Opportunities for Exploiting Problem Structure in Unit Commitment MIP Models

Ed Rothberg

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Gurobi Overview

- Founded in 2008 by Bob Bixby, Zonghao Gu, and Ed Rothberg
- Long history with the power industry
  - Bob Bixby often credited with sparking renewed interest in MIP models for unit commitment
    - September, 1999 Rutgers workshop
- Gurobi MIP solver widely used in the electrical power industry
  - At least 6 power industry software companies currently embed our solver

MIP In Unit Commitment

- MIP formulation and solution:

  "The long solution times and slow convergence characteristics of earlier MIP solvers is why many experienced people question the ability of MIP algorithms to solve these NP-Hard problems"

  "This effort has been so successful that virtually all of the current [MIP] problems solve to within reasonable convergence tolerances before entering the Branch and Bound phase"

  – *A Mixed Integer Programming Solution for Market Clearing and Reliability Analysis*, Streiffert, Philbrick, and Ott, 2005

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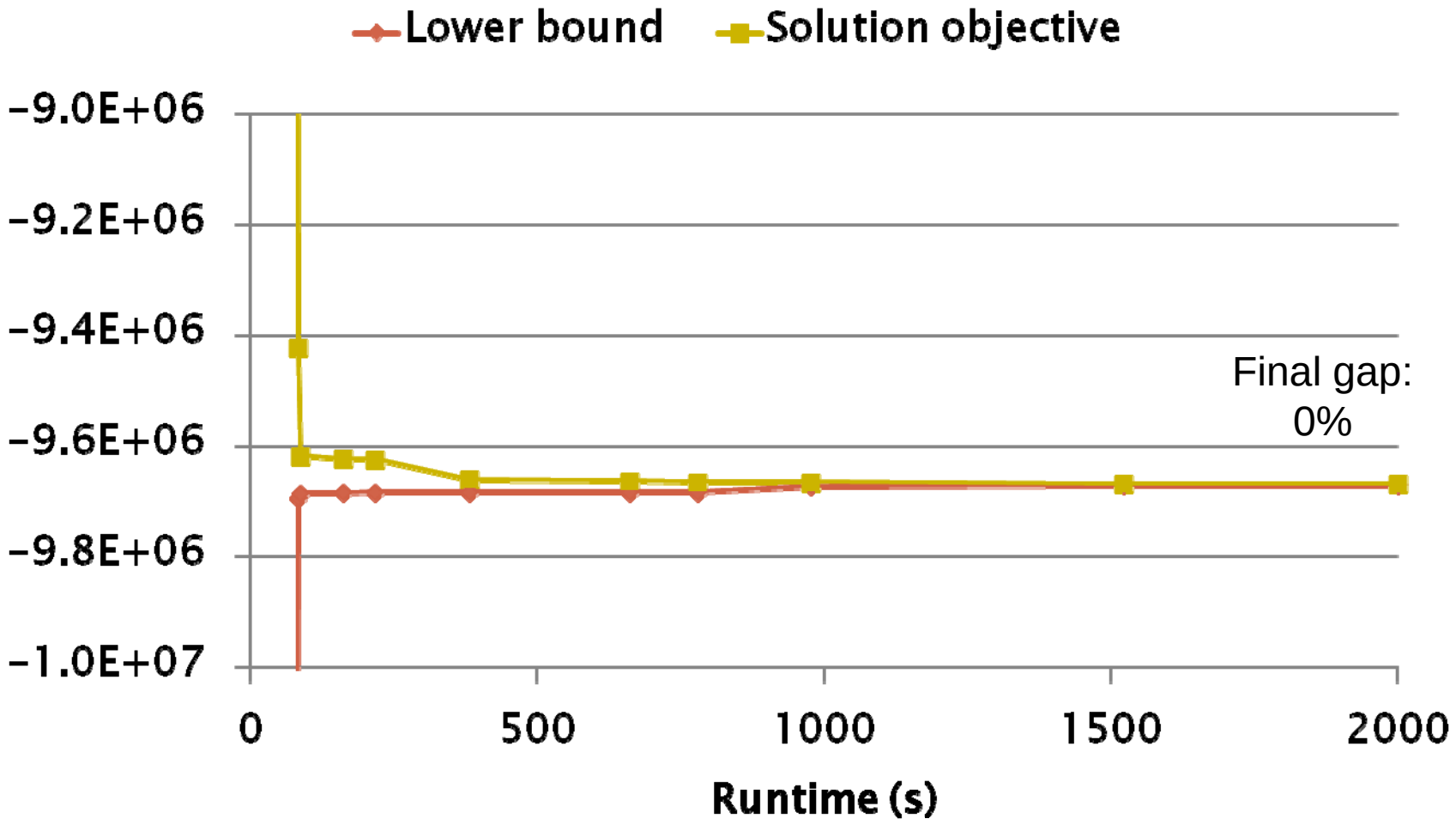
MIP Optimality Gap – Commercial UC Model



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Key MIP Solver Ingredients

- Key ingredients to this success…
  - Good model formulations
    - Tight linear relaxation
  - Cutting planes
    - Valid, redundant inequalities that tighten the linear relaxation
  - Heuristics
    - Inexpensive methods for converting a relaxation solution into an integer feasible solution

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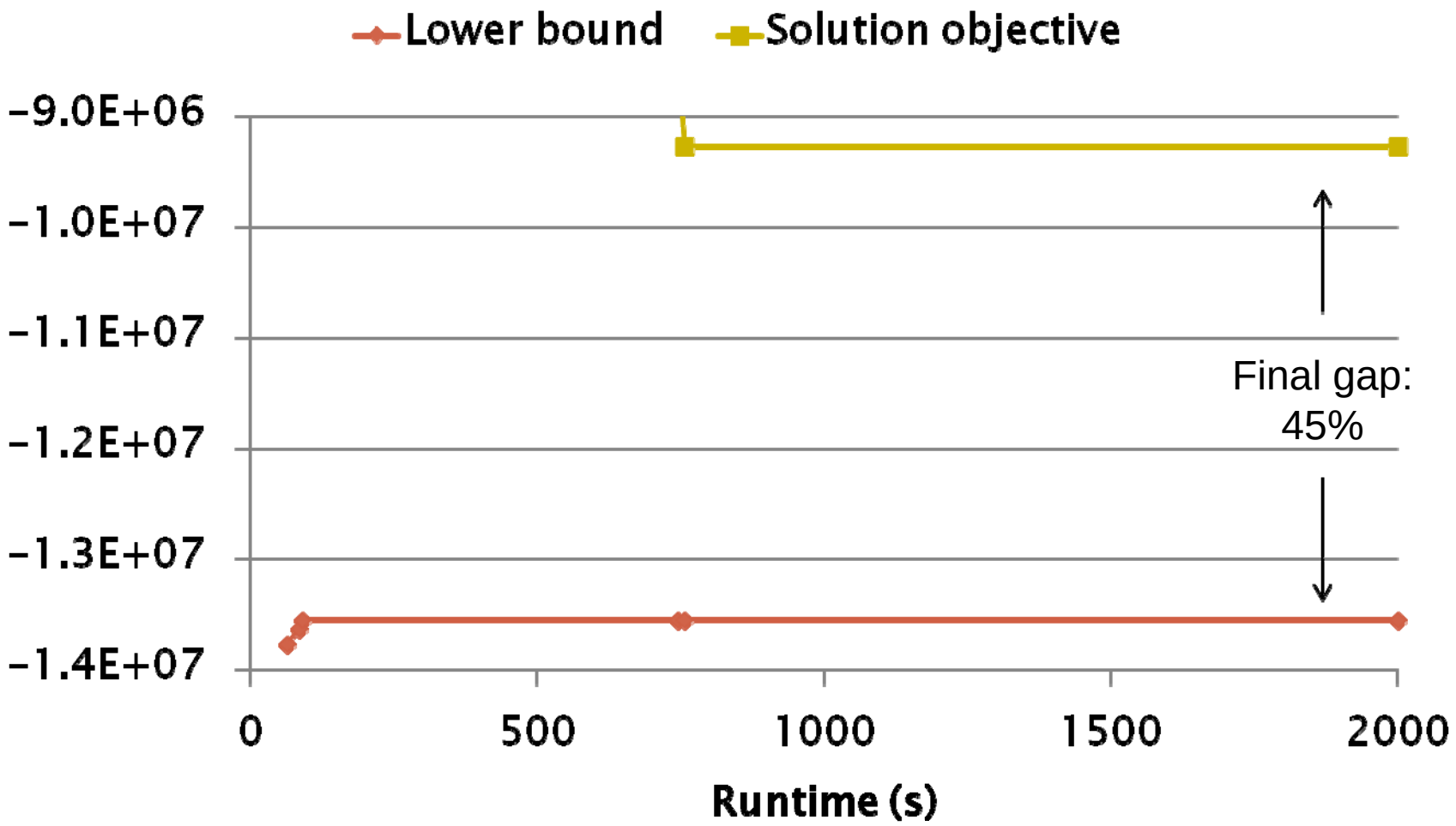
MIP Optimality Gap– Cuts and Heuristics Disabled



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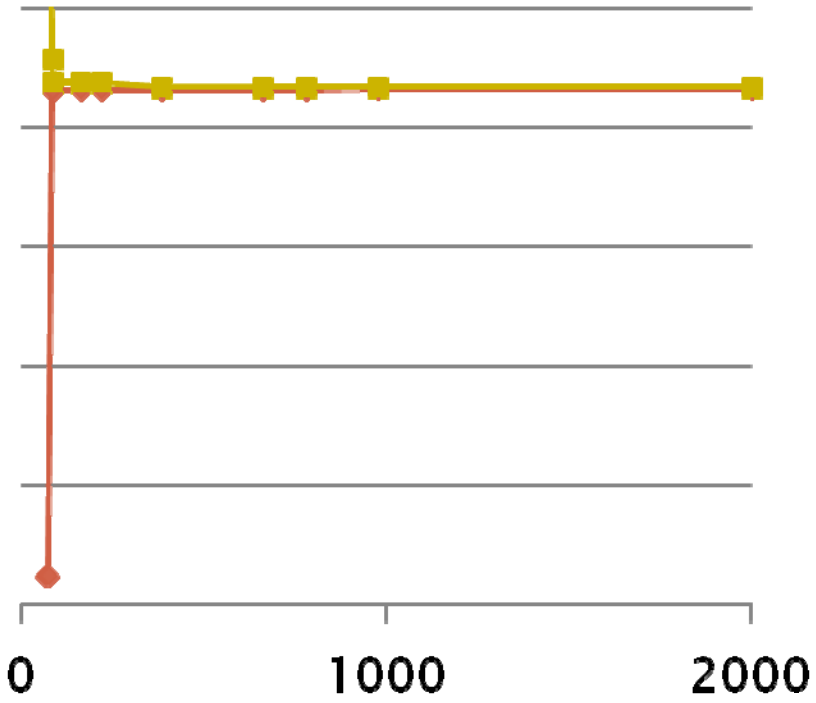
MIP Optimality Gap



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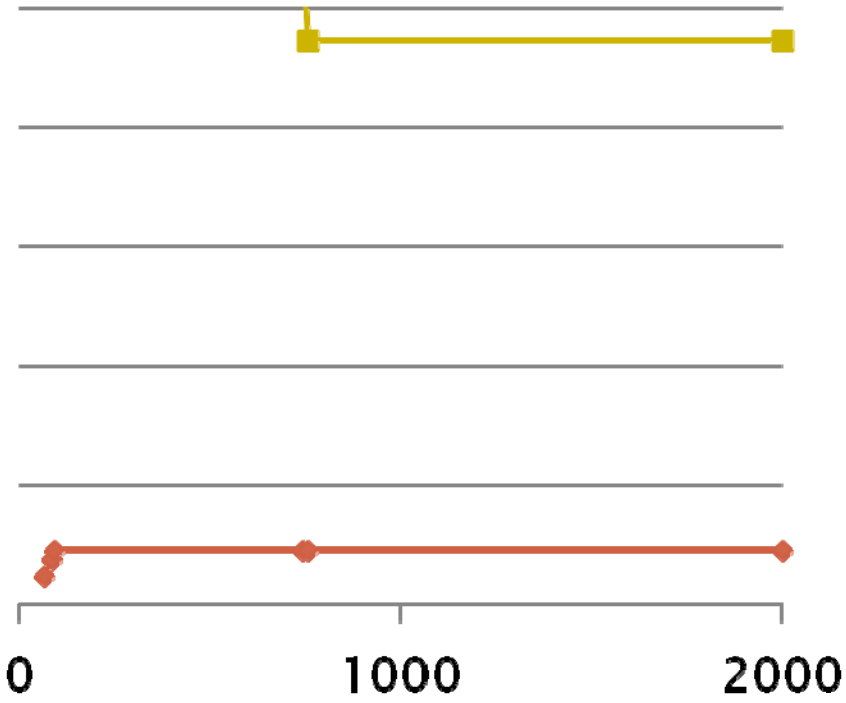

Cuts and Heuristics Disabled

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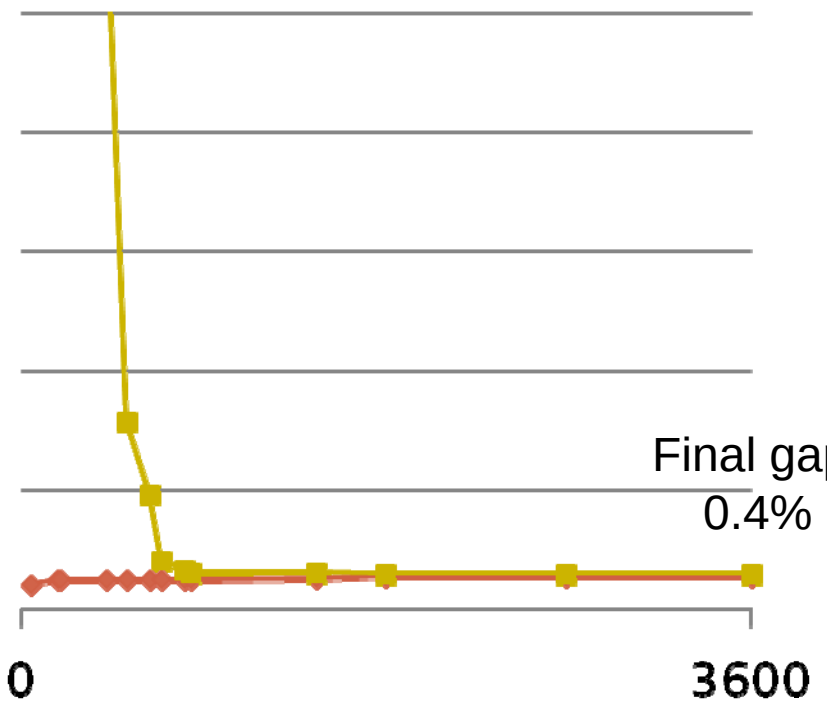
Larger Model – MIP Optimality Gap



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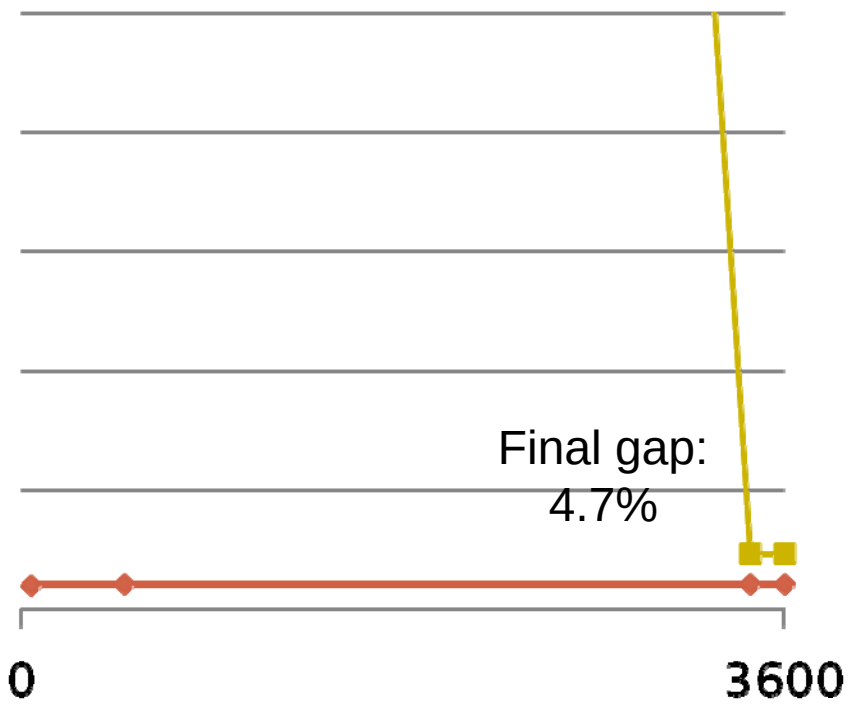

Cuts and Heuristics Disabled

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Can We Do Better?

- Is model structure being fully exploited?
- Would exploiting structure lead to…
  - …better cutting planes?
  - …better heuristics?

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Cutting Planes

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Cutting Planes

- Long history of cutting plane development
  - Gomory cuts date back to the 1950s
- Gurobi 5.0 includes 13 different cut types
  - Gomory, clique, knapsack, flow cover, MIR, …
  - Exploit different structure commonly found in MIP models

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Cutting Planes

- Exploiting unit-commitment structure
  - *Minimum Up/Down Polytopes of the Unit Commitment Problem With Start-Up Costs*, Rajan and Takriti, 2005
  - *Analyzing Valid Inequalities of the Generation Unit Commitment Problem*, Hedman, O'Neill, and Oren, 2008
  - *Tight Formulations for the Unit Commitment Problem*, Ostrowski, Anjos, and Vennelli, 2012
- Helpful in practice?
  - Tough to try on real data

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Cutting Planes

- Three main issues:
  - Complicated computational tradeoffs
  - Often redundant with existing cuts
  - Empirically: smaller scope for improvement than heuristics

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Heuristics

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Heuristics

- Very successful paradigm for finding high quality MIP solutions
  - Solution Construction
    - Find any reasonable quality solution
      - Very poor quality solution often good enough
  - Solution Improvement
    - Use local improvement to successively improve the solution

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Solution Construction

- For UC models…
  - Trivial feasible solution
    - Turn every generator on
  - MIP solver doesn't find it
    - Set every binary variable to 1?
    - Some binary variables capture transitions (startup/shutdown)
- Clear indication that structure is being lost
  - Simple decisions often obscured in MIP model

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Solution Improvement

- Basic framework
  - Solve a MIP model on a subset of the variables of the model
  - Fix other variables to current solution values
- Which subset?
  - Use the LP relaxation to choose a neighborhood
    - *Relaxation Induced Neighborhood Search (RINS)* [Danna, Rothberg, Le Pape, 2003]
  - Use meta-heuristics to choose neighborhoods
    - *Solution polishing* [Rothberg, 2007]

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Solution Improvement

- All rely on unstructured neighborhoods
  - No attempt to exploit problem structure
- Neighborhoods in unit commitment…

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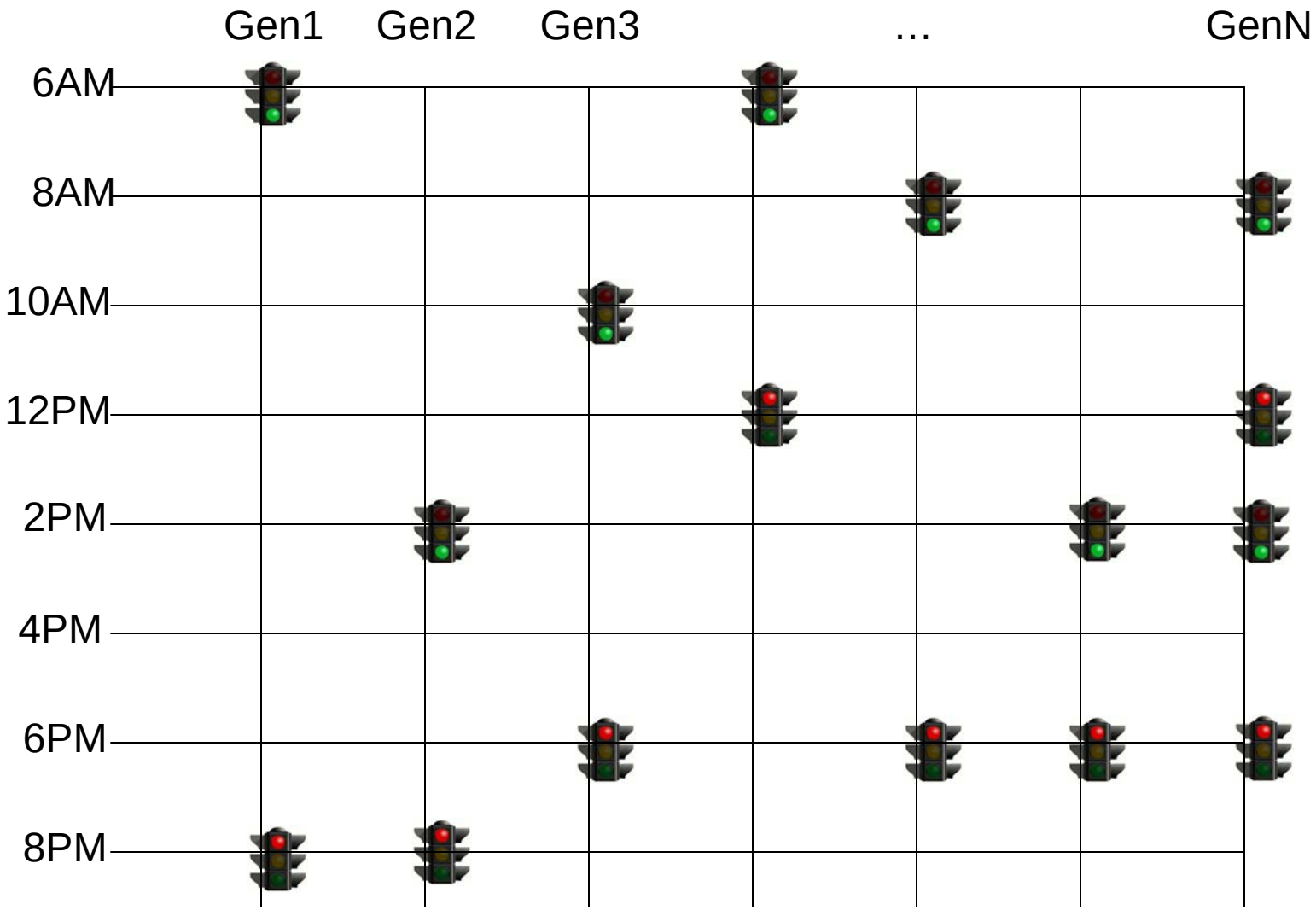
Unit Commitment Model



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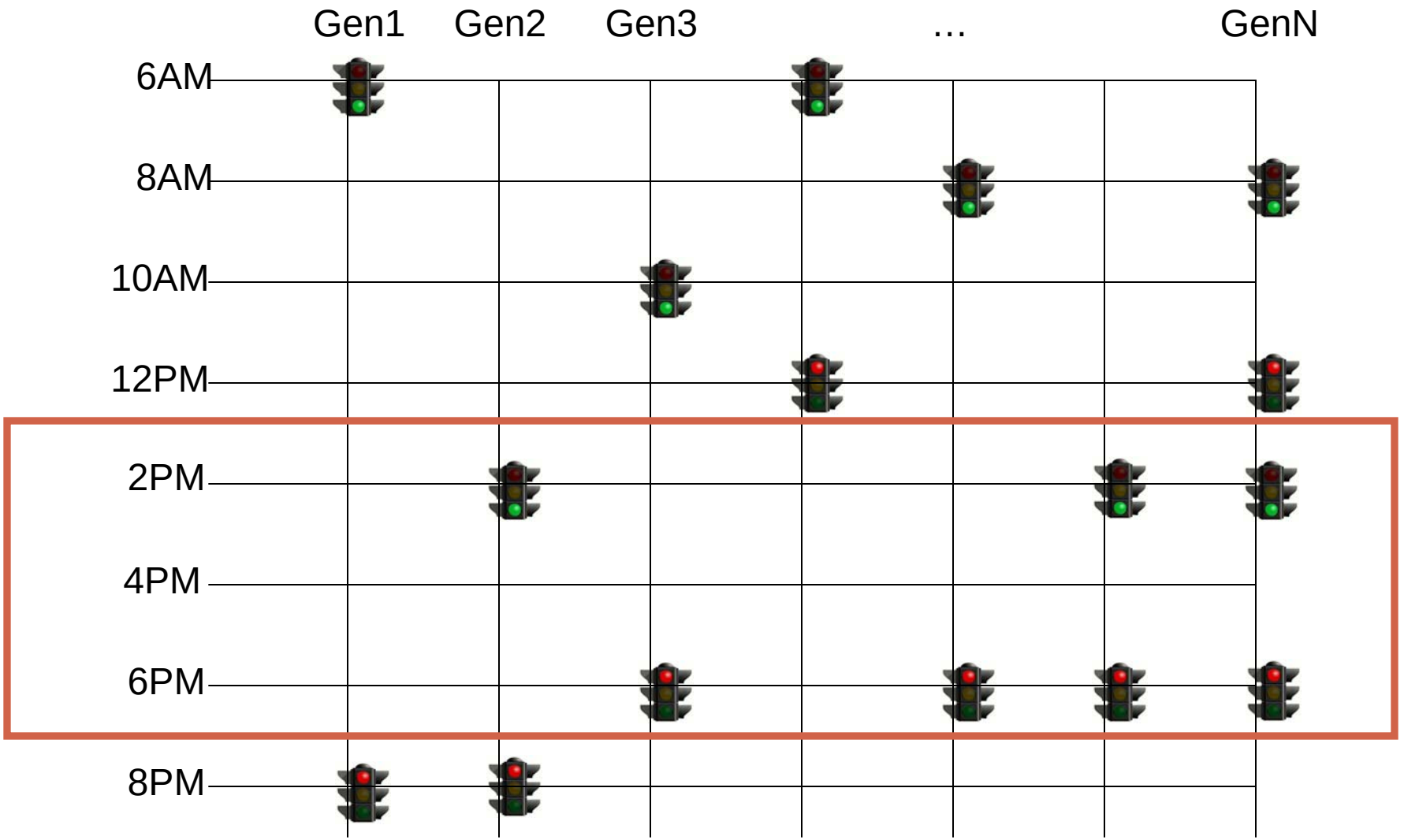
Structured Neighborhoods



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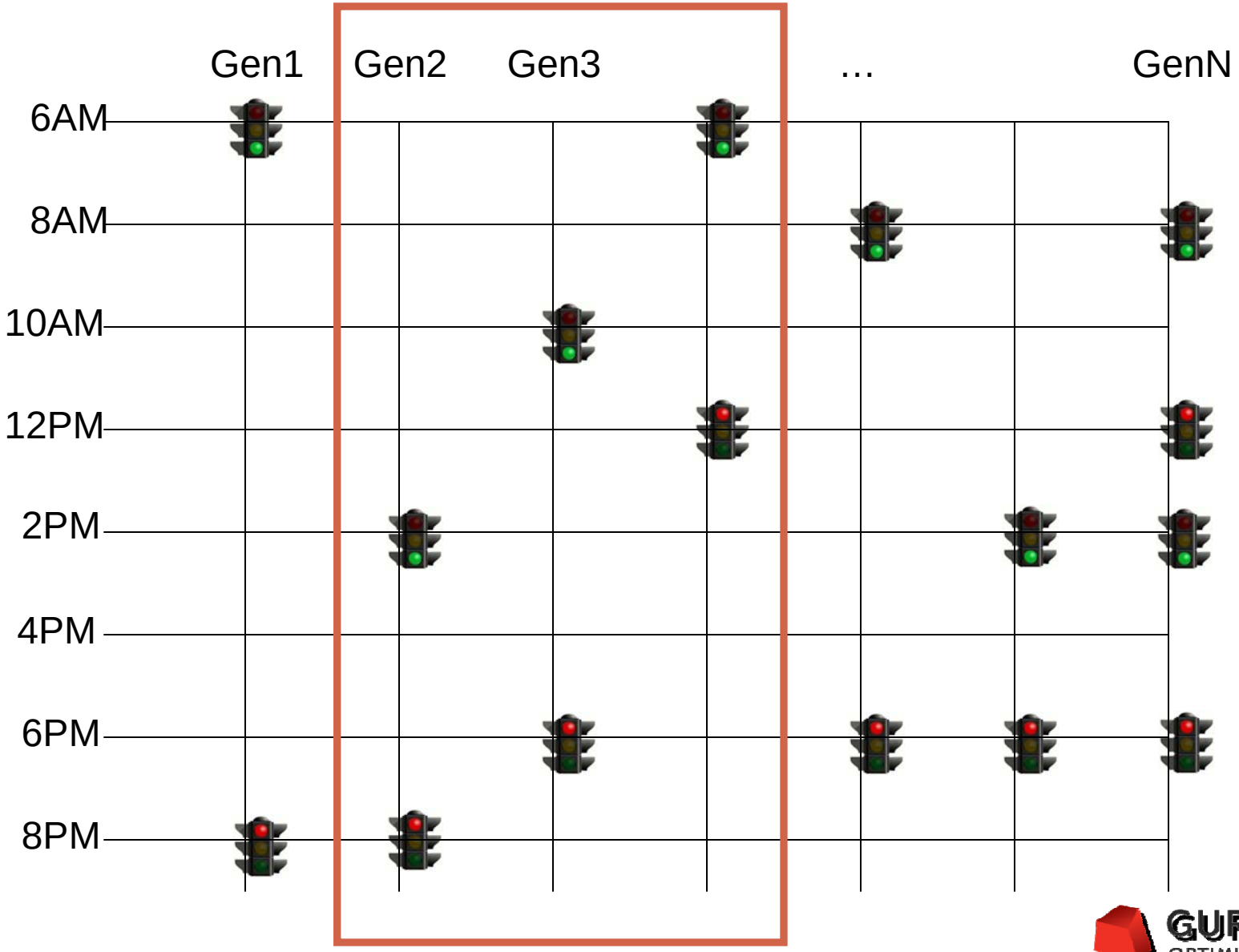
Structured Neighborhoods



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Conclusions

- MIP extremely effective for UC
- Model size and complexity has grown
  - Models don't solve at the root node any more
- Structure in UC models isn't being fully exploited
- Confident that results would significantly improve if we could understand/recognize model structure

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