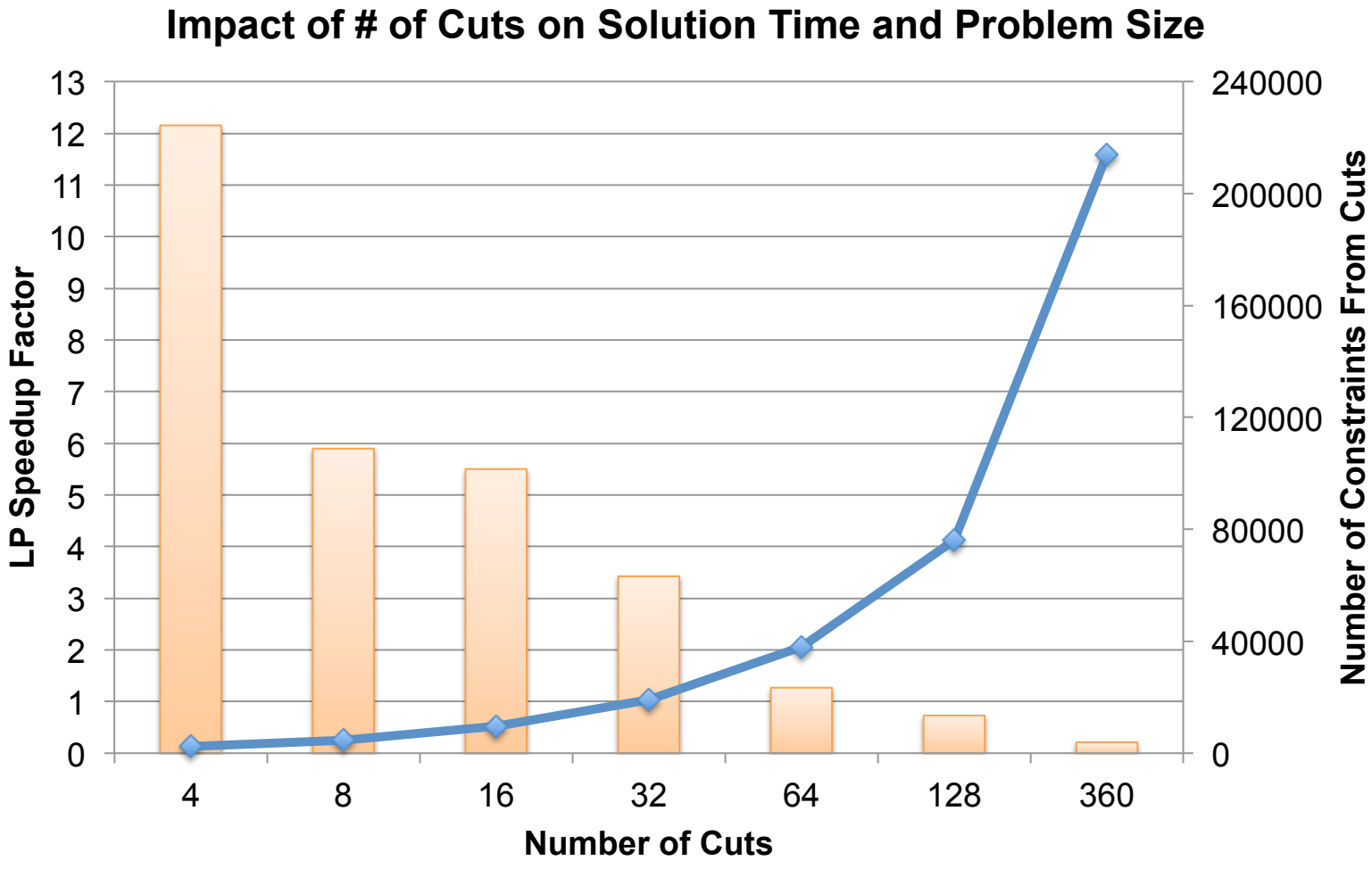


Voltage and Current Approximation: Outer Cuts



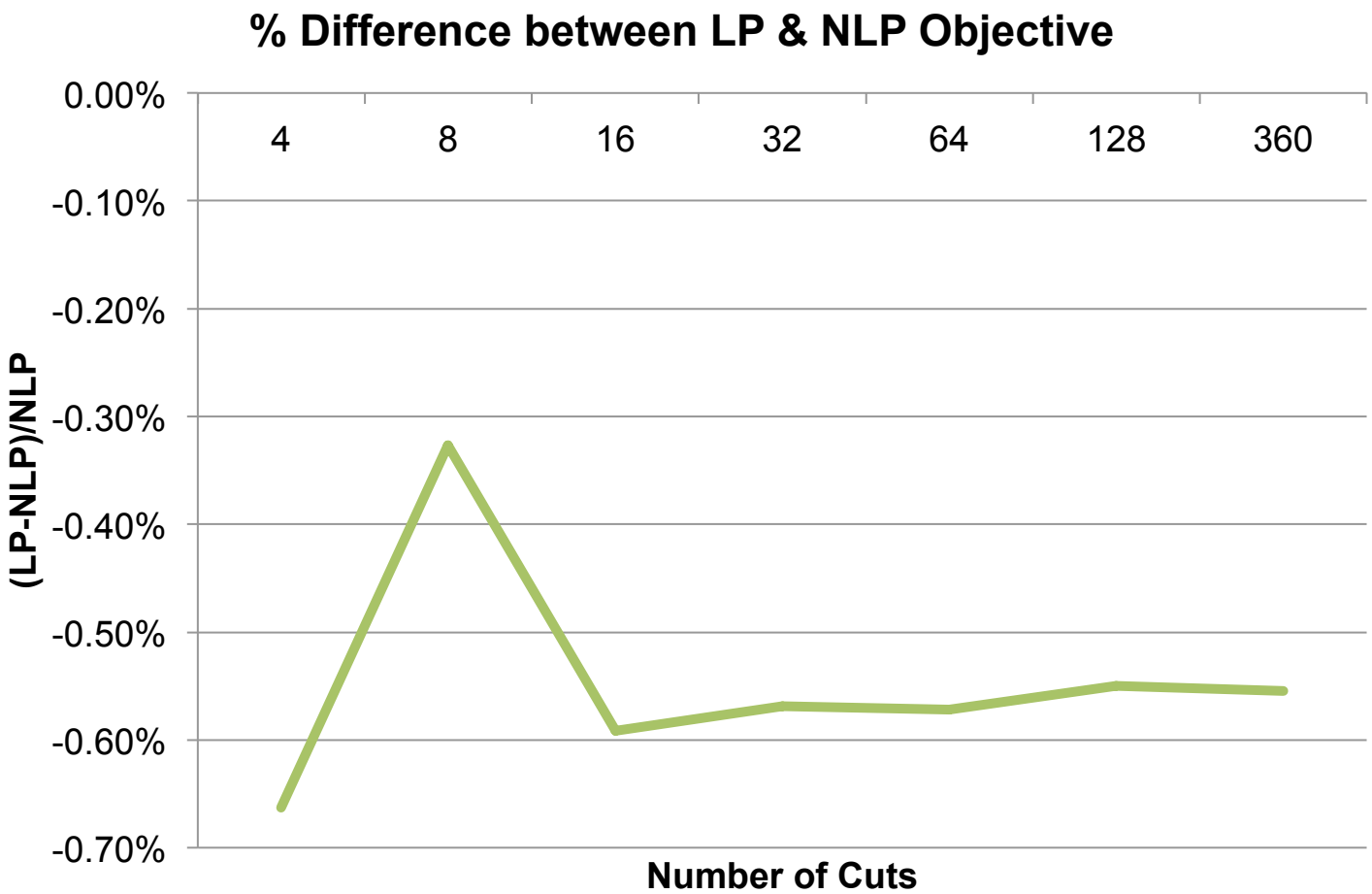


Voltage and Current Approximation Outer Cuts



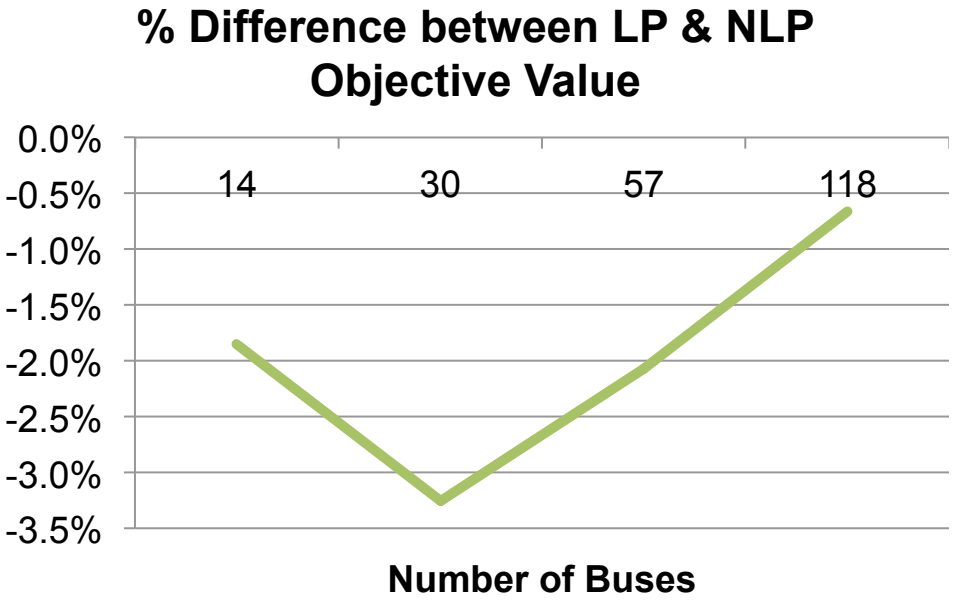
Test Case: IEEE 118 Buses, Current Tightly Constrained

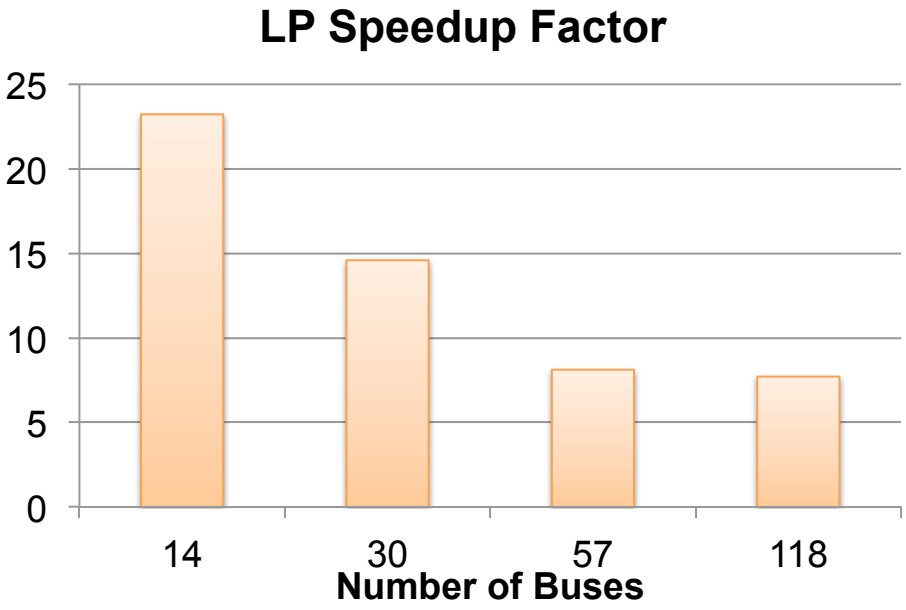
Voltage and Current Approximation Outer Cuts



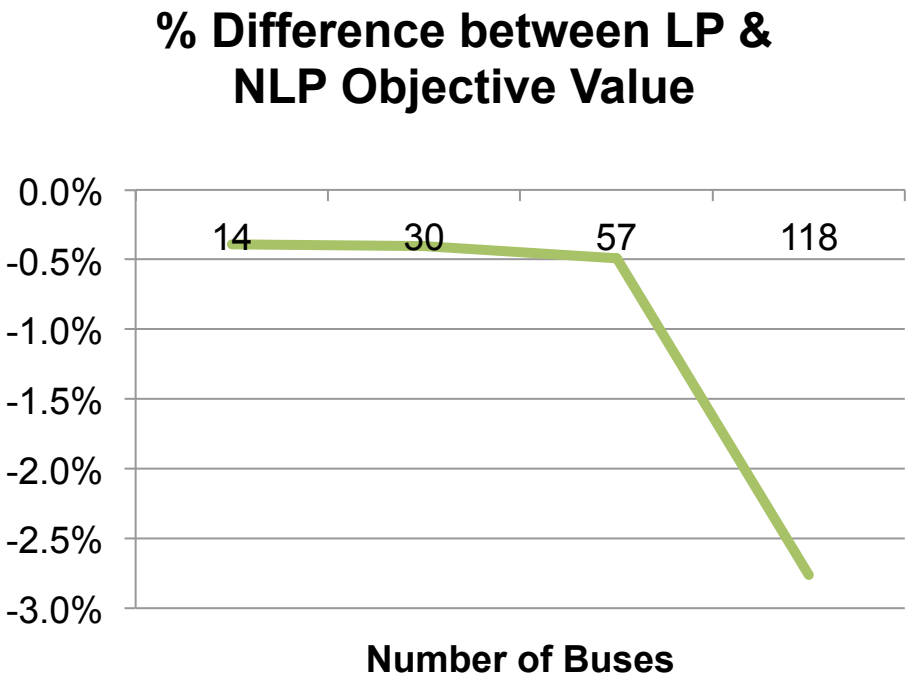
Test Case: IEEE 118 Buses, Current Tightly Constrained

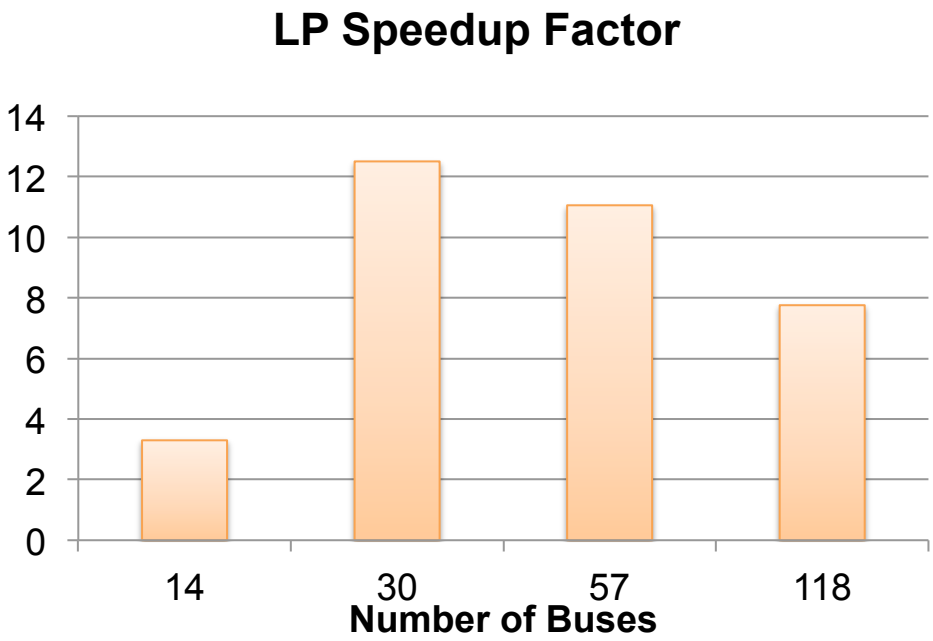
Results: Current Constrained Tightly





Results: Current Constrained Loosely





Transmission Switching Formulation: Modifications

z_{nm} = 1 if line *n-m* is switched off (no current flow); 0 otherwise

Replace

$$i^r_{nm} = g_{nm}(v^r_n - v^r_m) - b_{nm}(v^j_n - v^j_m)$$

$$i^j_{nm} = b_{nm}(v^r_n - v^r_m) + g_{nm}(v^j_n - v^j_m)$$

$$i^r_{nm}\cos\left(2\pi a/L\right) + i^j_{nm}\sin\left(2\pi a/L\right) \le i^{\max}_{nm}$$

With

$$i^r_{nm} \le g_{nm}(v^r_n - v^r_m) - b_{nm}(v^j_n - v^j_m) + z_{nm}M$$

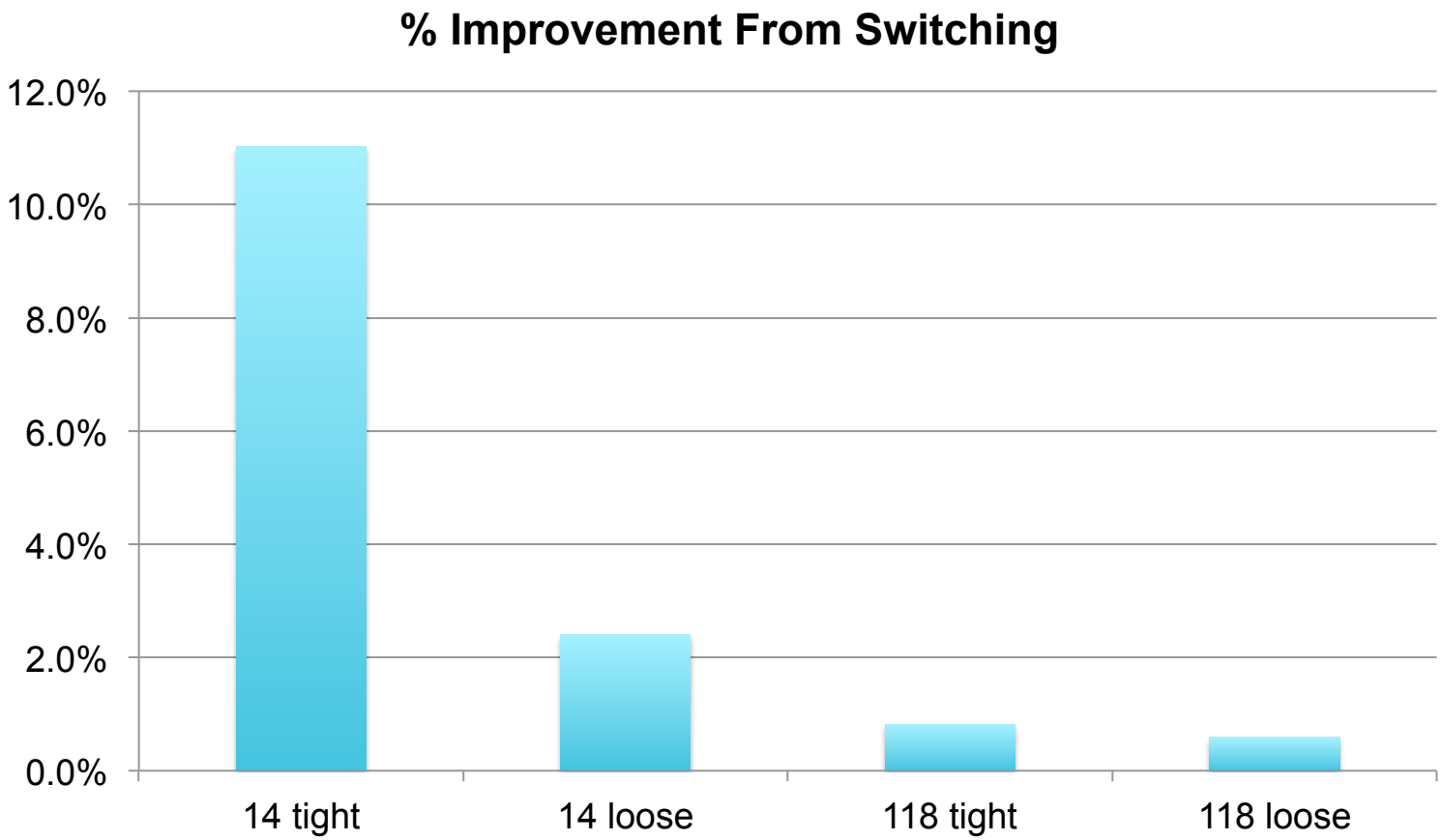
$$i^r_{nm} \ge g_{nm}(v^r_n - v^r_m) - b_{nm}(v^j_n - v^j_m) - z_{nm}M$$

$$i^j_{nm} \le b_{nm}(v^r_n - v^r_m) + g_{nm}(v^j_n - v^j_m) + z_{nm}M$$

$$i^j_{nm} \ge b_{nm}(v^r_n - v^r_m) + g_{nm}(v^j_n - v^j_m) - z_{nm}M$$

$$i^r_{nm}\cos\left(2\pi a/L\right) + i^j_{nm}\sin\left(2\pi a/L\right) \le \left(1 - z_{nm}\right) i^{\max}_{nm}$$

Transmission Switching Results



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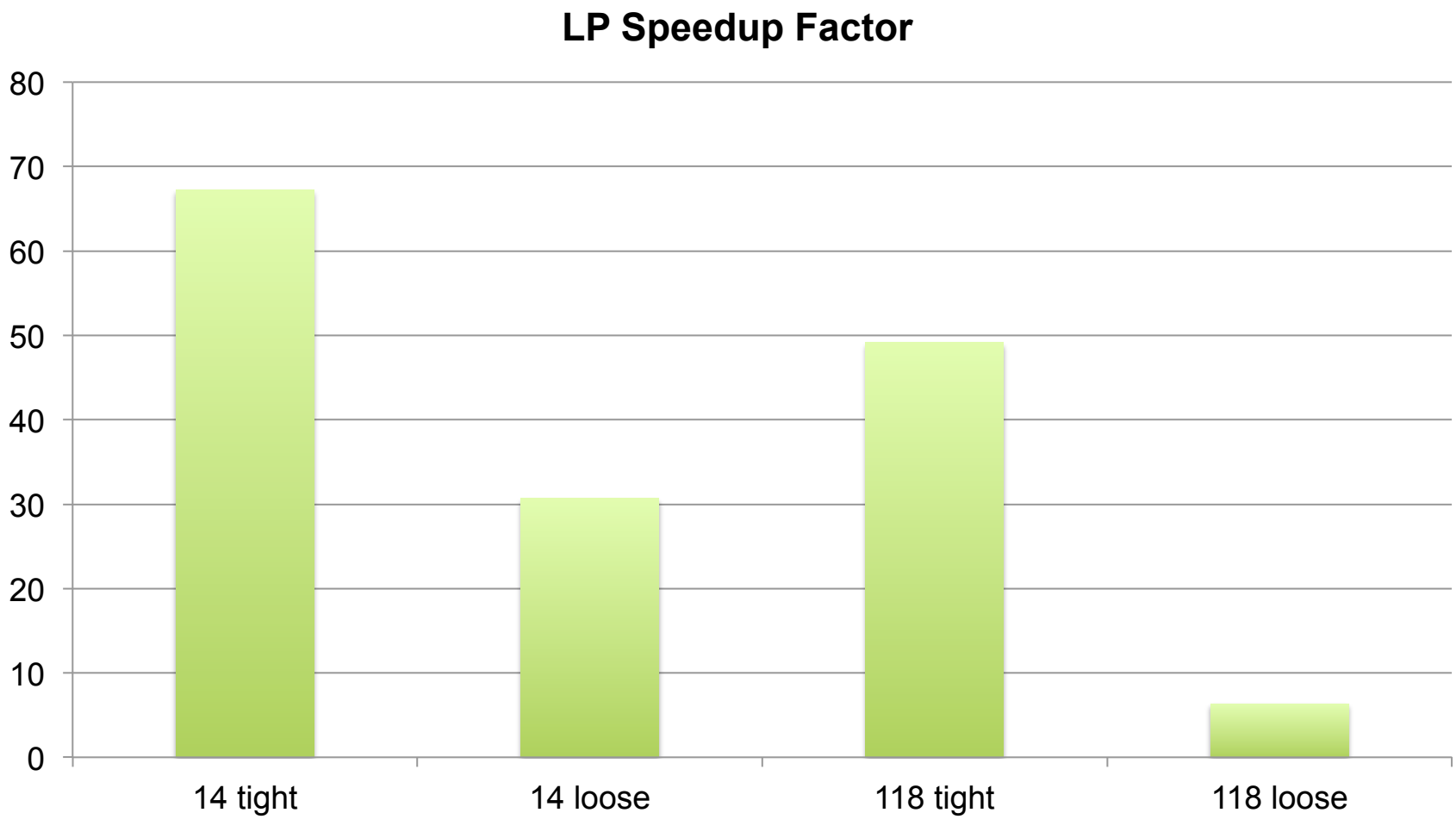
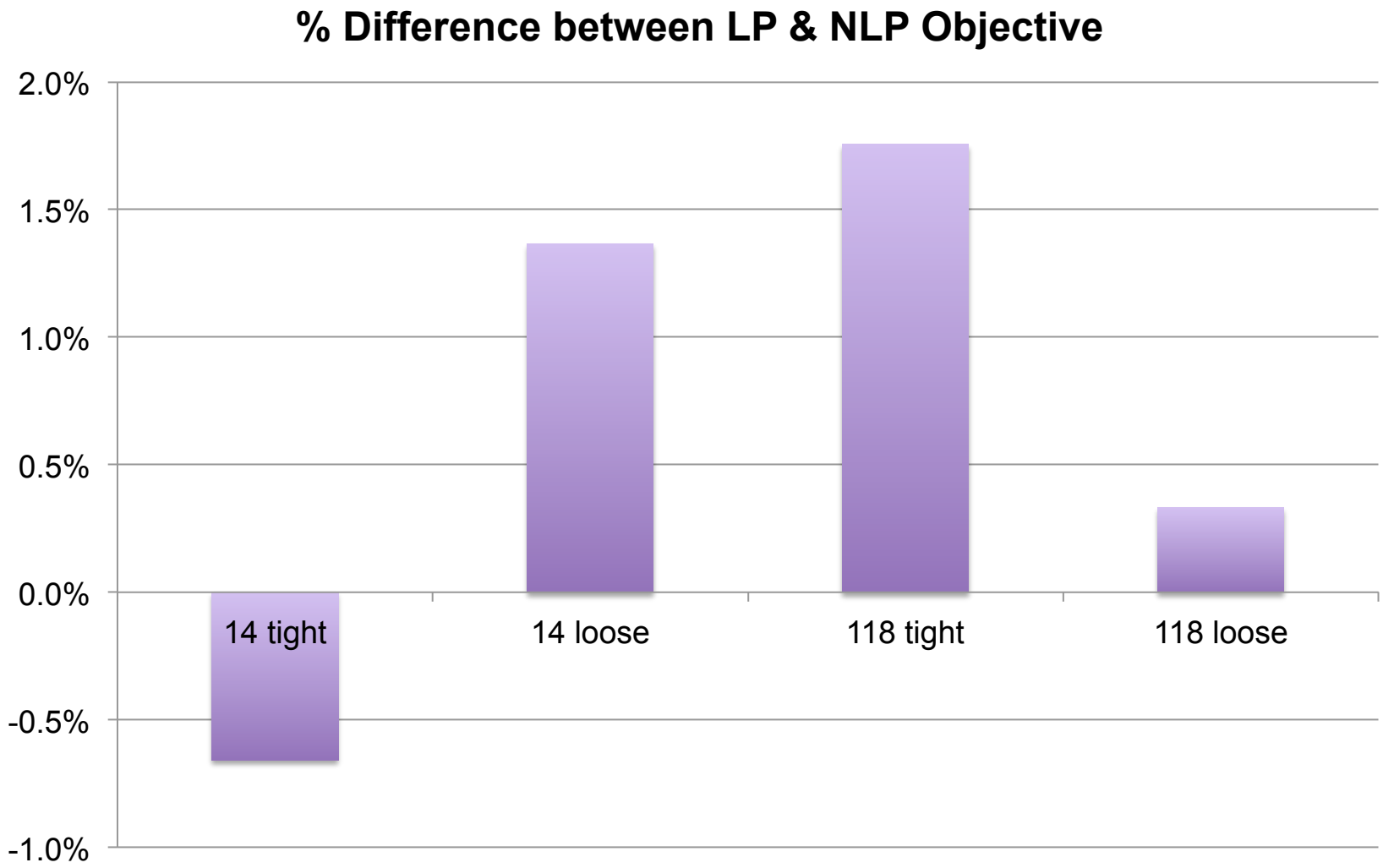


Transmission Switching Results



Conclusions

- I-V linear method solves the ACOPF with cost within 4% of nonlinear method
- I-V linear method is generally 5-10 times faster than the nonlinear method
- Transmission switching can lower cost significantly

References

- Paula Lipka, Richard O'Neill, and Shmuel Oren. "Developing Line Current Magnitude Constraints for IEEE Test Problems," FERC StaffPaper, 2012.
- Mehrdad Pirnia, Richard P. O'Neill, Paula A. Lipka, and Clay Campaigne. "A computational study of linear approximations to the convex constraints in the iterative linear IV-ACOPF Formulation," FERC Staff Paper, 2012.
- E.B. Fisher, R.P. O'Neill, and M.C. Ferris. "Optimal transmission switching," *IEEE Trans. Power Syst*, vol. 23, no. 3, pp. 1346-1355, Aug. 2008.
- K.W. Hedman, R.P. O'Neill, E.B. Fisher, and S.S. Oren, "Optimal transmission switching – sensitivity analysis and extensions," *IEEE Trans. Power Syst,* vol. 23, no. 3, pp. 1469-1479, Aug. 2008.
- K.W. Hedman, R.P. O'Neill, E.B. Fisher, and S.S. Oren, "Optimal transmission switching with contingency analysis," *IEEE Trans. Power Syst,* vol. 24, no. 3, pp. 1577-1586, Aug. 2009.
- T. Potluri and K. W. Hedman, "Impacts of Topology Control on the ACOPF," in *IEEE PES General Meeting 2011*, San Diego, CA, July 2012.

Questions?

Images

- https://www.midwestiso.org/marketsoperations/realtimemarketdata/pages/lmpcontourmap.aspx
- http://www.sciencedirect.com/science/article/pii/S0142061511002705\
- http://www.npr.org/2013/01/11/169153322/coal-loses-crown-as-king-of-power-generation
- http://techfest.youngsvilla.com/Electrical/ncer.aspx
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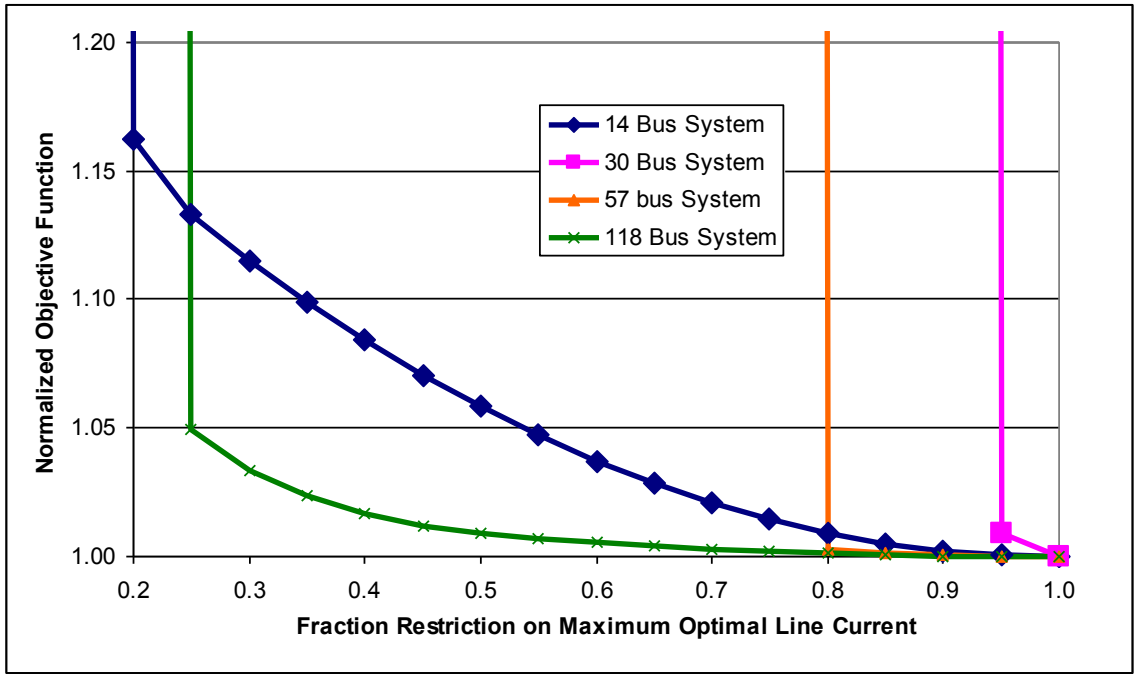
- http://www.google.com/imgres?um=1&client=firefox-a&hs=eg3&rls=org.mozilla:en-US:official&hl=en&biw=967&bih=833&tbm=isch&tbnid=klWqOwPngtm4UM:&imgrefurl=http://www.thesaleshuddle.com/huddledc1/&docid=kdHI4HSFN3UbwM&imgurl=http://www.thesaleshuddle.com/wp-content/uploads/2012/01/WashingtonDC_NightSkyline.jpg&

ISO/RTO Approach

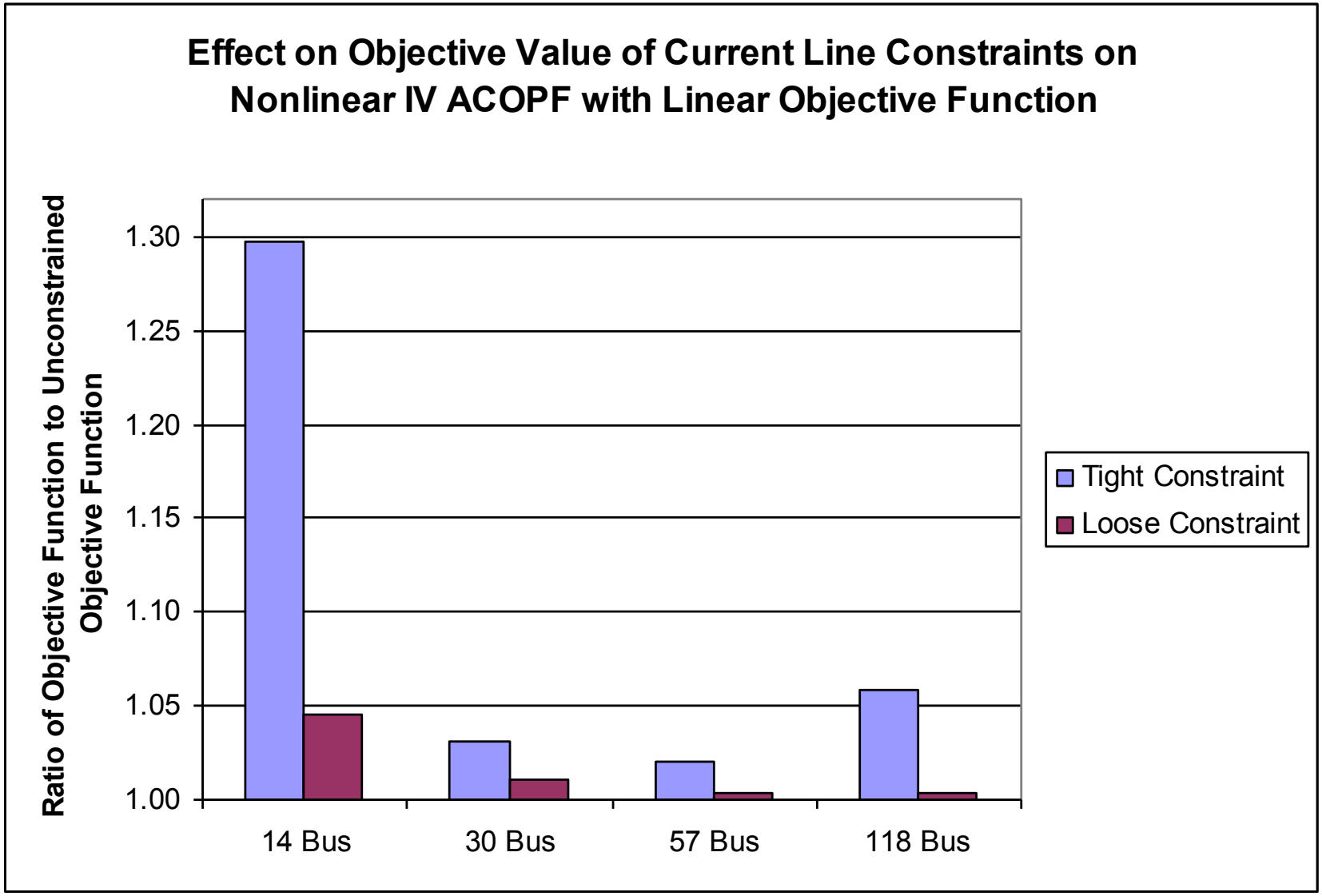
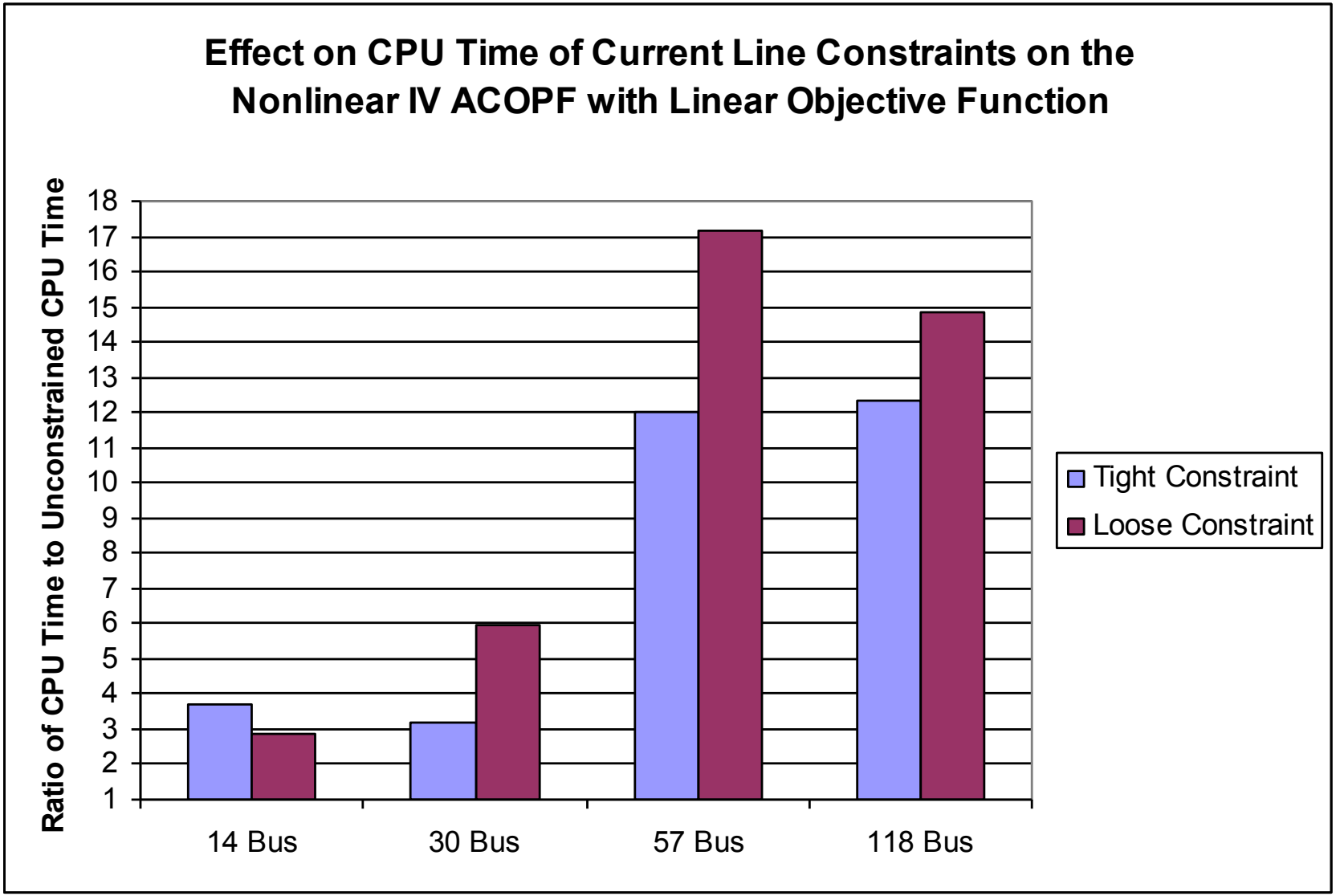
- Approach: Solve DC Approximation. If answer is AC-infeasible, tighten line limits.
- DC Approximation Assumptions:
  - Voltage Magnitudes Fixed to 1 p.u.
  - Resistance << Reactance
  - Small Angle Approximation
- Problems:
  - No guarantee of AC feasibility
  - If AC feasible, suboptimal!

Creating Current Constraints

- Solved ACOPF (w/o current constraints)
- i^{max*} = maximum {optimal line currents}
- $|i_{nmk}| \leq (f)\ i^{max*}\ ,\ 0 \leq f \leq 1$



Effect of Current Constraint Levels



Effect of Current Constraint Levels