# An Efficient Computational Method for Large-Scale Operations Planning

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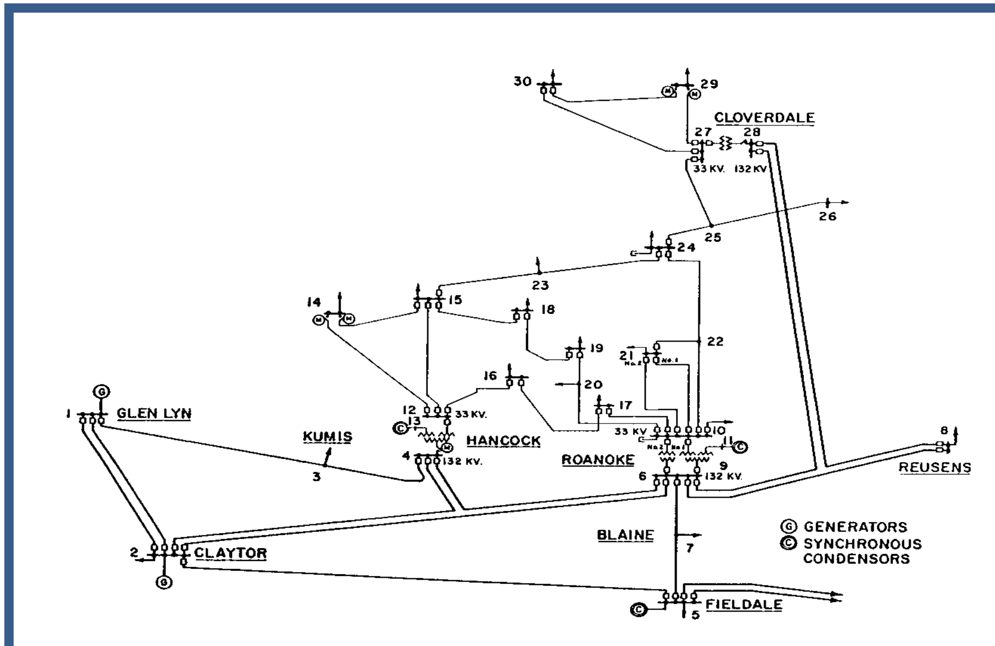
- J. Lavaei and S. Low, "Zero Duality Gap in Optimal Power Flow Problem," IEEE Transactions on Power Systems, 2012.
- J. Lavaei, "Zero Duality Gap for Classical OPF Problem Convexifies Fundamental Nonlinear Power Problems," in American Control Conference, 2011.
- J. Lavaei, D. Tse and B. Zhang, "Geometry of Power Flows in Tree Networks," in IEEE Power & Energy Society General Meeting, 2012.
- S. Sojoudi and J. Lavaei, "Physics of Power Networks Makes Hard Optimization Problems Easy To Solve," in IEEE Power & Energy Society General Meeting, 2012.
- J. Lavaei and S. Sojoudi, "Competitive Equilibria in Electricity Markets with Nonlinearities," in American Control Conference, 2012.
- M. Kraning, E. Chu, J. Lavaei and S. Boyd, "Message Passing for Dynamic Network Energy Management," Submitted for publication, 2012.

# Power Networks *(CDC 10, Allerton 10, ACC 11, TPS 11, ACC 12, PGM 12)*

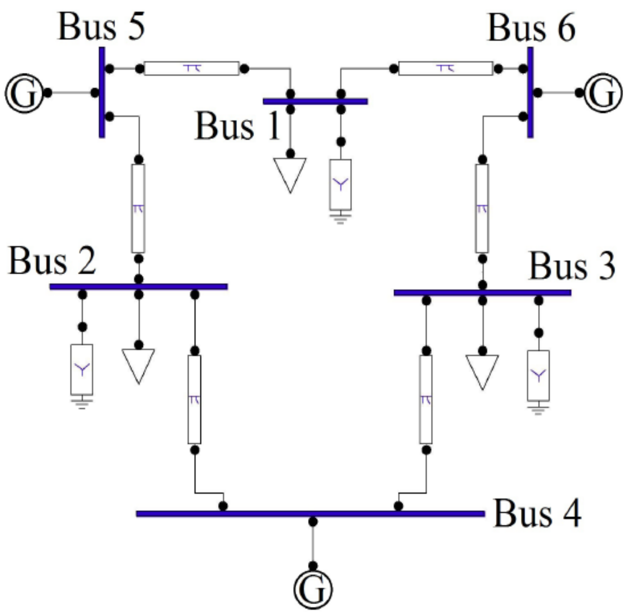
- ❑ **Optimizations:**
  - Resource allocation
  - State estimation
  - Scheduling




- ❑ **Issue:** Nonlinearities (power being quadratic in voltage)

- ❑ **Transition from traditional grid to smart grid:**
  - More variables (10X)
  - Time constraints (100X)

# Resource Allocation: Optimal Power Flow (OPF)



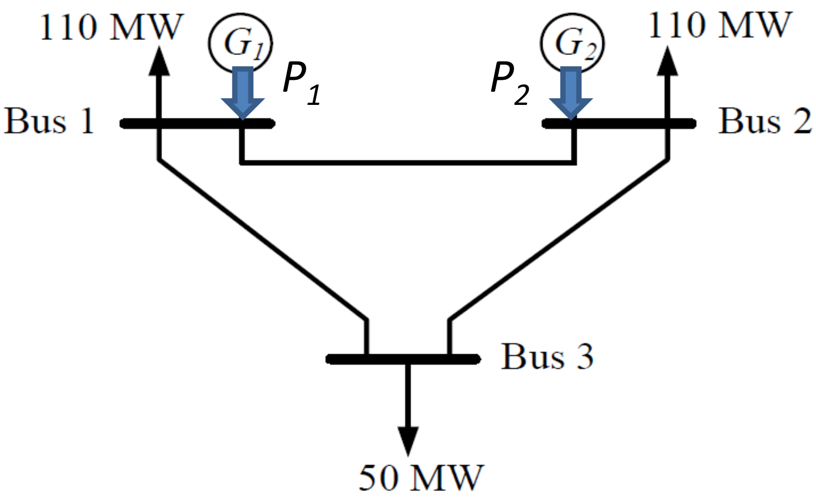

**OPF:** Given constant-power loads, find optimal $P$'s subject to:

- Demand constraints
- Constraints on $V$'s, $P$'s, and $Q$'s.

# Broad Interest in Optimal Power Flow

- OPF solved on different time scales:
  - Electricity market
  - Real-time operation
  - Security assessment
  - Transmission planning
- Existing methods based on linearization or local search algorithms
- Can save **$$$** if solved efficiently
- Huge literature since 1962 by power, OR and Econ people
  - Linear programming
  - Nonlinear programming
  - Lagrangian relaxation
  - Genetic algorithm …

# Local Solutions




**OPF** →

$$\text{minimize } f_1(P_1) + f_2(P_2)$$

- **Case 1:** $|V_1| = |V_2| = |V_3| = 0.8$

  $$\text{Local solution} = 1502.64, \qquad \text{Global solution} = 602.20$$

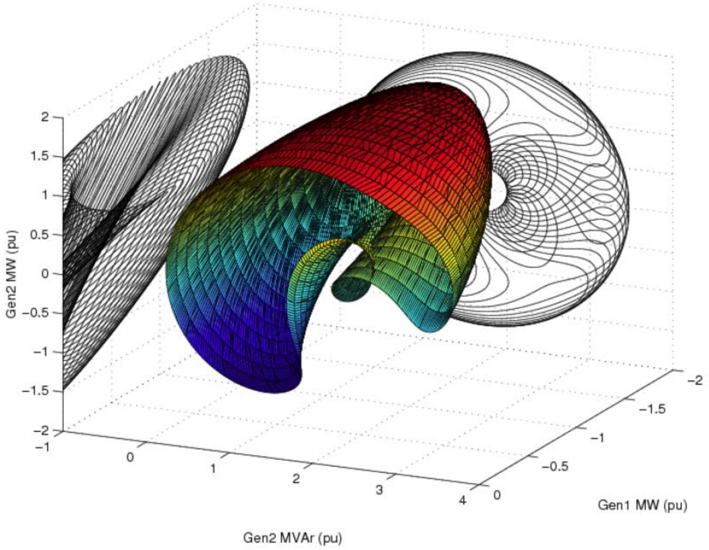
- **Case 2:** $0.8 \leq |V_1|, |V_2|, |V_3| \leq 1.2$

  $$\text{Local solution} = 1502.64, \qquad \text{Global solution} = 338.0$$

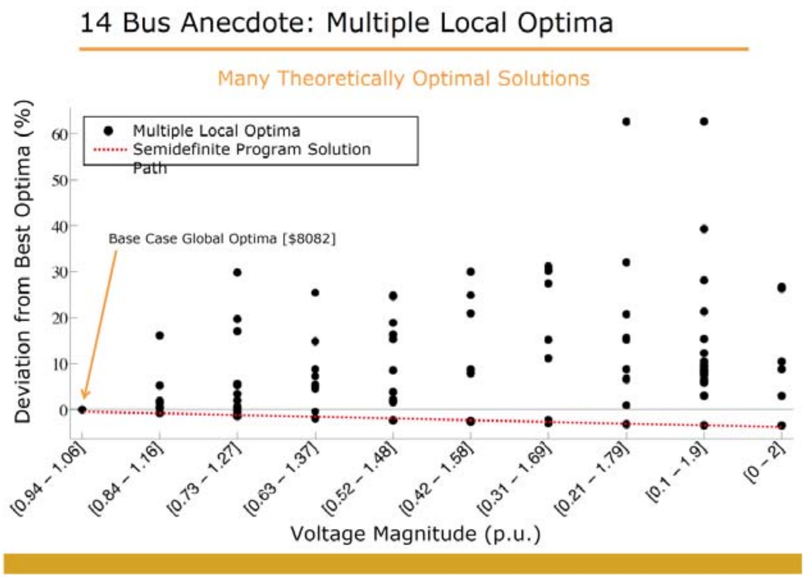
# Local Solutions

**Source of Difficulty:** Power is quadratic in terms of complex voltages.

Ian Hiskens from Umich:




*Anya Castillo et al.*



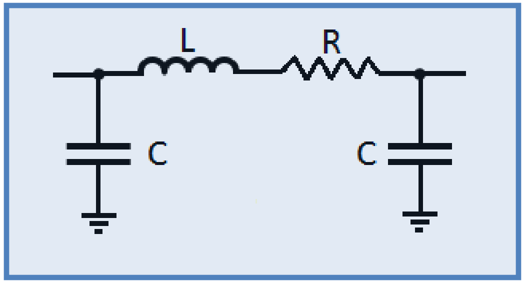

- ❑ Study of local solutions by Edinburgh's group

# Summary of Results

**Project 1: How to solve a given OPF in polynomial time? (joint work with Steven Low)**

- A sufficient condition to globally solve OPF:
  - Numerous randomly generated systems
  - IEEE systems with 14, 30, 57, 118, 300 buses
  - European grid
  - California grid with various load profiles

- **Various theories:** It holds widely in practice




- Generalizable to many optimizations in smart grids

# Summary of Results

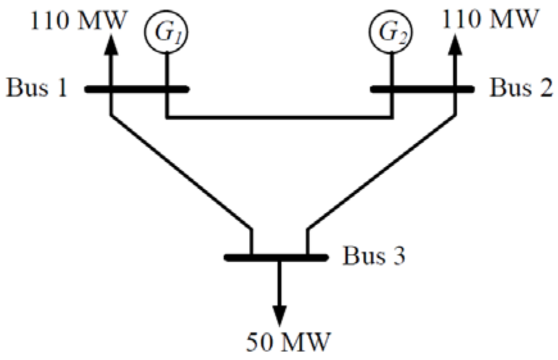
**Project 2:** **Find network topologies over which optimization is easy? (joint work with Somayeh Sojoudi, Baosen Zhang and David Tse)**

- ❑ Distribution networks are fine.
- ❑ Every transmission network can be turned into a good one.

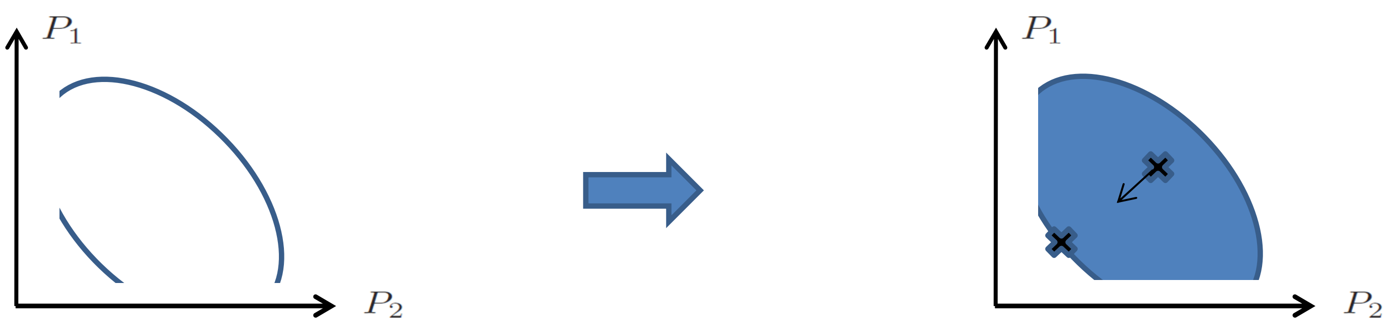
**Project 3:** **How to design a parallel algorithm for solving OPF? (joint work with Eric Chu, Matt Kraning and Stephen Boyd)**

- ❑ A practical (infinitely) parallelizable algorithm
- ❑ It solves 10,000-bus OPF in 0.85 seconds on a single core machine.

# Geometric Intuition: Two-Generator Network




$$\text{minimize } f_1(P_1) + f_2(P_2)$$
$$\text{subject to } (P_1, P_2) \in \mathcal{P}$$




$$\text{minimize } f_1(P_1) + f_2(P_2)$$
$$\text{subject to } (P_1, P_2) \in \mathcal{P}$$

$$\text{minimize } f_1(P_1) + f_2(P_2)$$
$$\text{subject to } (P_1, P_2) \in \text{conv}(\mathcal{P})$$

# Optimal Power Flow

$$\min_{\mathbf{V},\mathbf{P}_G,\mathbf{Q}_G} \quad \sum_{k\in\mathcal{G}} f_k(P_{G_k}) \qquad (1a)$$

Subject to

$$P_k^{\min} \le P_{G_k} \le P_k^{\max} \qquad (1b)$$

$$Q_k^{\min} \le Q_{G_k} \le Q_k^{\max} \qquad (1c)$$

$$V_k^{\min} \le |V_k| \le V_k^{\max} \qquad (1d)$$

$$\mathrm{Re}\left\{V_l(V_l - V_m)^* y_{lm}^*\right\} \le P_{lm}^{\max} \qquad (1e)$$

$$\mathrm{trace}\{\mathbf{V}\mathbf{V}^*\mathbf{Y}^* e_k e_k^*\} = P_{G_k} - P_{D_k} + (Q_{G_k} - Q_{D_k})\mathrm{i} \qquad (1f)$$

- (1a) → Cost
- (1b)–(1d) → Operation
- (1e) → Flow
- (1f) → Balance

❑ Extensions:

- Other objective (voltage support, reactive power, deviation)
- More variables, e.g. capacitor banks, transformers
- Preventive or corrective contingency constraints
- Multi-period OPF

❑ Conventional OPF captures common sources of non-convexity.

# Optimal Power Flow

- Express balance equations as inequalities:

$$\min_{\mathbf{V},\mathbf{P}_G,\mathbf{Q}_G} \quad \sum_{k\in\mathcal{G}} f_k(P_{G_k}) \tag{2a}$$

$$\text{Subject to} \quad P_k^{\min} \leq P_{G_k} \leq P_k^{\max} \tag{2b}$$

$$Q_k^{\min} \leq Q_{G_k} \leq Q_k^{\max} \tag{2c}$$

$$V_k^{\min} \leq |V_k| \leq V_k^{\max} \tag{2d}$$

$$\text{Re}\{V_l(V_l - V_m)^* y_{lm}^*\} \leq P_{lm}^{\max} \tag{2e}$$

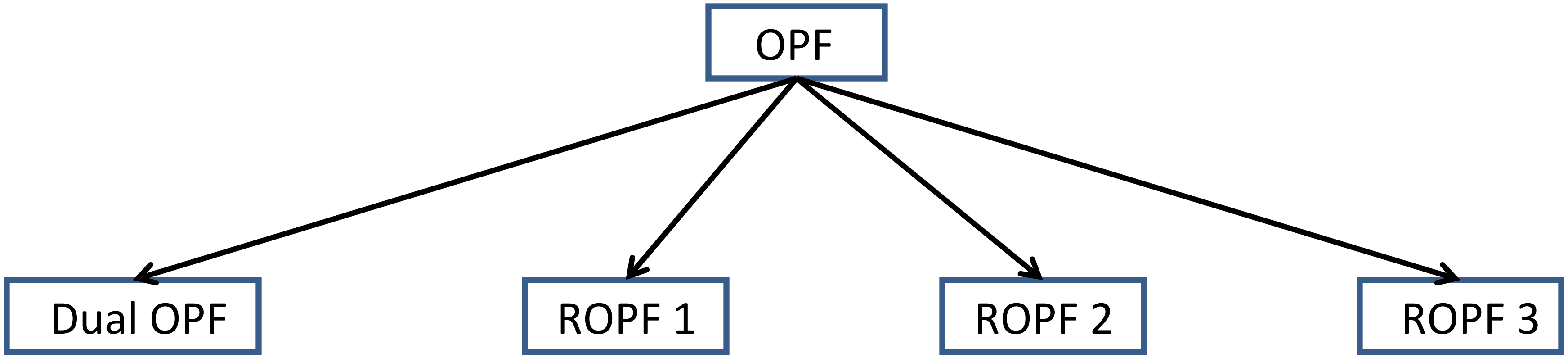
$$\text{trace}\{\mathbf{V}\mathbf{V}^*\mathbf{Y}^* e_k e_k^*\} \leq P_{G_k} - P_{D_k} + (Q_{G_k} - Q_{D_k})i \tag{2f}$$

- Proof without this change involves geometric techniques.

- Allow power over-delivery or assume positive LMPs.

**Trick:** Replace $\mathbf{V}\mathbf{V}^*$ with a matrix $\mathbf{W} \succeq 0$ subject to $\text{rank}\{\mathbf{W}\} = 1$.

# Various Relaxations



# Zero Duality Gap

- OPF:
  - Real-valued (DC)
  - Complex-valued (AC)

- Networks:
  - Distribution (acyclic)
  - Transmission (cyclic)

**Theorem**

*Zero duality gap for OPF in two cases:*

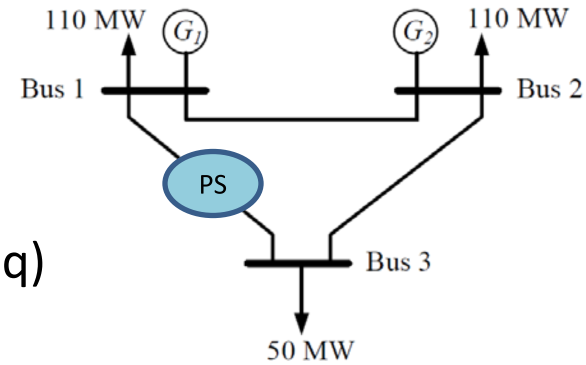
- *DC/AC distribution networks*
- *DC transmission networks*

- DC lines are becoming more deployed (Nordic circuit).
- Theory applies to scheduling of EVs charging, control of PV invertors,...

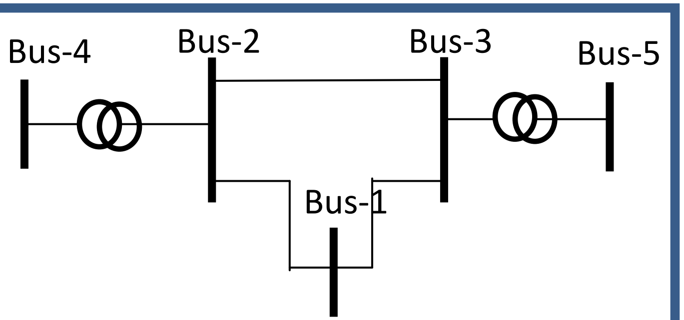
# AC Transmission Networks

- ❑ **How about AC transmission networks?**
  - May not be true for every network
  - Various sufficient conditions

❑ **Result 1:** AC transmission network manipulation:
- High performance (lower generation cost)
- Easy optimization
- Easy market (positive LMPs and existence of Eq)




❑ **Result 2:** Reduced computational complexity



# Simulations

## Simulations:

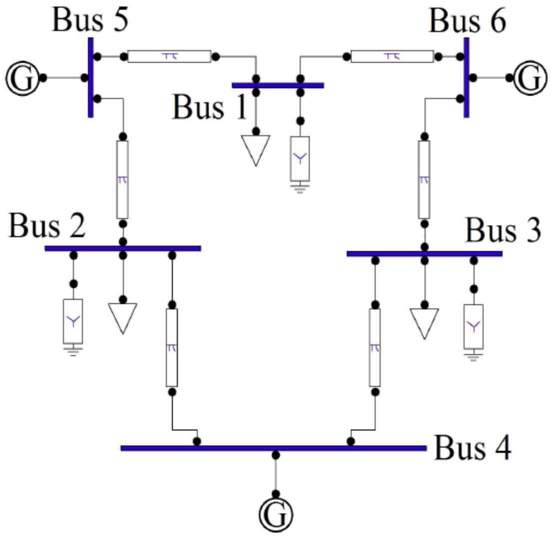
- Zero duality gap for IEEE 30-bus system
- Guarantee zero duality gap for all possible load profiles?
- **Theoretical side:** Add 12 phase shifters
- **Practical side:** 2 phase shifters are enough
- IEEE 118-bus system needs no phase shifters (power loss case)

❖ Don't have phase shifters:

- ➢ **Add virtual phase shifters and then do a local search**
- ➢ **Add virtual phase shifters and penalize their effect**
- ➢ **Take a direct approach**

# Conclusions

- ❑ **Focus:** OPF with a 50-year history
- ❑ **Goal:** Find a global solution efficiently

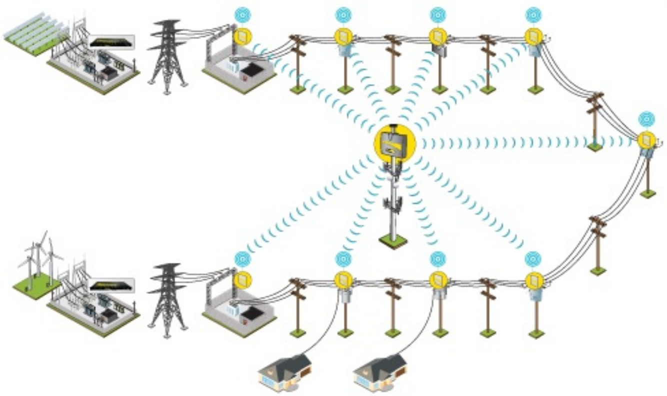



- ❑ Obtained provably global solutions for many practical OPFs
- ❑ Developed various theories for distribution and transmission networks
- ❑ Talked about parallel implementation via ADMM

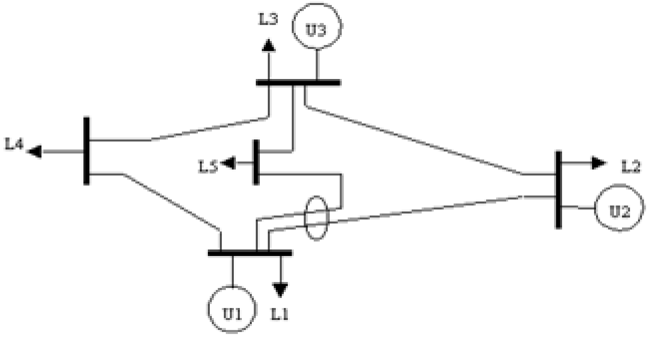
# Future Directions on OPF

- **Dynamics related to smart grid:**
  - Renewable, EVs

- **Effect of communication**



- **Discrete variables:**
  - Unit commitment, switches




- **Network design:**
  - Phase shifter placement