Branch Flow Model

relaxations, convexification, computation

Masoud Farivar Steven Low

Subhonmesh Bose Lina Li Mani Chandy

Computing + Math Sciences

Electrical Engineering

Caltech

June 2012

Motivations

Large active network of DER

	#nodes				remarks
SCE					Rooftop
C					ofs
CA					Portfolio
US	--			2035	Obama's goal for clean energy

Millions of active endpoints introducing rapid large random fluctuations in supply and demand

DER: PVs, wind turbines, EVs, batteries, DR loads

Implications

Current control paradigm works well today

- Low uncertainty, few active assets to control
- Centralized, open-loop, human-in-loop, worst-case preventive
- Schedule supplies to match loads

Future needs

- Fast computation to cope with rapid, random, large fluctuations in supply, demand, voltage, freq
- Simple algorithms to scale to large networks of active DER
- Real-time data for adaptive control, e.g. real-time DR

Our approach: endpoint control

Endpoint based control

- Self-manage through local sensing, communication, control
- Real-time, scalable, closed-loop, distributed, robust

Local algorithms with global perspective

- Holistic framework with global objectives
- Decompose global objectives into local algorithms

Control and optimization framework

- Theoretical foundation for a holistic framework that integrates engineering + economics
- Systematic algorithm design

GENI project:

Key challenges

Nonconvexity

- Convex relaxations

Large scale

- Distributed algorithms

Uncertainty

- Risk-limiting approach

Why is convexity important

Foundation of LMP

- Convexity justifies the use of Lagrange multipliers as various prices
- Critical for efficient market theory

Efficient computation

- Convexity delineates computational efficiency and intractability

A lot rides on (assumed) convexity structure

•engineering, economics, regulatory

Optimal power flow (OPF)

Problem formulation

- Carpentier 1962

Computational techniques

- Dommel & Tinney 1968
- Surveys: Huneault et al 1991, Momoh et al 2001, Pandya et al 2008

Bus injection model: SDP relaxation

- Bai et al 2008, 2009, Lavaei et al 2010, 2012
- Bose et al 2011, Zhang et al 2011, Sojoudi et al 2012
- Lesieutre et al 2011

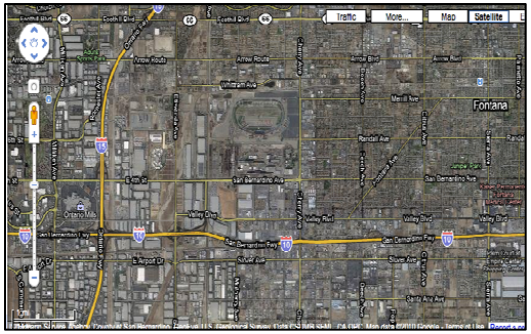
Branch flow model: SOCP relaxation

- Baran & Wu 1989, Chiang & Baran 1990, Taylor 2011, Farivar et al 2011

Application: Volt/VAR control

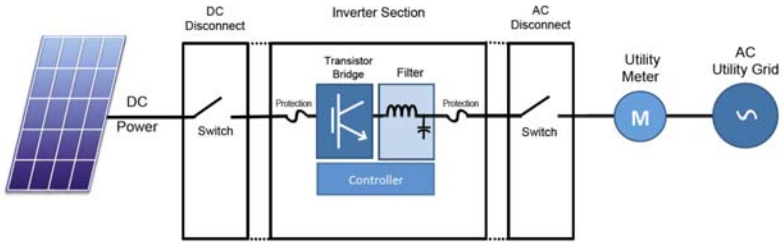
Motivation

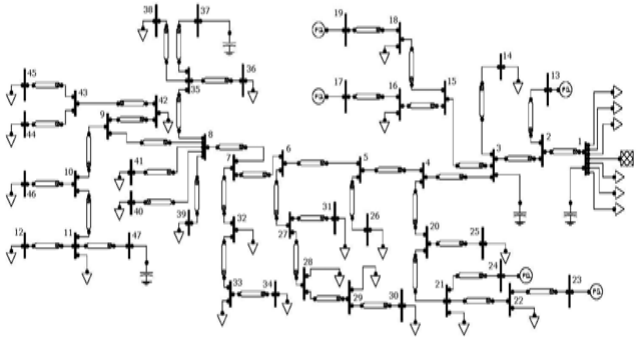
- Static capacitor control cannot cope with rapid random fluctuations of PVs on distr circuits



Inverter control

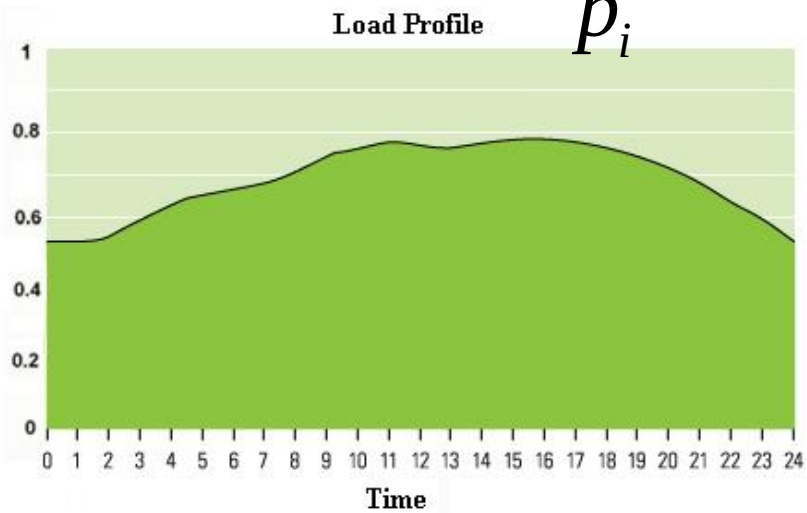
- Much faster & more frequent
- IEEE 1547 does not optimize VAR currently (unity PF)



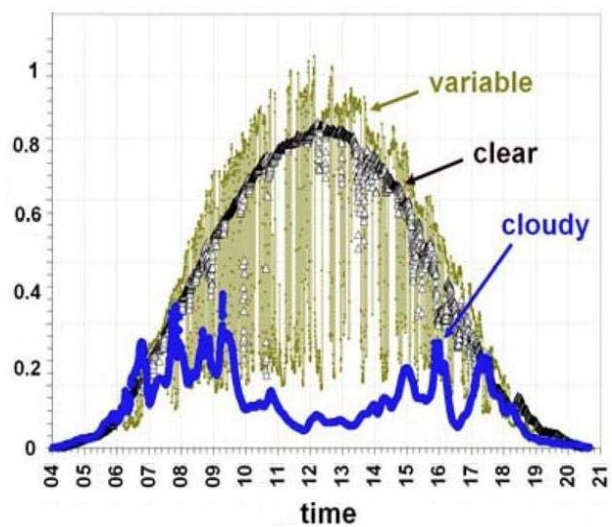


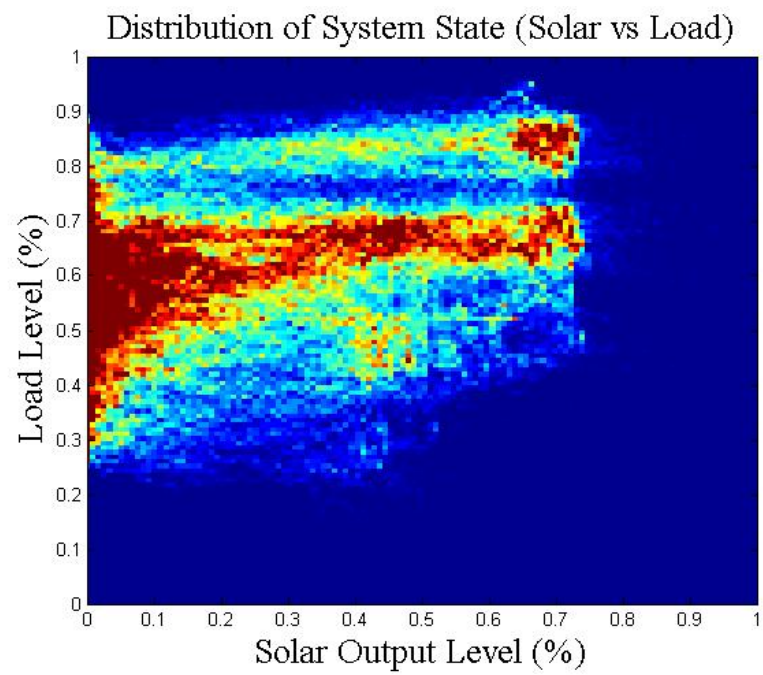
Load and Solar Variation

p_i^c



p_i^g





Empirical distribution
of (load, solar) for Calabash

Summary

RESULTS FOR SOME VOLTAGE TOLERANCE THRESHOLDS

Voltage Drop Tolerance(pu)	Annual Hours Saved Spending Outside Feasibility Region	Average Power Saving (%)
0.03	842.9	3.93%
0.04	160.7	3.67%
0.05	14.5	3.62%

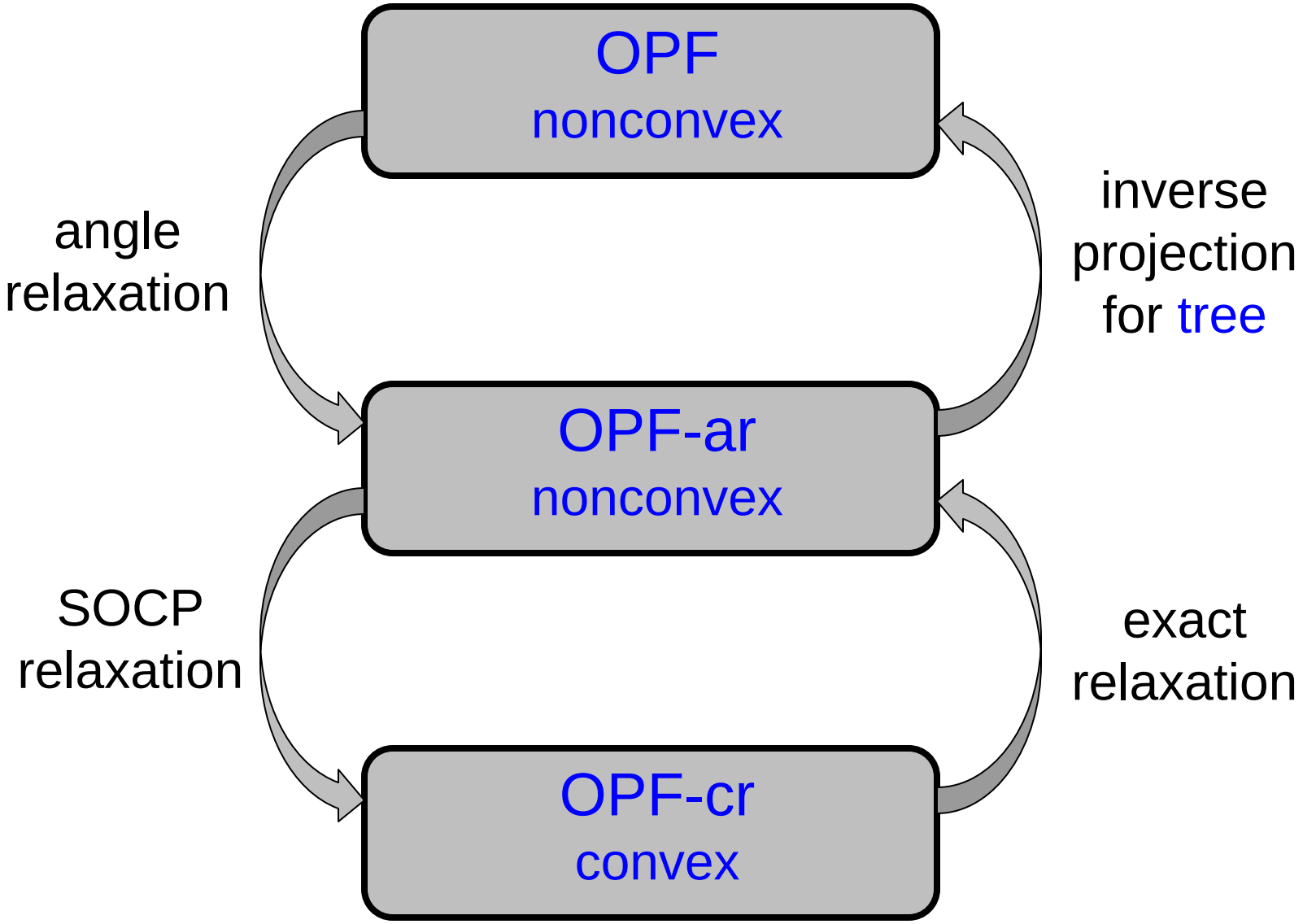
- More reliable operation
- Energy savings

Theory

summary

details

Solution strategy



OPF solution

Solve OPF-cr

SOCP

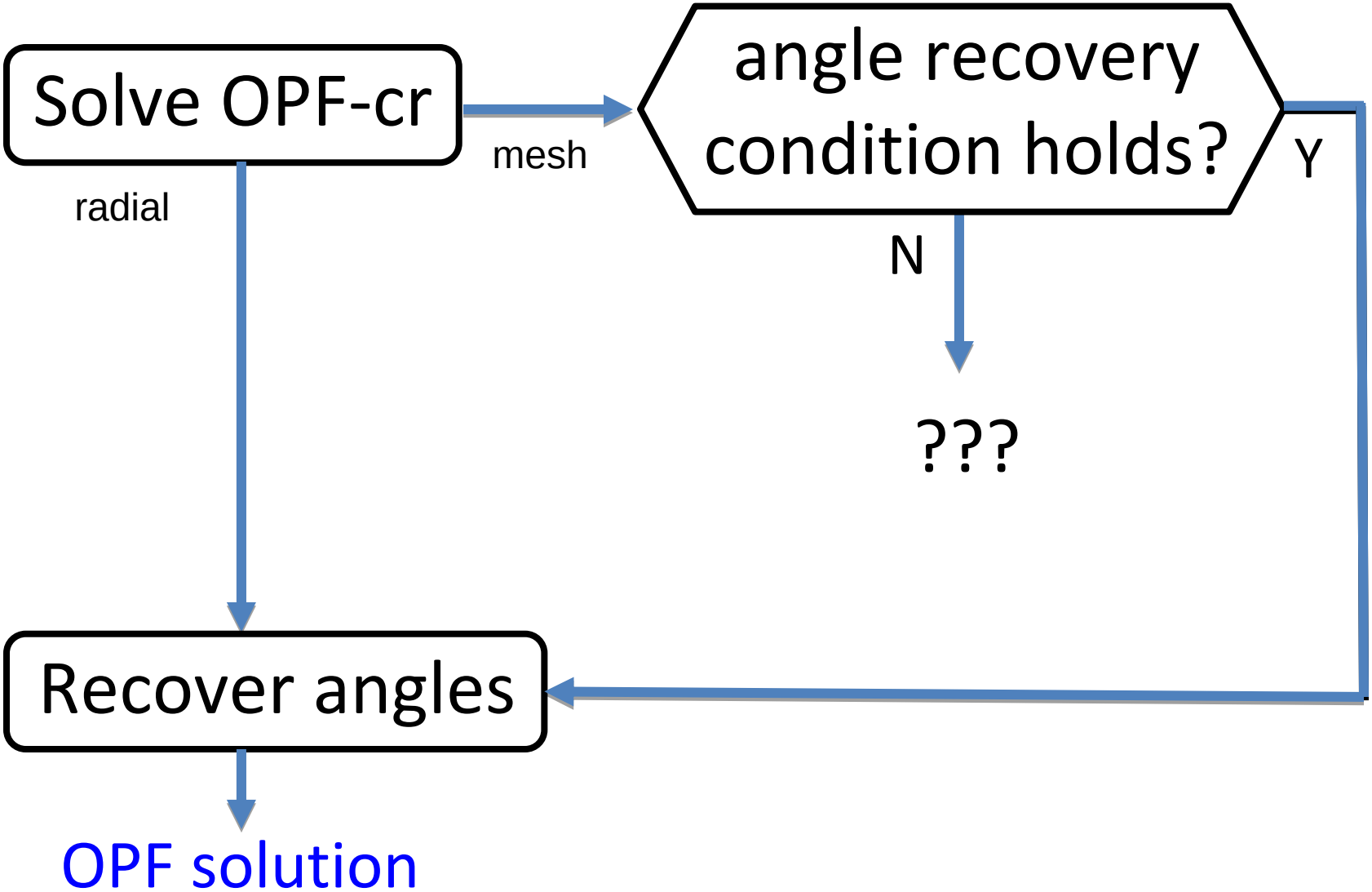
radial

Recover angles

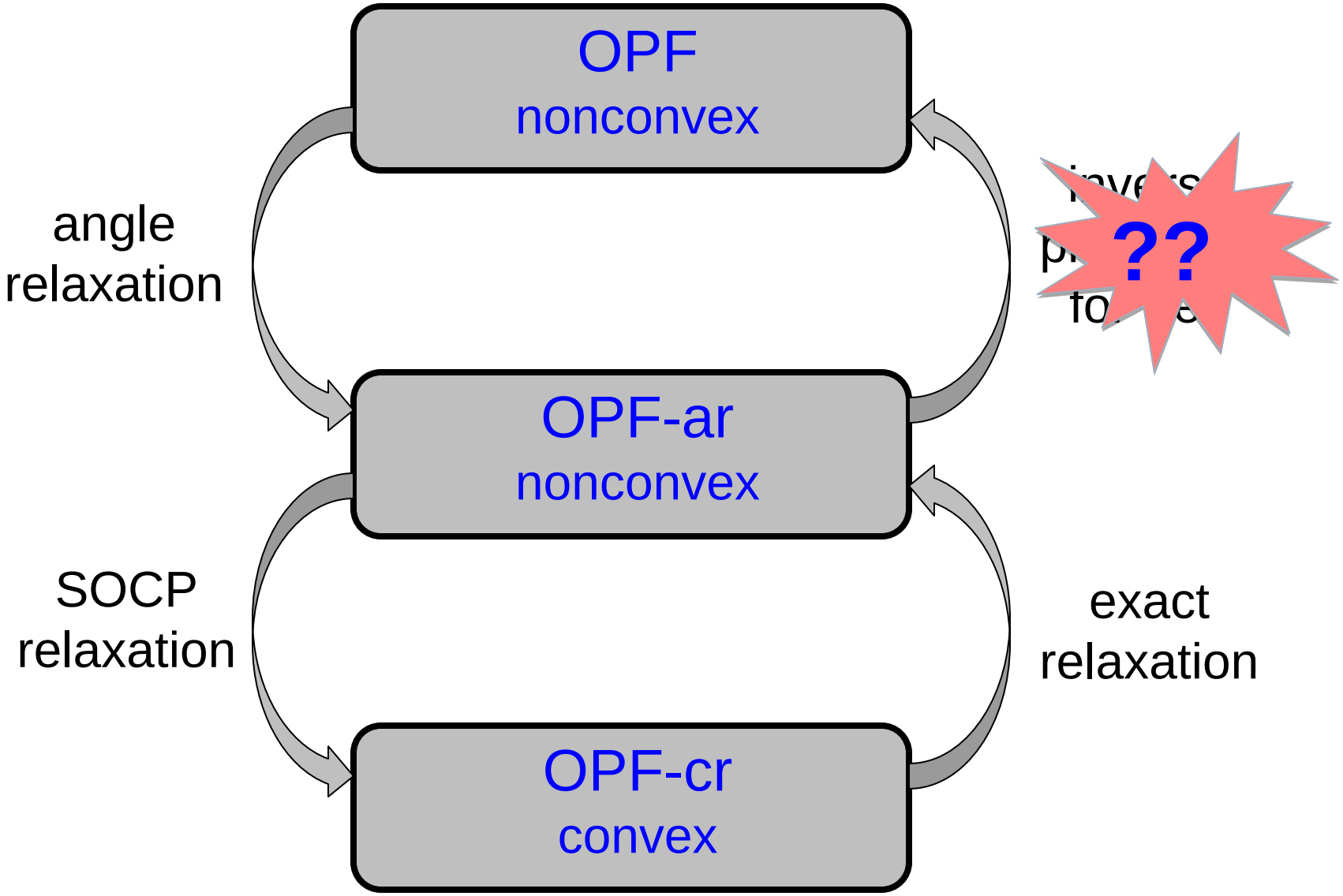
- explicit formula
- distributed alg

OPF solution

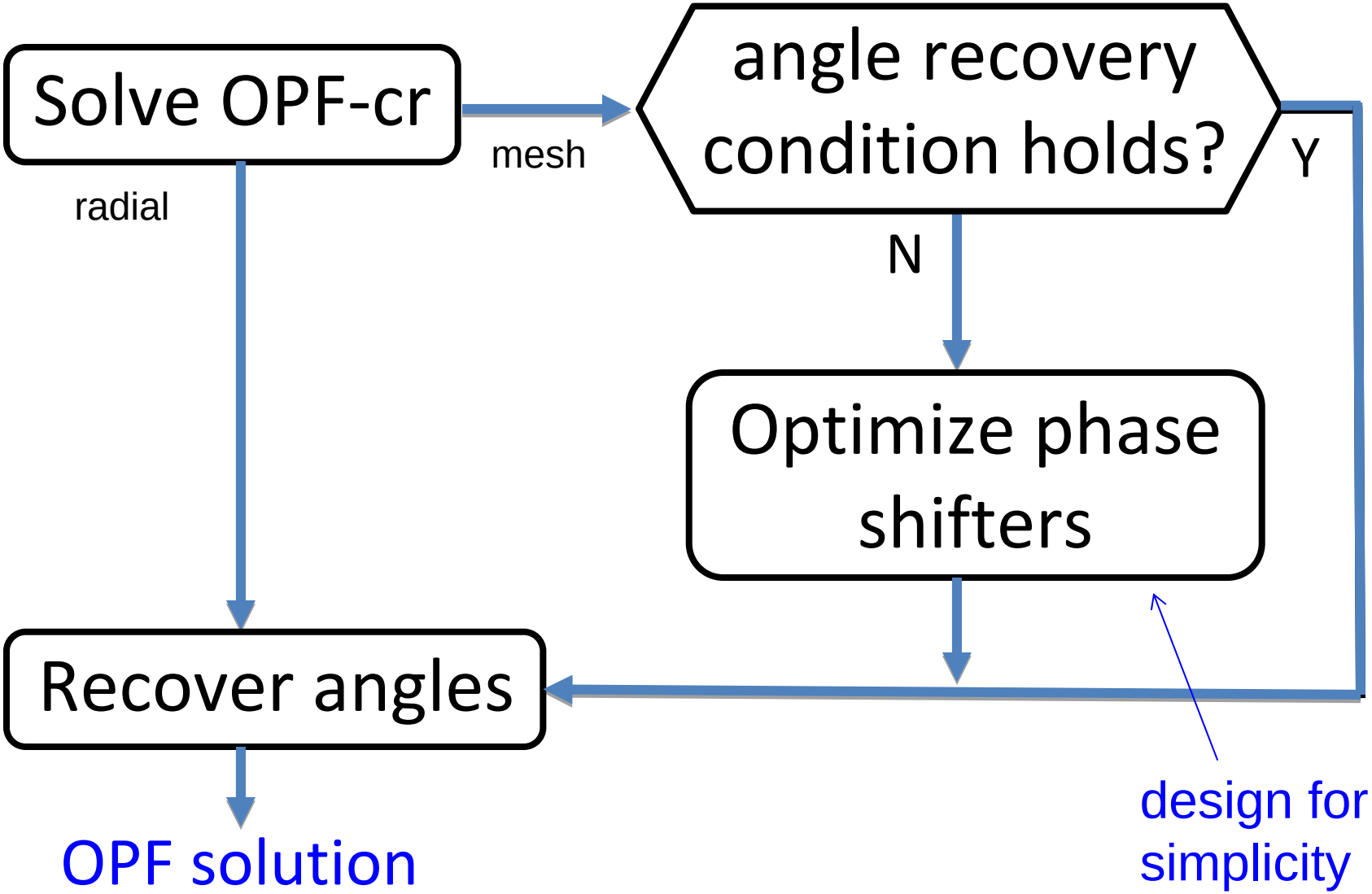
OPF solution



Recap: solution strategy



OPF solution



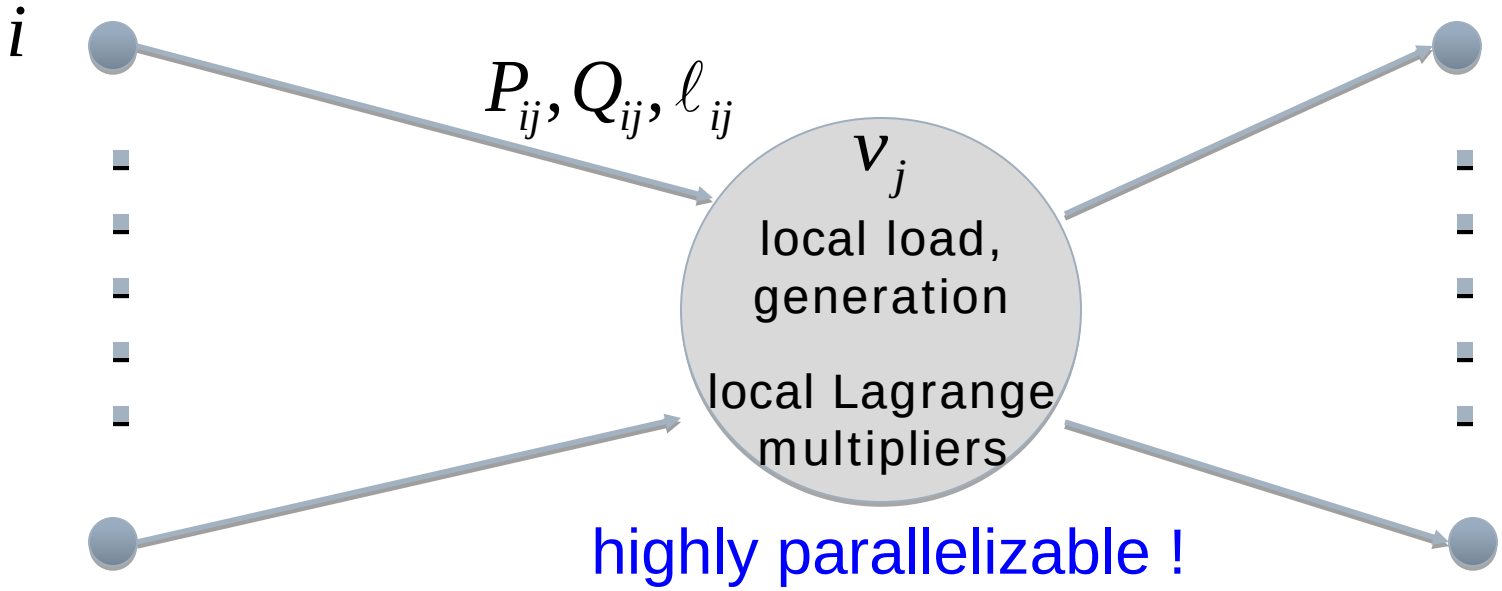
Examples

		No PS	With PS
Test cases	# links (m)	Min loss (OPF, MW)	Min loss (OPF-cr, MW)
IEEE 14-Bus	20	0.546	0.545
IEEE 30-Bus	41	1.372	1.239
IEEE 57-Bus	80	11.302	10.910
IEEE 118-Bus	186	9.232	8.728
IEEE 300-Bus	411	211.871	197.387
New England 39-Bus	46	29.915	28.901
Polish (case2383wp)	2,896	433.019	385.894
Polish (case2737sop)	3,506	130.145	109.905

Examples

Test cases	# links (m)	# active PS $\lvert\phi_i\rvert > 0.1^\circ$	Angle range ($^\circ$) $[\phi_{\min}, \phi_{\max}]$
IEEE 14-Bus	20	2 (10%)	$[-2.1,\ 0.1]$
IEEE 30-Bus	41	3 (7%)	$[-0.2,\ 4.5]$
IEEE 57-Bus	80	19 (24%)	$[-3.5,\ 3.2]$
IEEE 118-Bus	186	36 (19%)	$[-1.9,\ 2.0]$
IEEE 300-Bus	411	101 (25%)	$[-11.9,\ 9.4]$
New England 39-Bus	46	7 (15%)	$[-0.2,\ 2.2]$
Polish (case2383wp)	2,896	376 (13%)	$[-20.1,\ 16.8]$
Polish (case2737sop)	3,506	433 (12%)	$[-21.9,\ 21.7]$

Extension: distributed solution

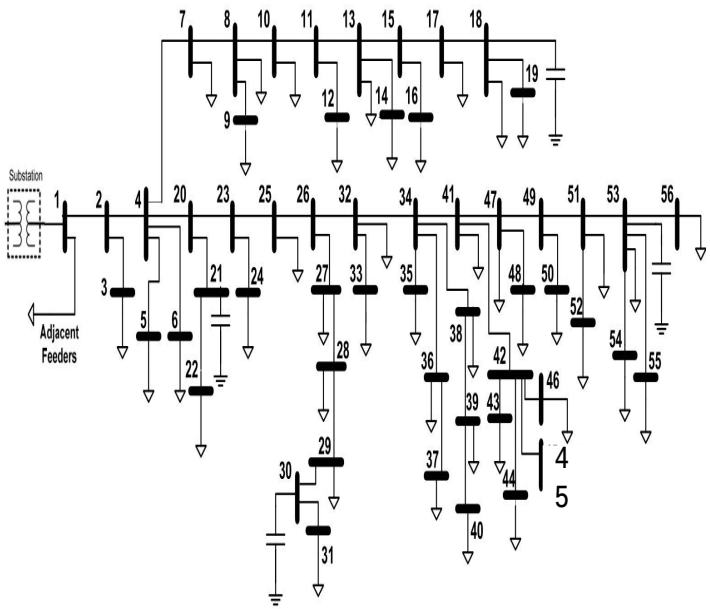


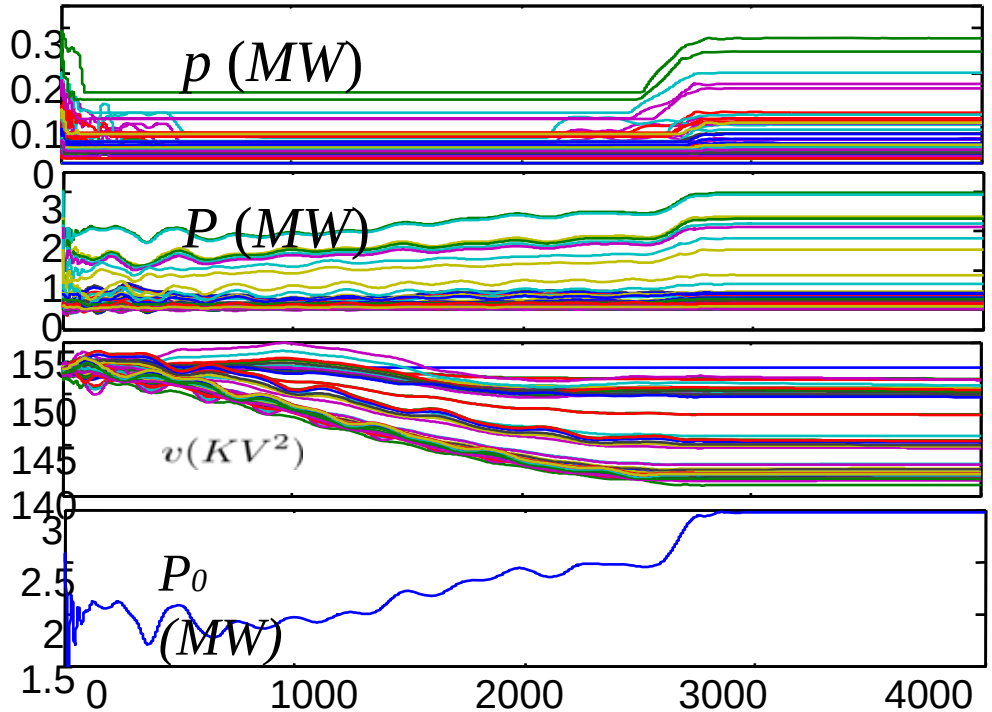
Local algorithm at bus j

- update local variables based on Lagrange multipliers from children
- send Lagrange multipliers to parents

Extension: distributed solution

SCE distribution circuit





Theorem

Distributed algorithm converges

- to global optimal for radial networks
- to global optimal for convexified mesh networks
- to approximate/optimal for general mesh networks

Work in progress with Lina Li, Lingwen Gan, Caltech

Key messages

Radial networks computationally simple

- Exploit tree graph & convex relaxation
- Real-time scalable control promising

Mesh networks can be convexified

- Design for simplicity
- Need few (?) phase shifters (sparse topology)
- Efficient algorithms for mesh without phase shifters (work in progress)

Theory: details

Branch flow model and OPF

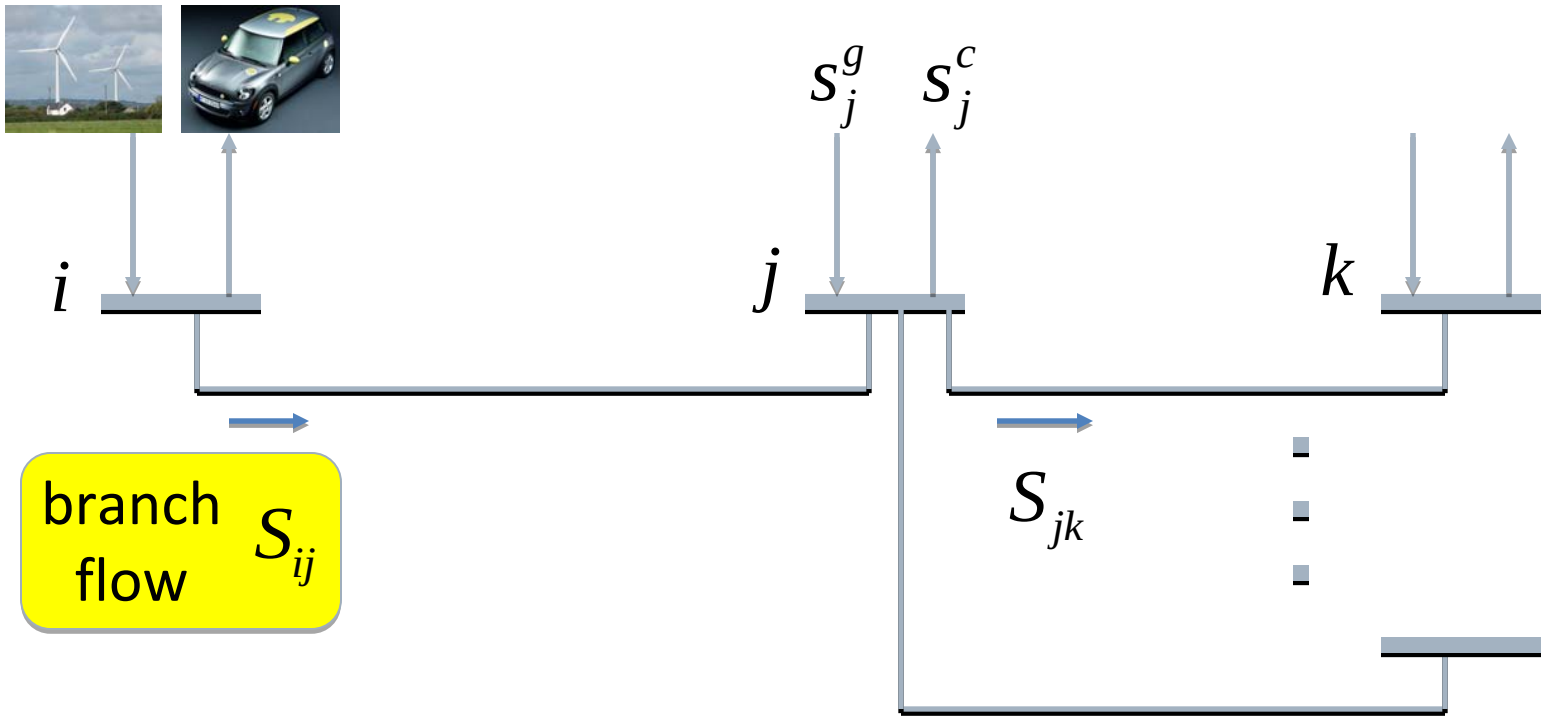
Solution strategy: two relaxations

- Angle relaxation
- SOCP relaxation

Convexification for mesh networks

Extensions

Two models

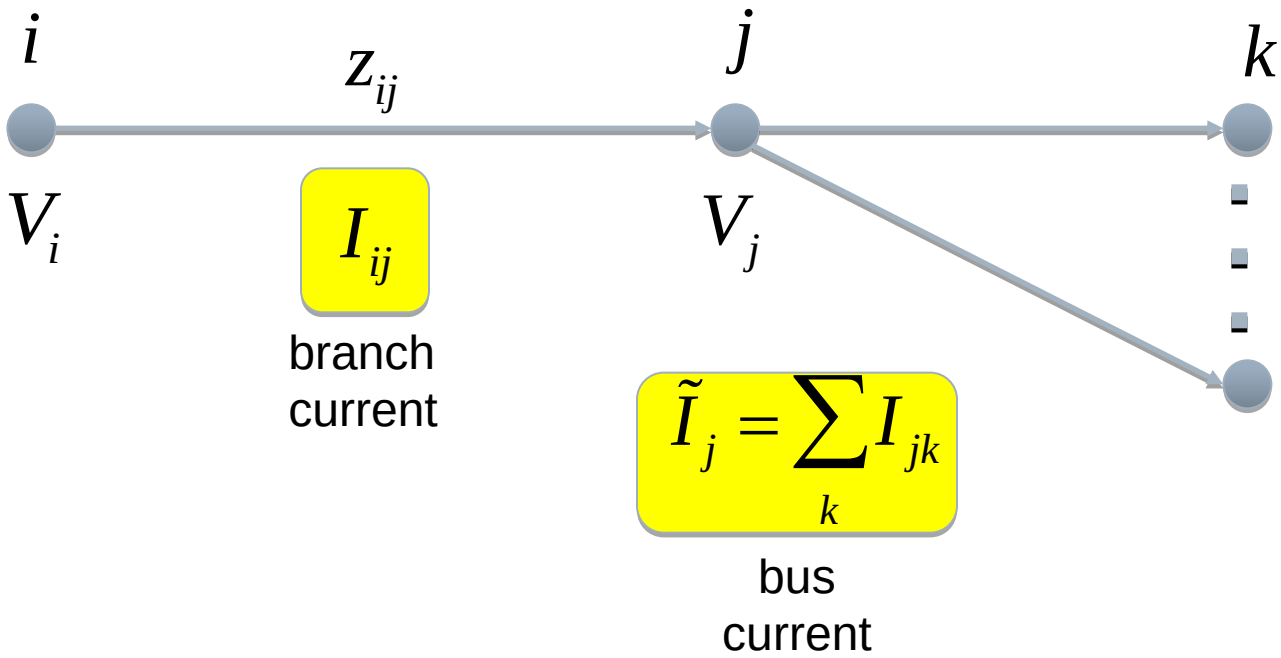


branch flow S_{ij}

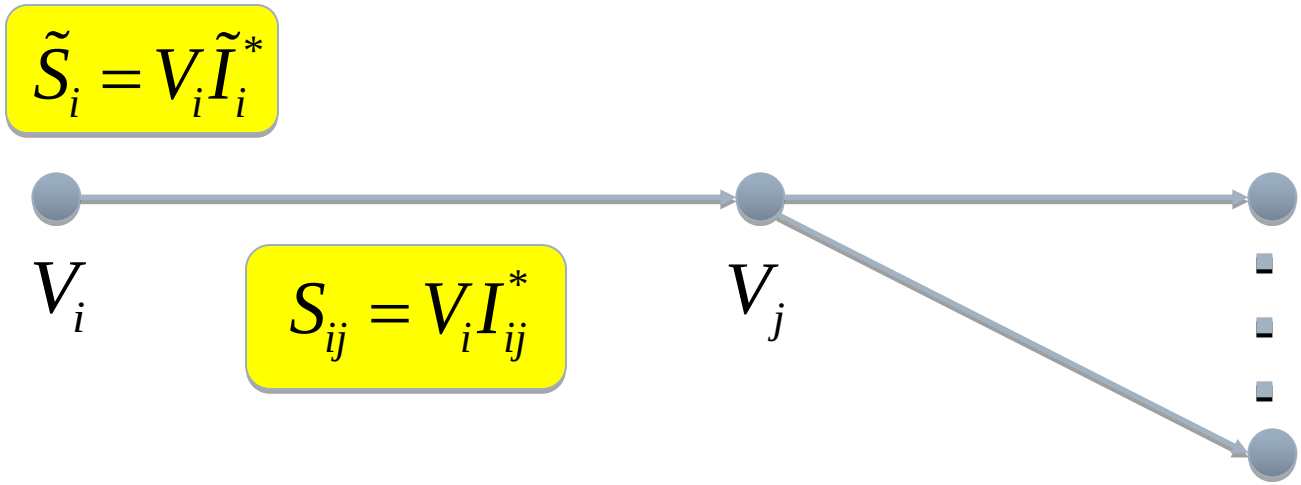
$$\tilde{S}_j = \sum_k S_{jk}$$

bus injection

Two models


i
z_{ij}
j
k
V_i
I_{ij}
V_j
branch current
$\tilde{I}_j = \sum_k I_{jk}$
bus current

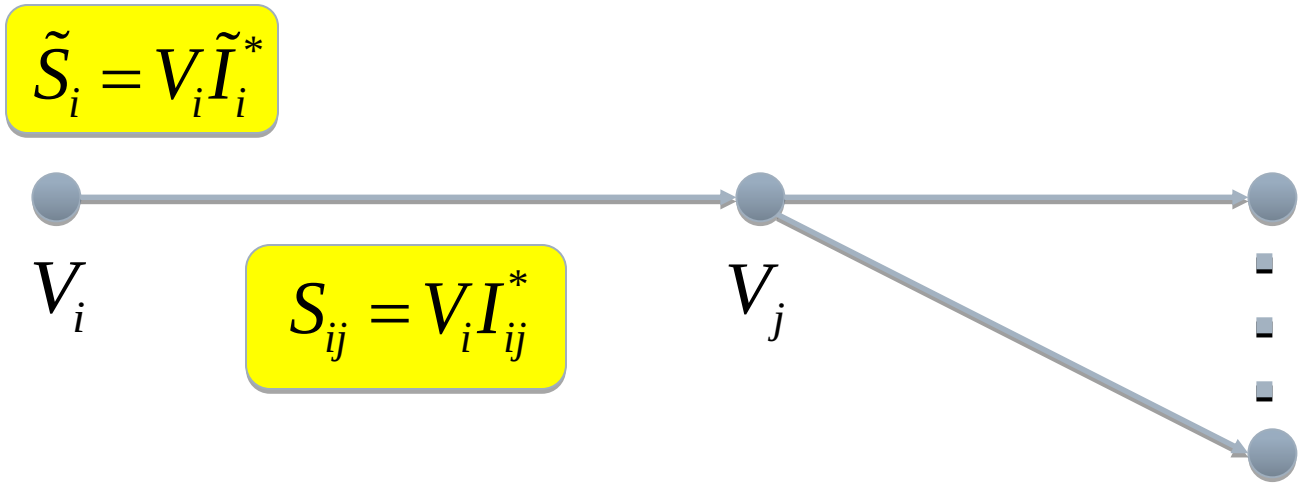
Two models



Equivalent models of Kirchhoff laws

- Bus injection model focuses on nodal vars
- Branch flow model focuses on branch vars

Two models



1. What is the model?
2. What is OPF in the model?
3. What is the solution strategy?

let’s start with
something familiar

Bus injection model

$$\tilde{S}_j = V_j \tilde{I}_j^* \qquad \text{for all } j$$ power definition

$$\tilde{I} = YV$$ Kirchhoff law

$$\tilde{S}_j = -s_j \qquad \text{for all } j$$ power balance

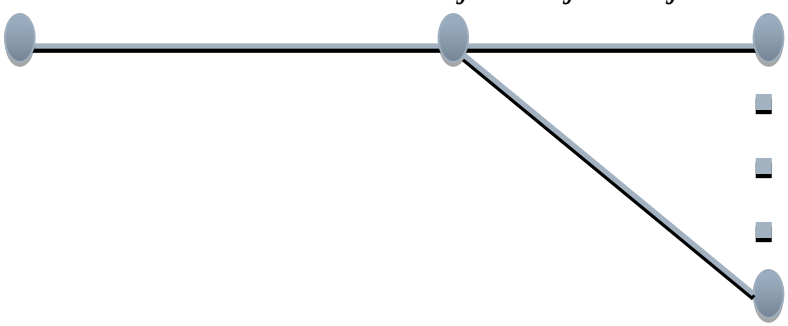
---

admittance matrix:

$$Y_{ij} := \begin{cases} \sum_{k \sim i} y_{ik} & \text{if } i = j \\ -y_{ij} & \text{if } i \sim j \\ 0 & \text{else} \end{cases}$$

$$\boxed{\tilde{S}_j = V_j \tilde{I}_j^*}$$

$$s_j = s_j^c - s_j^g$$



Bus injection model: OPF

$$\min \quad \sum_j f_j\left(\mathrm{Re}\left(\tilde{S}_j\right)\right)$$

e.g. generation cost

$$\text{over} \quad \tilde{x} := \left(\tilde{S}, \tilde{I}, V, s\right)$$

$$\text{subject to} \quad \tilde{I} = YV$$

Kirchhoff law

$$\tilde{S}_j = -s_j \qquad \tilde{S}_j = V_j \tilde{I}_j^*$$

power balance

$$\underline{s}_j \le s_j \le \overline{s}_j$$

$$\underline{V}_k \le |V_k| \le \overline{V}_k$$

Bus injection model: OPF

$$\min \quad \sum_{k \in G} \operatorname{tr} M_k W$$

over $\quad W$ positive semidefinite matrix

$$\text{s.t.} \quad \underline{P}_k \le \operatorname{tr} \Phi_k W \le \overline{P}_k$$

$$\underline{Q}_k \le \operatorname{tr} \Psi_k W \le \overline{Q}_k$$

$$\underline{V}_k^2 \le \operatorname{tr} J_k W \le \overline{V}_k^2$$

$$W \ge 0, \quad \cancel{\operatorname{rank} W = 1}$$

- only variable is matrix W
- obj & constraints are <u>linear</u> in W
- … except …

convex relaxation: SDP

Bus injection model: SDR

Non-convex QCQP

↓

Rank-constrained SDP

↓

Relax the rank constraint and solve the SDP — Bai 2008

↓

Does the optimal solution satisfy the rank-constraint?

yes → We are done!

Lavaei 2010, 2012
Radial: Bose 2011, Zhang 2011
Sojoudi 2011

no → Solution may not be meaningful

Lesiertre 2011

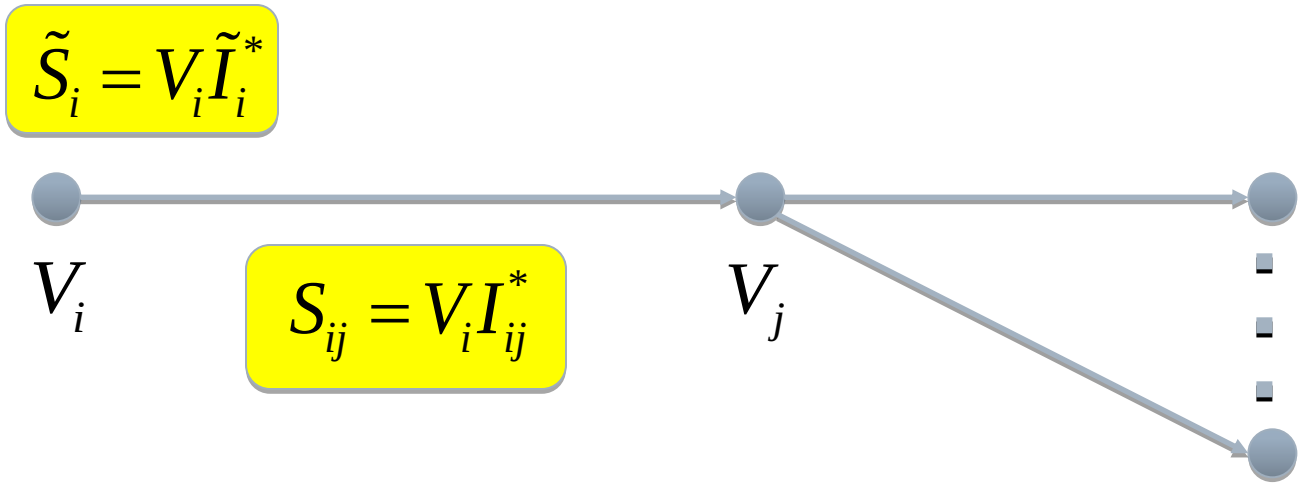
Bus injection model: summary

OPF = rank constrained SDP

Sufficient conditions for SDP to be exact

- Whether a solution is globally optimal is always easily checkable
- Mesh: must solve SDP to check
- Tree: depends only on constraint pattern or r/x ratios

Two models



1. What is the model?
2. What is OPF in the model?
3. What is the solution strategy?

Branch flow model

$$S_{ij} = V_i I_{ij}^* \qquad \text{for all } i \to j$$ power def

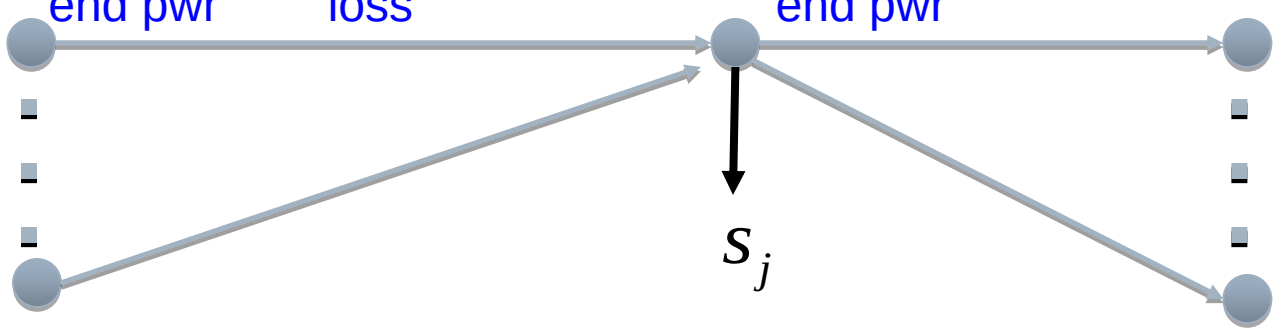
$$V_i - V_j = z_{ij} I_{ij} \qquad \text{for all } i \to j$$ Ohm's law

$$\sum_{i \to j} \left(S_{ij} - z_{ij} \left| I_{ij} \right|^2 \right) - \sum_{j \to k} S_{jk} = s_j \qquad \text{for all } j$$ power balance

sending end pwr

loss

sending end pwr



Branch flow model: OPF

$$\min \quad \sum_{i \sim j} r_{ij} \left| I_{ij} \right|^2 + \sum_{i} \alpha_i \left| V_i \right|^2$$

real power loss

CVR (conservation voltage reduction)

OPF

$$\min \quad f\left(h(x)\right)$$

$$\text{over} \quad x := (S, I, V, s^g, s^c)$$

$$\text{s. t.} \quad \underline{s}_i^g \le s_i^g \le \overline{s}_i^g \qquad \underline{s}_i \le s_i^c \qquad \underline{v}_i \le v_i \le \overline{v}_i$$

generation, VAR control

branch flow model

$$\sum_{i \to j} \left(S_{ij} - z_{ij} \left| I_{ij} \right|^2 \right) - \sum_{j \to k} S_{jk} = s_j^c - s_j^g$$

$$V_j = V_i - z_{ij} I_{ij} \qquad S_{ij} = V_i I_{ij}^*$$

OPF

$$\min \quad f\left(h(x)\right)$$

$$\text{over} \quad x := (S, I, V, s^g, s^c)$$

$$\text{s. t.} \quad \underline{s}_i^g \le s_i^g \le \overline{s}_i^g \qquad \underline{s}_i \le s_i^c \qquad \underline{v}_i \le v_i \le \overline{v}_i$$

demand response

branch flow model

$$\sum_{i \to j} \left(S_{ij} - z_{ij} \left| I_{ij} \right|^2 \right) - \sum_{j \to k} S_{jk} = s_j^c - s_j^g$$

$$V_j = V_i - z_{ij} I_{ij} \qquad S_{ij} = V_i I_{ij}^*$$

Theory

Branch flow model and OPF

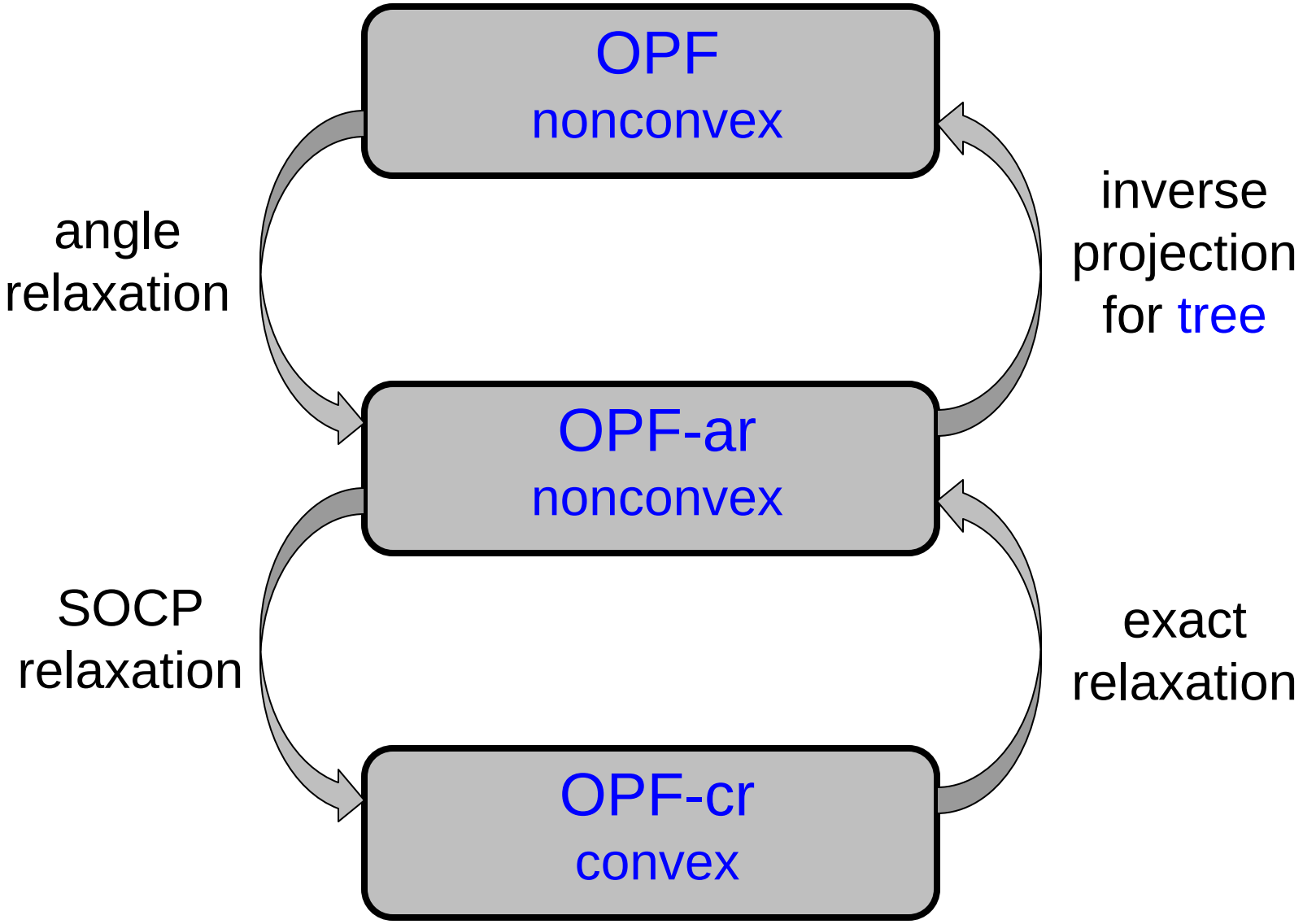
Solution strategy: two relaxations

- Angle relaxation
- SOCP relaxation

Convexification for mesh networks

Extensions

Solution strategy



OPF

$$\min \quad f\left(h(x)\right)$$

$$\text{over} \quad x := (S, I, V, s^g, s^c)$$

$$\text{s. t.} \quad \underline{s}_i^g \le s_i^g \le \overline{s}_i^g \qquad \underline{s}_i \le s_i^c \qquad \underline{v}_i \le v_i \le \overline{v}_i$$

branch flow model

$$\left\{ \begin{array}{l} \sum_{i \to j} \left(S_{ij} - z_{ij} \left| I_{ij} \right|^2 \right) - \sum_{j \to k} S_{jk} = s_j^c - s_j^g \\ V_j = V_i - z_{ij} I_{ij} \qquad S_{ij} = V_i I_{ij}^* \end{array} \right.$$

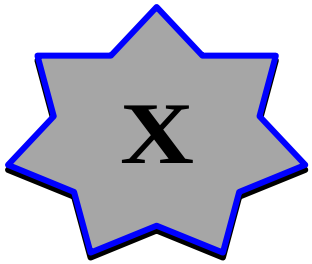
OPF

$$\min \quad f\left(h(x)\right)$$

$$\text{over} \quad x := (S, I, V, s^g, s^c)$$

$$\text{s. t.} \quad \underline{s}_i^g \le s_i^g \le \overline{s}_i^g \qquad \underline{s}_i \le s_i^c \qquad \underline{v}_i \le v_i \le \overline{v}_i$$

$$x \in \mathbf{X}$$



OPF-ar

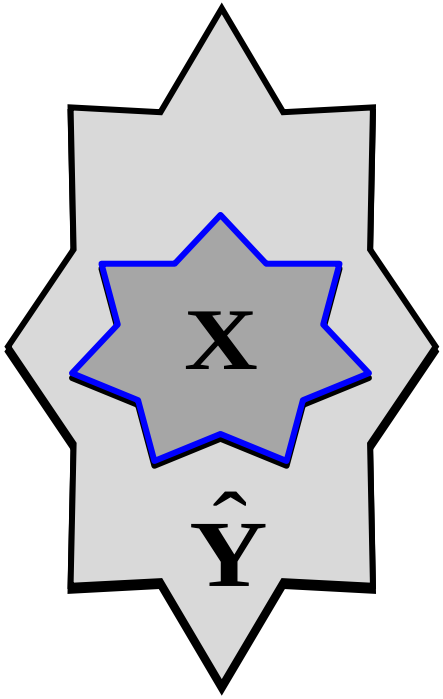
$$\min \quad f(\hat{y})$$

$$\text{over} \quad \hat{y} := (S, \ell, v, s^g, s^c)$$

$$\text{s. t.} \quad \underline{s}_i^g \le s_i^g \le \overline{s}_i^g \qquad \underline{s}_i \le s_i^c \qquad \underline{v}_i \le v_i \le \overline{v}_i$$

$$\hat{y} := h(x) \in \hat{\mathbf{Y}}$$

relax each voltage/current from a point in complex plane into a circle



OPF-cr

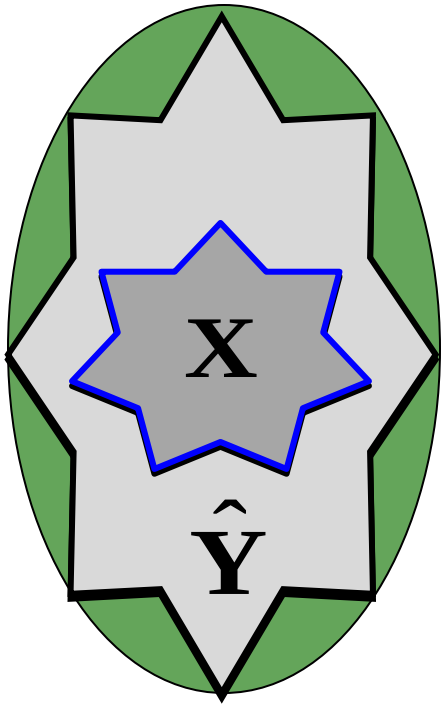
$$\min \quad f(\hat{y})$$

$$\text{over} \quad \hat{y} := (S, \ell, v, s^g, s^c)$$

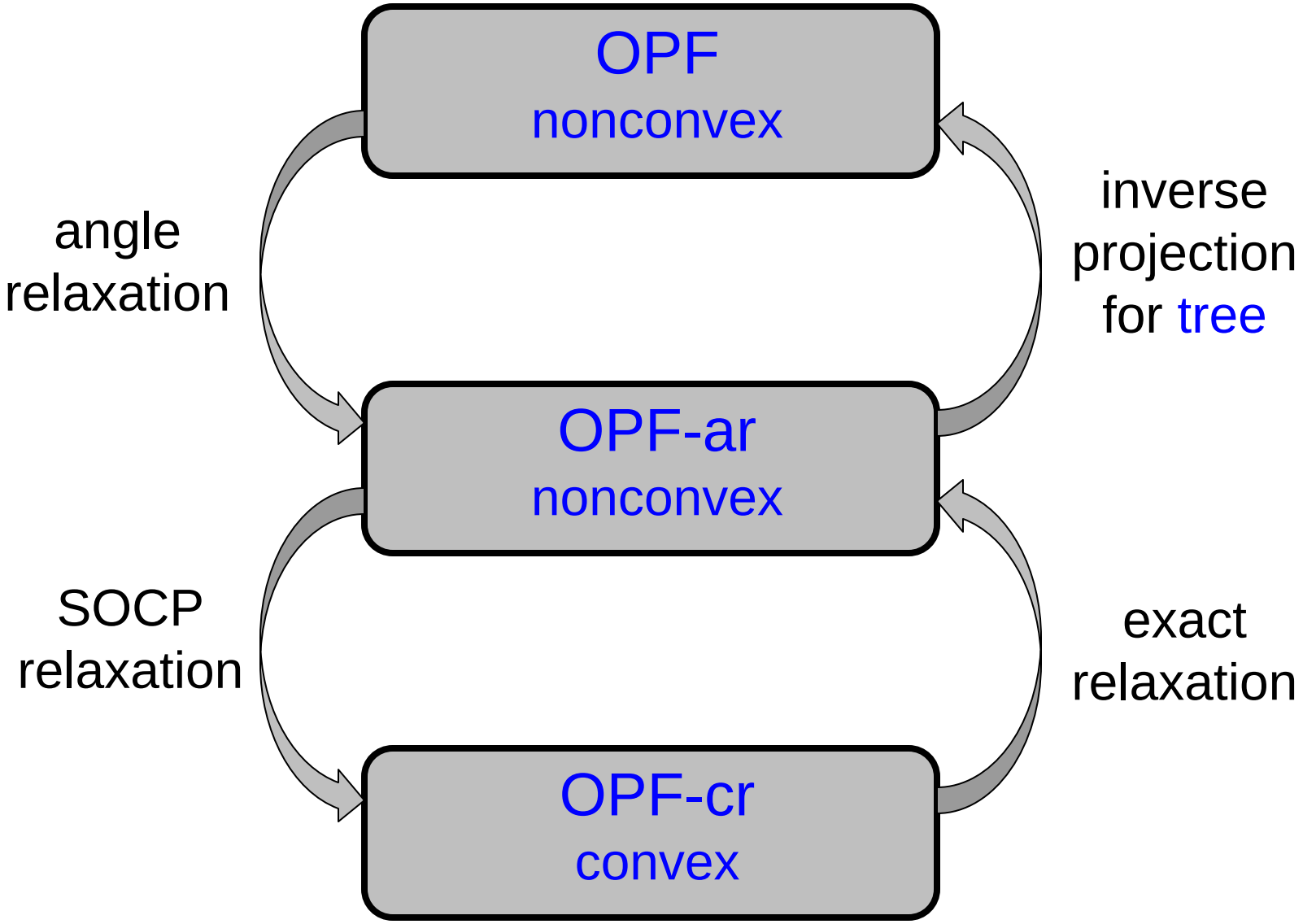
$$\text{s. t.} \quad \underline{s}_i^g \le s_i^g \le \overline{s}_i^g \qquad \underline{s}_i \le s_i^c \qquad \underline{v}_i \le v_i \le \overline{v}_i$$

$$\hat{y} \in \operatorname{conv} \hat{\mathbf{Y}}$$

relax to convex hull
(SOCP)



Recap so far ...


OPF
nonconvex
angle
relaxation
inverse
projection
for tree
OPF-ar
nonconvex
SOCP
relaxation
exact
relaxation
OPF-cr
convex

OPF-cr is exact relaxation

Theorem

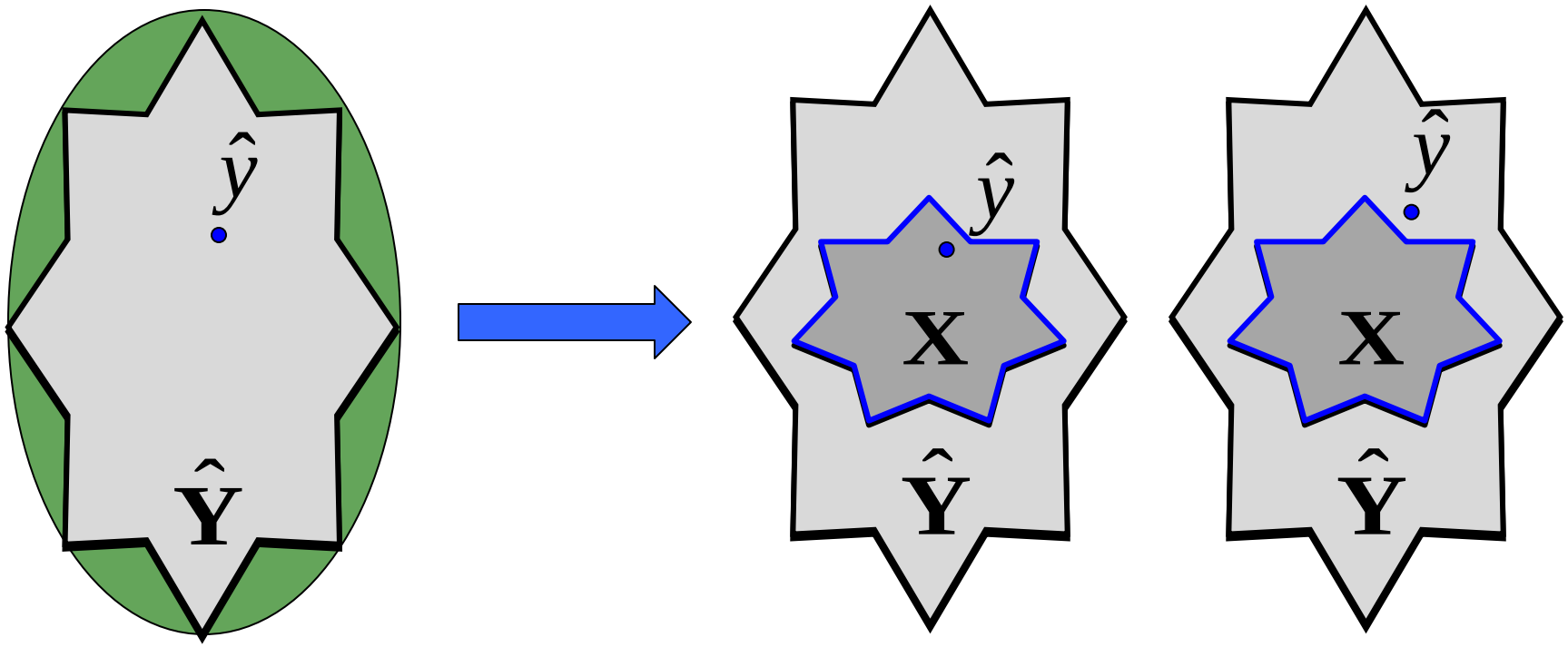
OPF-cr is convex (SOCP)

- much simpler than SDP

OPF-cr is exact

- optimal of OPF-cr is also optimal for OPF-ar
- for mesh as well as radial networks
- real & reactive powers, but volt/current mags

Angle recovery



OPF-ar

does there exist θ s.t.

$$h_\theta^{-1}(\hat{y}) \in \mathbf{X} \ ?$$

Angle recovery

<u>Theorem</u>

Inverse projection exist iff $\exists! \theta$ s.t.

$$B\theta = \beta(\hat{y})$$

incidence matrix;
depends on topology

depends on
OPF-ar solution

Two simple angle recovery algorithms

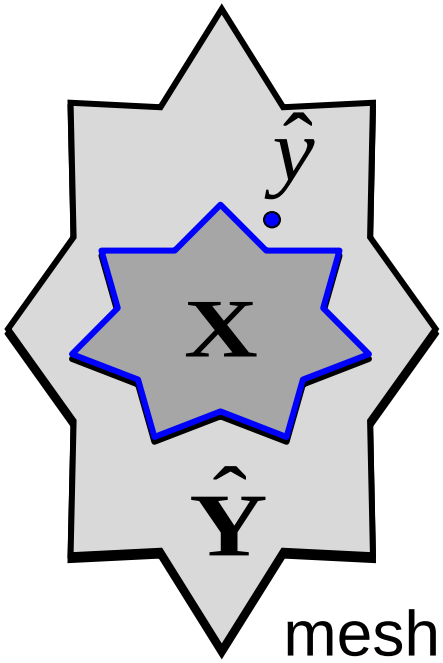
- centralized: explicit formula
- decentralized: recursive alg

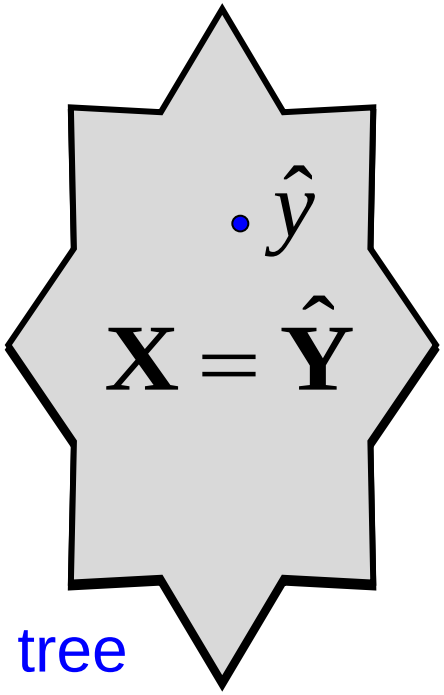
Angle recovery

<u>Theorem</u>

For radial network: $\exists!\,\theta$

$$B\theta = \beta(\hat{y})$$





OPF solution

Solve OPF-cr

SOCP

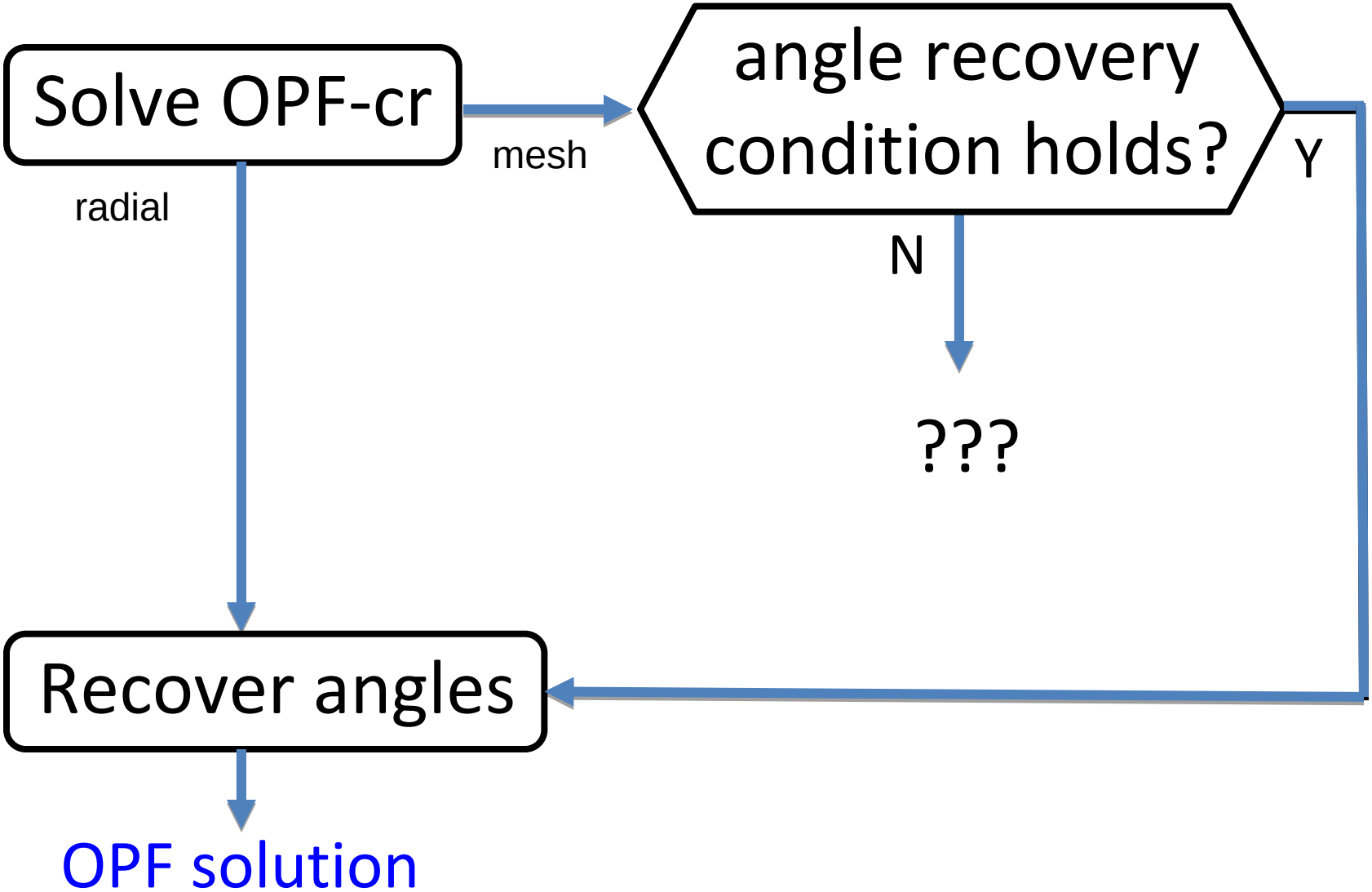
radial

Recover angles

- explicit formula
- distributed alg

OPF solution

OPF solution



Outline

Branch flow model and OPF

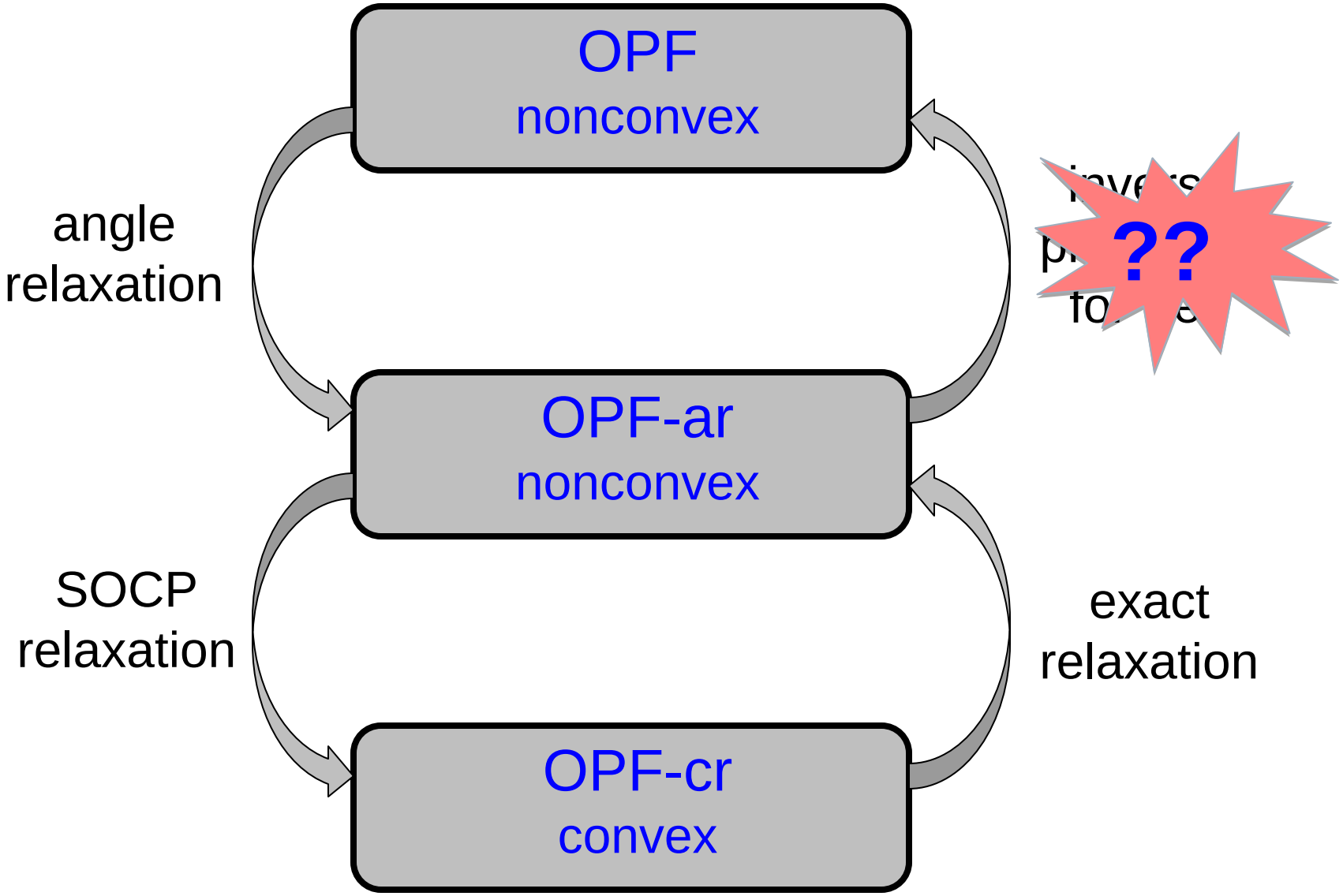
Solution strategy: two relaxations

- Angle relaxation
- SOCP relaxation

Convexification for mesh networks

Extensions

Recap: solution strategy



Convexification of mesh networks

OPF $\quad \min_{x} \; f\left(h(x)\right) \quad \text{s.t.} \quad x \in \mathbf{X}$

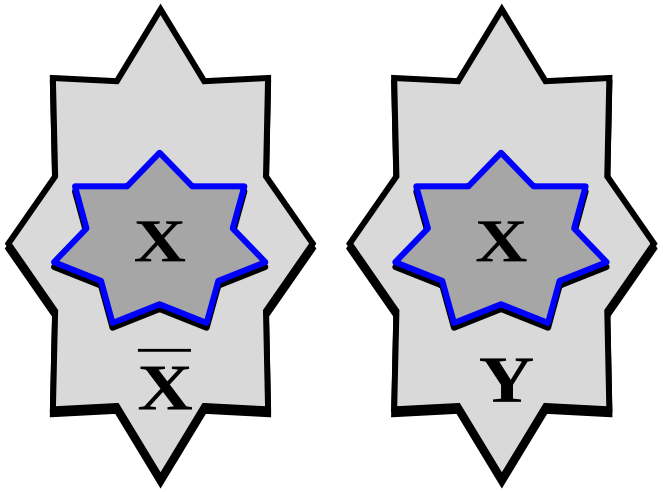
OPF-ar $\quad \min_{x} \; f\left(h(x)\right) \quad \text{s.t.} \quad x \in \mathbf{Y}$

OPF-ps $\quad \min_{x,\phi} \; f\left(h(x)\right) \quad \text{s.t.} \quad \left(x,\theta\right) \in \overline{\mathbf{X}}$

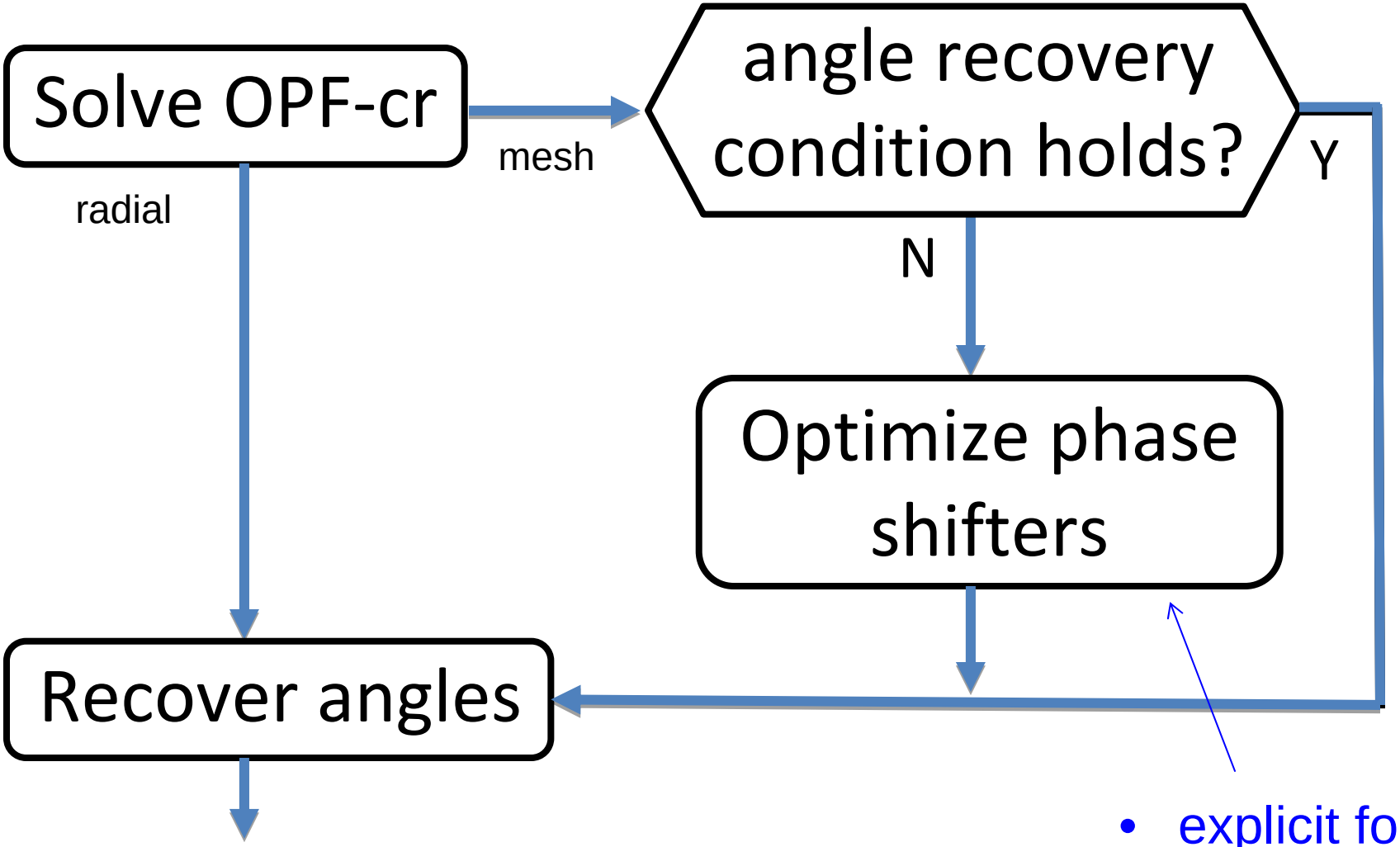
optimize over phase shifters as well

Theorem

- $\overline{\mathbf{X}} = \mathbf{Y}$
- Need phase shifters only outside spanning tree



OPF solution



- explicit formula
- distributed alg

Outline

Branch flow model and OPF

Solution strategy: two relaxations

- Angle relaxation
- SOCP relaxation

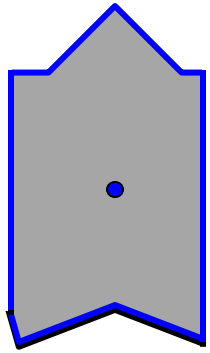
Convexification for mesh networks

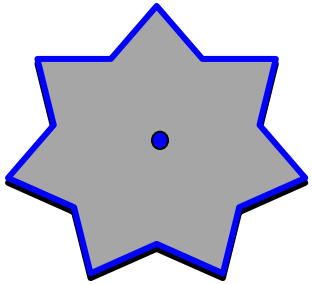
Extensions

Extension: equivalence

$$\tilde{\mathbf{X}} := \left\{ \tilde{x} = \left(\tilde{S}, \tilde{I}, V \right) \middle| \text{BI model} \right\}$$

$$\mathbf{X} := \left\{ x = \left(S, I, V \right) \middle| \text{BF model} \right\}$$



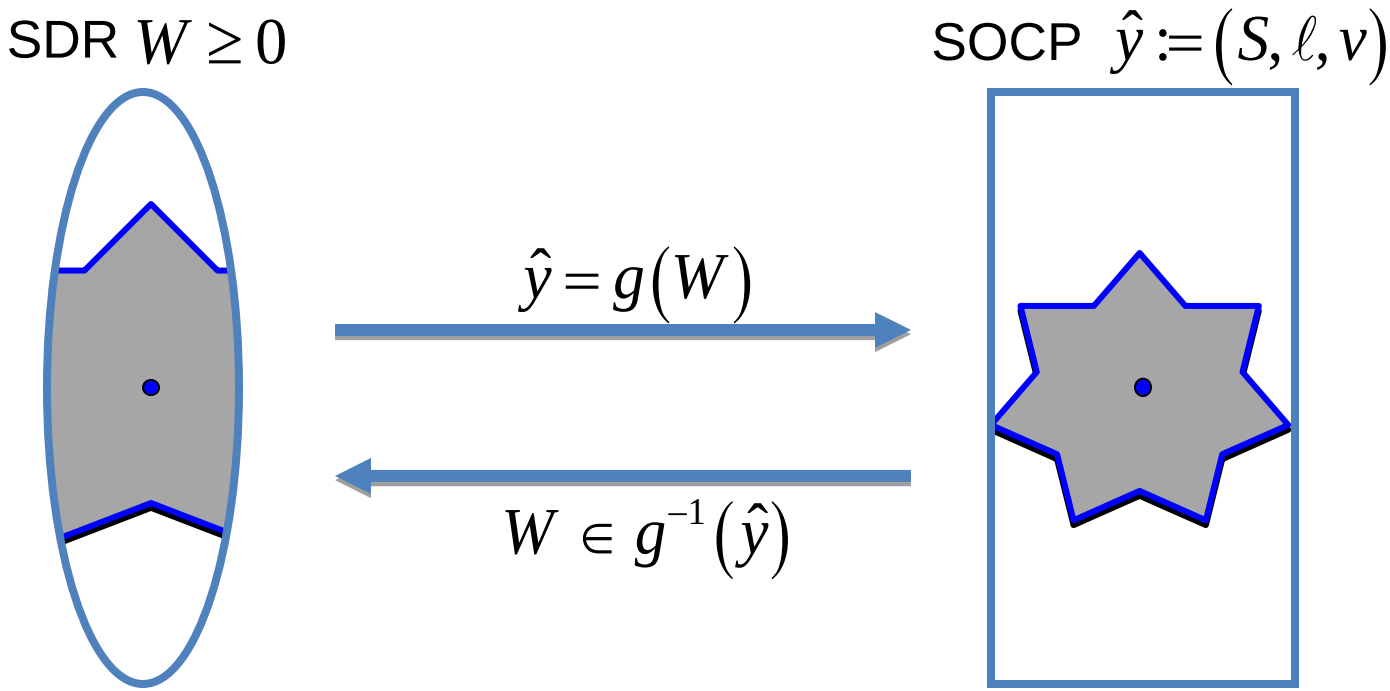


<u>Theorem</u>

BI and BF model are equivalent

(there is a bijection between $\tilde{\mathbf{X}}$ and $\mathbf{X}$)

Extension: equivalence



<u>Theorem</u>: radial networks

- $\tilde{y}$ in SOCP $\Longleftrightarrow$ W in SDR
- $\tilde{y}$ satisfies angle cond $\Longleftrightarrow$ W has rank 1

Work in progress with Subhonmesh Bose, Mani Chandy

Backup Slides

PCPM algorithm

Problem: $$\min_{x \in X, z \in Z} \quad f(x) + g(z)$$
$$\text{s.t.} \quad Ax = z$$

Idea: Introduce Lagaranian variables y for $Ax=z$

Solve the saddle point of the Lagaranian function

$L(x,y,z)$ $$L(x, y, z) = f(x) + g(z) + y^T(Ax - z)$$

Algorithm: For each step k,

Predictor: $$p^k = y^k + \lambda(Ax^k - z^k)$$

$$x^{k+1} = \arg\min\{f(x) + < p^{k+1}, Ax > + (1/(2\lambda))||x - x^k||^2\}$$
$$z^{k+1} = \arg\min\{g(z) - < p^{k+1}, z > + (1/(2\lambda))||z - z^k||^2\}$$

Corrector: $$y^{k+1} = y^k + \lambda(Ax^{k+1} - z^{k+1})$$

Predictor Corrector Proximal Multiplier (PCPM) method

using branch flow in tree networks

$$\max_{P,Q,l,v,p,q} \quad \sum_{i=1}^{n} f_i(p_i) - \rho \sum_{(i,j)\in E} r_{i,j} l_{i,j} - C(P_0)$$

$$s.t. \quad \boxed{\begin{aligned} P_{i,j} &= \sum_{h:(j,h)\in E} P_{j,h} + r_{i,j}\ell_{i,j} + p_j, \\ Q_{i,j} &= \sum_{h:(j,h)\in E} Q_{j,h} + x_{i,j}\ell_{i,j} + q_j, \\ v_i - v_j &= 2(r_{i,j}P_{i,j} + x_{i,j}Q_{i,j}) - (r_{i,j}^2 + x_{i,j}^2)\ell_{i,j}, \\ P_0 &= \sum_{h:(0,h)\in E} P_{0,h} \end{aligned}}$$

$$\boxed{\begin{aligned} & P_0 \le \bar{P}_0 \\ & \frac{P_{i,j}^2 + Q_{i,j}^2}{v_i} \le l_{i,j}, \\ & \underline{v}_i \le v_i \le \bar{v}_i, \underline{q}_i \le q_i \le \bar{q}_i, \ \underline{p}_i \le p_i \le \bar{p}_i, \end{aligned}}$$

Linear coupled local constraints Ax=z

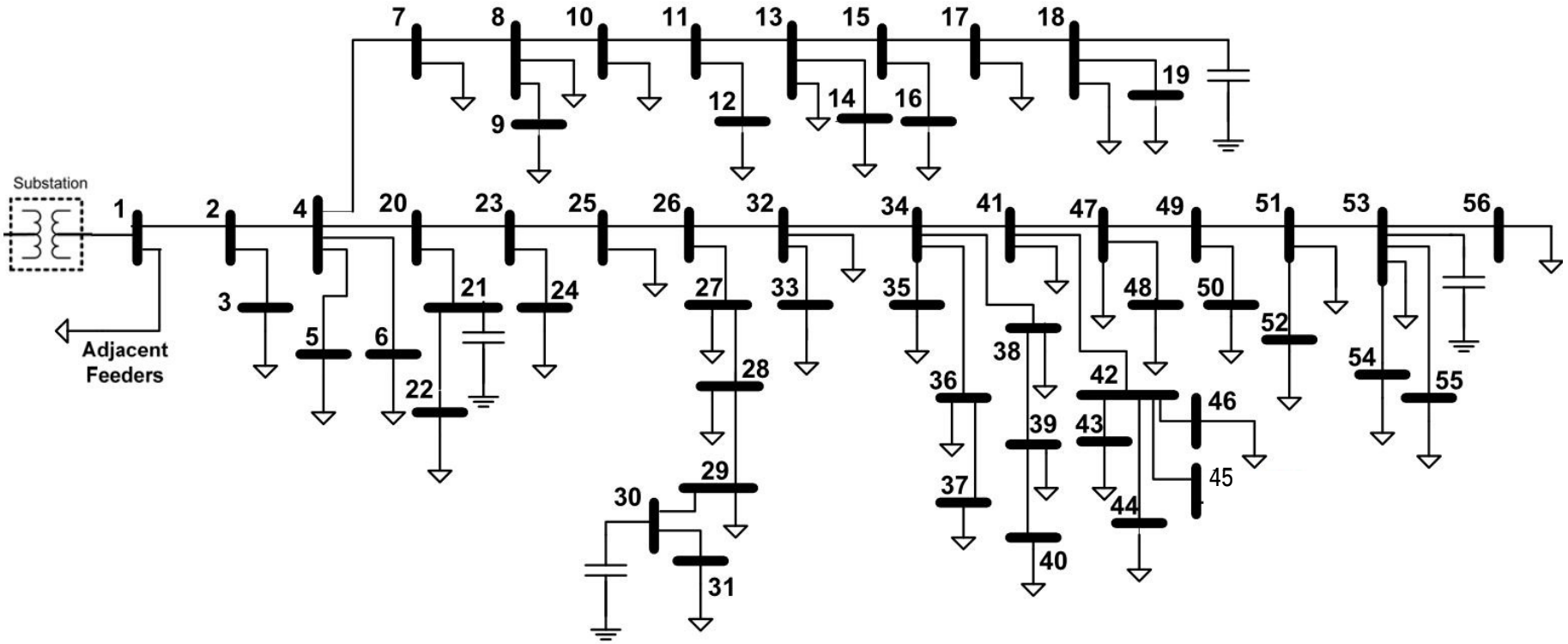
Local constraints

Local variables for each bus i:

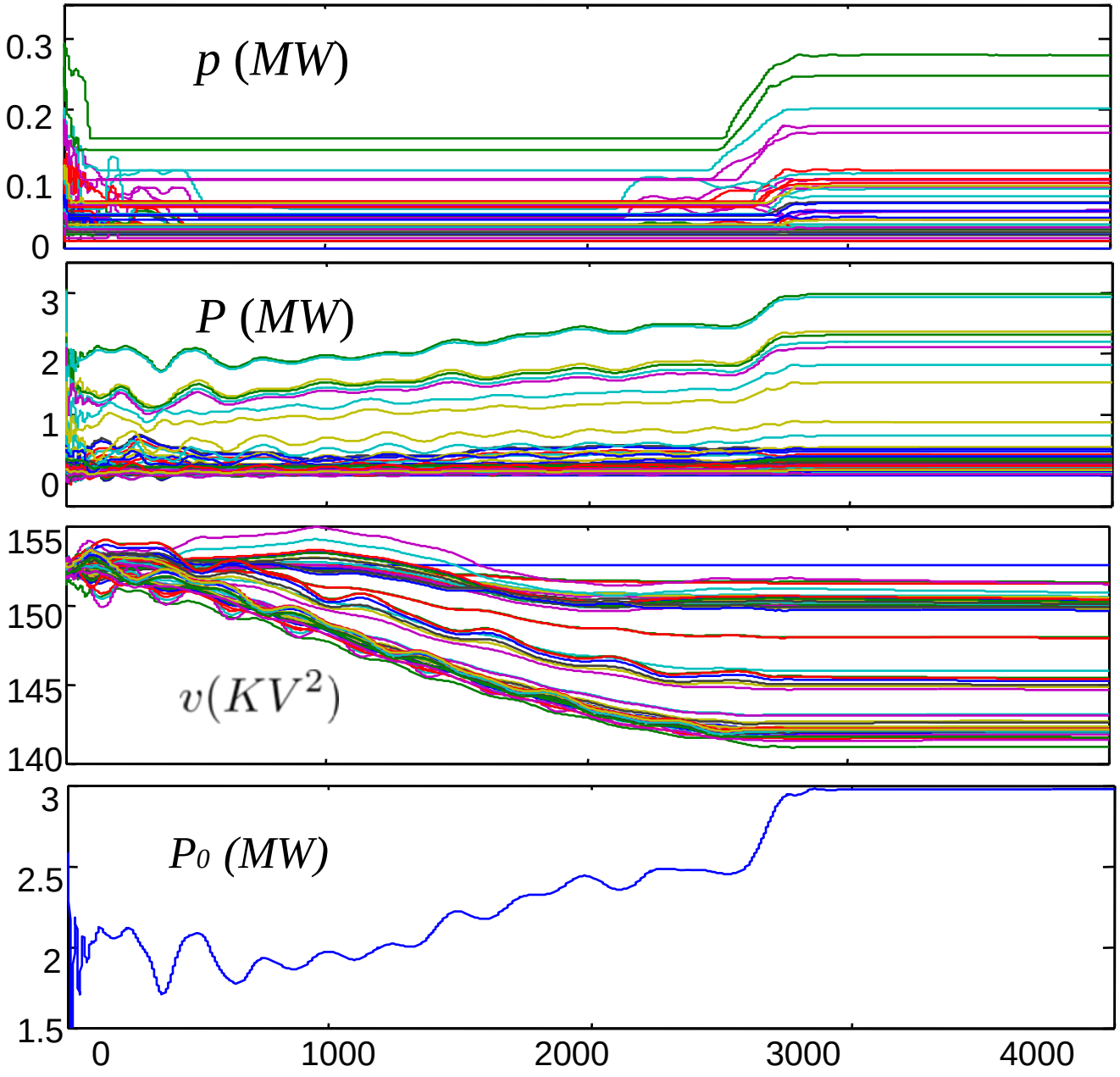
$$P_{\pi(i),i}, Q_{\pi i,i}, \ell_{\pi(i),i}, p_i, q_i, v_i$$

Corresponding local Lagrangian multipliers

Case study



Schematic Diagram of a SCE distribution System


p (MW)
0.3
0.2
0.1
0
P (MW)
3
2
1
0
155
150
145
140
$v(KV^2)$
P0 (MW)
3
2.5
2
1.5
0
1000
2000
3000
4000