
Uncertainty of renewable power, stochastic unit commitment, and implication on DA market

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Disclaimer

- The views and opinions expressed in this presentation are those of the author and do not necessarily reflect the official opinion and position of ABB.
Objective

- Not to propose any specific solution method for solving stochastic unit commitment
- Discuss stochastic optimization in unit commitment under uncertainty caused by high level of renewable power

Issues to examine

- Uncertainty characterization, problem dimension, data requirement
- Choice of objectives and constraints, and risk attitude
- Two stage or multi-stage formulation?
- Evaluation of stochastic solutions
- Implications of stochastic unit commitment on DA market
Deterministic SCUC

- Decisions to make
  - Commitment status and dispatch level of dispatchable resources for each of time interval (e.g. 24) intervals

- Loads are assumed to be known at each node over the scheduling horizon

- Network constraints under normal or contingency conditions

\[
\begin{align*}
\min_{p_{j,k}} \sum_{j \in \mathcal{N}} \sum_{k \in \mathcal{T}} c_j^d + c_j^g + c_j^f \\
\text{s.t.} \sum_{j \in \mathcal{N}} p_{j,k} + \sum_{j \in \mathcal{N}_{\text{wind}}} \mathbb{E}[p_{j,k}^{\text{wind}}] &= D_k, \ k \in \mathcal{T} \\
\sum_{j \in \mathcal{N}} p_{j,k} + \sum_{j \in \mathcal{N}_{\text{wind}}} \mathbb{E}[p_{j,k}^{\text{wind}}] &\geq D_k + R_k, \ k \in \mathcal{T} \\
\end{align*}
\]

Source: 2009 ANL Report on Wind Energy Integration

- Network constraints are affected not only by total load but the spatial distribution of load and generations

- If the total load remains unchanged but the spatial distribution changes, the schedule decision are likely to change

- This property limits our ability to aggregate renewable generation scenarios
Wind power forecast is characterized by the cone of uncertainty – smaller uncertainty with shorter lead time.

Aggregation could reduce randomness if level of correlation among wind plants is low.

Aggregation not possible if network constraints are to be considered (location matters).
There are non-software technology to address the stochastic property of wind power.
Wind power uncertainty modeling for SCUC

- Conceptually, the stochastic processes of wind power can be described by joint p.d.f., embodying the marginal and conditional distributions.

- In practice, we have to consider discrete approximation:
  - Number of uncertain parameters $M$ – the number of wind farms
  - Number of stages $T$ – the number of hours in SCUC
  - Number of outcomes of each uncertain parameter $N=3$ (highly underestimated, and rough very approximation)
  - Number of Scenarios $S = (N^M)^T$ (assuming independence between wind speeds at different wind farms)

- The number of scenarios grows exponentially, astronomical for very conservative assumptions of number of wind farms.
### Wind power uncertainty modeling for SCUC

- **Sampling techniques may be used**
- **Even with very, very intelligent and sparse sampling, the number of samples are still astronomical, unless the sampling method has exponentially growing effectiveness**
- **If sampling is used, how can we verify the solution is good? Even if the optimal values are close, are the market dispatch and LMP stable and fair?**

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Decision making under uncertainty

- Existence of uncertainty parameters, such as (load uncertainty, wind power output)
- Decisions have to be made before the outcomes of some or all of the uncertain parameters are revealed
- The outcome of the decisions could be revealed all at one time (single stage) or in steps (multiple stages)
- Decisions that can be changed before becoming history are as good as not made
Decision Making Under Uncertainty
- Two Stage Model

- Stage 1 decision
- Stage 2 decision

- Decision horizon

Min $x, y_1, ..., y_K$
\[ c^T x + \sum_{k=1}^{K} p_k q_k^T y_k \]
\[ s.t. \quad Ax = b \]
\[ T_k x + W_k y_k = h_k, k = 1, ..., K \]

Min $x_1, ..., x_K, y_1, ..., y_K$
\[ \sum_{k=1}^{K} p_k (c^T x_k + q_k^T y_k) \]
\[ s.t. \quad Ax_k = b, k = 1, ..., K \]
\[ T_k x_k + W_k y_k = h_k, k = 1, ..., K \]
\[ x_i = x_{i+1}, i = 1, ..., K - 1 \]
Decision Making Under Uncertainty
- Multi Stage (Four) Model

- Stage 1 decision
- Stage 2 decision
- Stage 3 decision
- Stage 4 decision

- Decision horizon

- Decision BEFORE stochastic outcome is known
- Stochastic outcome is revealed
- Decision AFTER stochastic outcome is known
Two Stage or Multi Stage Model for SCUC?

- The choice is neither arbitrary nor expedient
- Primary considerations
  - stochastic information structure – Do the outcomes of the uncertain parameters reveal themselves in one stage or multiple stages?
  - Finality of decisions – at what point are decisions becomes irrevocable?
- The decision for hour i does not have to be made until hour i-1 when the uncertainty in hour 1 to i-1 are known, and the range of uncertainty for the remaining hours are getting smaller (not getting larger)
- It is neither necessary nor optimal to fix commitment decisions for the entire 24 hours before the first hour begins
- Multi stage SCUC provide better optimal solution due to decision deferral and shirking uncertainty
Stochastic optimization formulation - objectives

- Risk neutral – optimize the expected value of outcomes, need p.d.f.

\[
\min_x \mathbb{E}(F(x, \omega))
\]

- Risk aversion - including variance term to expected value, need p.d.f.

\[
\min_x \mathbb{E}(F(x, \omega)) + k \psi(\text{Var}(F(x, \omega)))
\]

- Extreme risk aversion - optimize outcome in the worst case, p.d.f. not needed

\[
\min_x \max_\omega F(x, \omega)
\]

- Other formulations
Stochastic optimization formulation - constraints

- **Deterministic constraints** - All constraints must be met under all outcome scenarios – may be impossible to achieve
  \[ g(x, \omega) \geq 0 \]

- **Expected constraints** - Constrained are satisfied on average,
  \[ \mathbb{E}_{\omega}(g(x, \omega)) \geq 0 \]

- Is half umbrella good for 50% chance of rain?

- No unique way to define stochastic unit commitment problem
- Different choice leads to very different decisions
- The ’correct’ formulation depends on the risk attitude
- Whose risk attitude should be considered?, Should ISO choose the risk attitude for every market participant?
Multi Stage (25 stage) Stochastic Unit Commitment

- Stochastic parameter vector

\[ \bar{\xi}_{1...k} = (h_1, \ldots, h_k) \]

- Recursive formulation (risk neutral)

\[
Q_1 = \min_{x_1} \left\{ c_1^T x_1 + \mathbb{E}\left( Q_2(x_1, y_1, \xi_1) \right) \right\}
\]

\[
Q_k = \min_{x_k, y_{k-1}} \left\{ c_k^T x_k + d_{k-1}^T y_{k-1} + \mathbb{E}\left( Q_{k+1}(x_1, \ldots, x_k, y_1, \ldots, y_k, \xi_k | \xi_{1\ldots(k-1)}) \right) \right\},
\]

\[ k = 2, \ldots, 24 \]

\[
Q_{25} = \min_{y_{24}} \left\{ d_{24}^T y_{24} \right\}
\]

- Model state

\[(x_1, \ldots, x_k, y_1, \ldots, y_k)\]

- Information state

\[(h_1, \ldots, h_k)\]
Multi Stage (25 stage) Stochastic Unit Commitment - Nested Formulation

- Expected cost

\[
Q_1 = \min_{x_1} \left[ c_1^T x_1 + E \left( \min_{x_2} \left[ c_2^T x_2 + E \left( \min_{x_3} \left[ c_3^T x_3 + \ldots + E \left( \min_{x_{24}} \left[ c_{24}^T x_{24} + E \left( \min_{y_{24}} \left[ d_{24}^T y_{24} \right] \right) \right] \right) \right] \right) \right) \right] \]

- Extreme risk aversion (some call it robust formulation)

\[
Q_1 = \min_{x_1} \left[ c_1^T x_1 + \max \left( \min_{x_2} \left[ c_2^T x_2 + \max \left( \min_{x_3} \left[ c_3^T x_3 + \ldots + \max \left( \min_{x_{24}} \left[ c_{24}^T x_{24} + \max \left( \min_{y_{24}} \left[ d_{24}^T y_{24} \right] \right) \right] \right) \right) \right) \right] \right] \]
Evaluation of stochastic solutions

- Stochastic solution is optimal in the expectation, if the identical experiment is repeated many times
- Monte Carlo simulation
- Exhaustive enumeration (something for the super computer or cloud computing)

<table>
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<tr>
<th>Scenarios</th>
<th>S1</th>
<th>S2</th>
<th>...</th>
<th>SK</th>
<th>Expected Value</th>
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<td>P2</td>
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<td>PK</td>
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<td>Solution A</td>
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<td>Solution C</td>
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Stochastic SCUC and DA market

- DA market is financially binding and provides deterministic LMP for market settlement, hours before the first hour of the operating day.
- Stochastic unit commitment does not produce one deterministic schedule, but many schedules contingent upon uncertain outcomes.
- The schedule except for first hour commitment is not final until the last hour of the market.

![Diagram showing the process of commitment schedules with decision stages.]
LMP Calculation with Stochastic SCUC

- As the operating day unfolds, one scenario is realized gradually based on the outcomes of uncertain parameters.
- Before the operating days starts, multiple potential schedules produce multiple LMPs.
- How to settle the DA market? Expected LMP? What needs to be done to ensure physical feasibility and revenue adequacy?

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<th>Scenario</th>
<th>L</th>
<th>M</th>
<th>H</th>
<th>Expected Value</th>
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<td>LMP ($/MWh)</td>
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<td>$5</td>
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<tr>
<td>Single Cap Block at $27/MWh</td>
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<td>100</td>
<td>0</td>
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<td>Double weighted LMP</td>
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More questions

- Does expected LMP send the right signal to the market?
- Does it neutralize the purpose of stochastic SCUC?
- Is stochastic SCUC compatible with DA market?
- Regardless of the LMP calculation methods, a more fundamental question is - who should pay the price of unpredictability in wind power?
Summary

- For moderate number of wind power farms, the scenarios over 24 hours is huge, determining the probability is no easy task.

- Network constraint consideration in SCUC precludes strategy of wind power aggregation

- Choice of stochastic unit commitment model depends on risk attitude, how can we incorporate and balance different risk attitudes of the market operator and participants?

- Stochastic SCUC schedule is contingent upon uncertain outcomes, settling the market before operating day begins require finalizing LMP/generation

- Ensuring feasibility (min output, min up/down time) and revenue adequacy can be tricky

- Conceptual challenges must be resolved before computational challenges
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