

# AUCTION DESIGN FOR WHOLESALE ELECTRICITY MARKETS

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by  
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# Auction Design for Wholesale Electricity Markets *Outline*



- Electricity Market Design
  - Structure/concepts/objectives
- Market Commodity Clearing
  - Economic efficiency
  - Fenchel decomposition
  - Coordination process and convergence criterion
- Market Commodity Pricing
  - Vickrey payments and participation tickets
  - Incentive compatibility
  - Voluntary participation
  - Market budget balancing
- Summary



# Electricity Market Design

## *Proposal History*

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- 1986: Economic Reform of Electricity Industry
  - Generators, loads and network as atomic economic entities
- 1990: Existence and Optimality of Market Equilibrium
  - Walras model and Pareto optimality
- 1995: Fenchel Duality and Decomposition
  - Convex and continuous market model formulation
- 1999: Standard Market Design
  - Pricing scheme: worse of ex-ante LMP or ex-post as-bid price
- 2005: MIP Co-Optimization Engine
  - Industry leading performances and functionality
- 2010: Auction Design
  - Efficient, incentive compatible, rational and budget balanced



# Electricity Market Design

## *Theoretical Concepts*

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- Microeconomics
  - Mathematical modeling instead of “economic lab”
  - Existence and optimality of competitive equilibrium
- Game Theory
  - Information availability and game strategy concepts
  - Existence and value of core of game
- Auction theory
  - Incentive compatibility and volunteer participation
  - Allocation efficiency and budget balance
- Convex Analysis
  - Duality theory and decomposition
  - Optimization software technology

Note: • All these concepts converged into  
Algorithmic Mechanism Design



# Electricity Market Design

## *Auction Design Principles*

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### 1. Economic Efficiency

- Maximal overall market benefits; *optimal equilibrium*

### 2. Incentive Compatibility

- Truthful submission of operating costs; *no market power*

### 3. Volunteer Participation

- No negative market outcomes; *no need for uplift*

### 4. Market Balance

- Market commodity and budget balance; *no market subsidiary*

### □ *Impossibility Theorem*

*It is not possible to comply with all four principles under general conditions with three or more market participants*



# Electricity Market Design

## *LMP Market vs. Design Principles*

### LMP-Based Market

#### Market efficiency maximization

- Clearing non-truthful bids/offers

#### Market power always exists

- Market power mitigation is necessary

#### No guaranteed profitability

- Uplift payments are necessary

#### Market budget deficit

- Due to out-of-market payments



### Design Principles

#### 1. Economic Efficiency

- Maximize total market profit

#### 2. Incentive Compatibility

- Truthful submissions

#### 3. Volunteer Participation

- Non-negative market outcomes

#### 4. Market Balance

- Commodity and budget balance

*Needed: Payment System Improvements*

- *Incentive compatible pricing scheme*
- *Energy block pricing*



# Market Commodity Clearing

## *Economic Efficiency*

Maximize economic efficiency of overall electricity market

$$\max_{x^G, x^L} \left\{ \sum_L f^L(x^L) - \sum_G f^G(x^G) \mid (x^G, x^L) \in (X^G \times X^L) \cap X^N \right\}$$

$$f^L(x^L) = \begin{aligned} & -\text{Load Curtailment Costs} \\ & + \text{Min En Demand Block Benefits} \\ & + \text{Inc En Demand Benefits} \\ & - \text{AS Costs} \end{aligned}$$

$$f^G(x^G) = \begin{aligned} & \text{Generation Startup Costs} \\ & + \text{Min En Supply Block Benefits} \\ & + \text{Inc En Supply Benefits} \\ & + \text{AS Costs} \end{aligned}$$



# Market Commodity Clearing *Technology Sets*

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- Supply Technology Set

$$X^G = \text{conv} \left\{ \begin{array}{l} \textit{Power Output Limits : } x_{\min}^G \leq x^G \leq x_{\max}^G \\ \textit{Ramping Limits : } R_{dn}^G \leq \Delta x^G \leq R_{up}^G \\ \textit{Minimum ON / OFF Time} \\ \textit{Max Number of Starts} \end{array} \right.$$

- Demand Technology Set

$$X^L = \text{conv} \left\{ \begin{array}{l} \textit{Power Demand Limits : } x_{\min}^L \leq x^L \leq x_{\max}^L \\ \textit{Ramping Limits : } R_{dn}^L \leq \Delta x^L \leq R_{up}^L \\ \textit{Maximum Curtailment Time} \\ \textit{Max Number of Curtailments} \end{array} \right.$$





# Market Commodity Clearing

## *Technology Sets cont.*

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- Network Technology Set

$$X^N = \text{conv} \left\{ \begin{array}{l} \text{Active Power Balance:} \quad P_i^G - P_i^L = \sum_{j \rightarrow i} P^{ij}(V^i, V^j, \theta^{ij}) \\ \text{Reactive Power Balance:} \quad Q_i^G - Q_i^L = \sum_{j \rightarrow i} Q^{ij}(V^i, V^j, \theta^{ij}) \\ \text{Transmission Constraints:} \quad |S^{ij}| = \sqrt{P^{ij^2} + Q^{ij^2}} \leq S_{ij}^{\max} \\ \text{Voltage Limits:} \quad V_{\min}^i \leq V^i \leq V_{\max}^i \end{array} \right.$$

- *Note*

*The supply, demand and transmission network technology sets are extended to form convex hulls*



# Market Commodity Clearing

## *Fenchel Decomposition*

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$$\max_{x^G, x^L} f(x^G, x^L) = \max_{x^G, x^L} \left\{ \sum_L f^L(x^L) - \sum_G f^G(x^G) \mid (x^G, x^L) \in (X^G \times X^L) \cap X^N \right\} =$$

$$\min_{c^G, c^L} \left\{ \sum_G \varphi^G(c^G) + \sum_{G,L} \varphi^N(c^G, c^L) + \sum_L \varphi^L(c^L) \right\} = \min_{c^G, c^L} \left\{ \varphi(c^G, c^L) \right\}$$

$$\varphi^G(c^G) = \max_{x^G} \left\{ c^G \cdot x^G - f^G(x^G) \mid x^G \in X^G \right\}$$

$$\varphi^N(c^G, c^L) = \max_{x^G, x^L} \left\{ c^L \cdot x^L - c^G \cdot x^G \mid (x^G, x^L) \in X^N \right\}$$

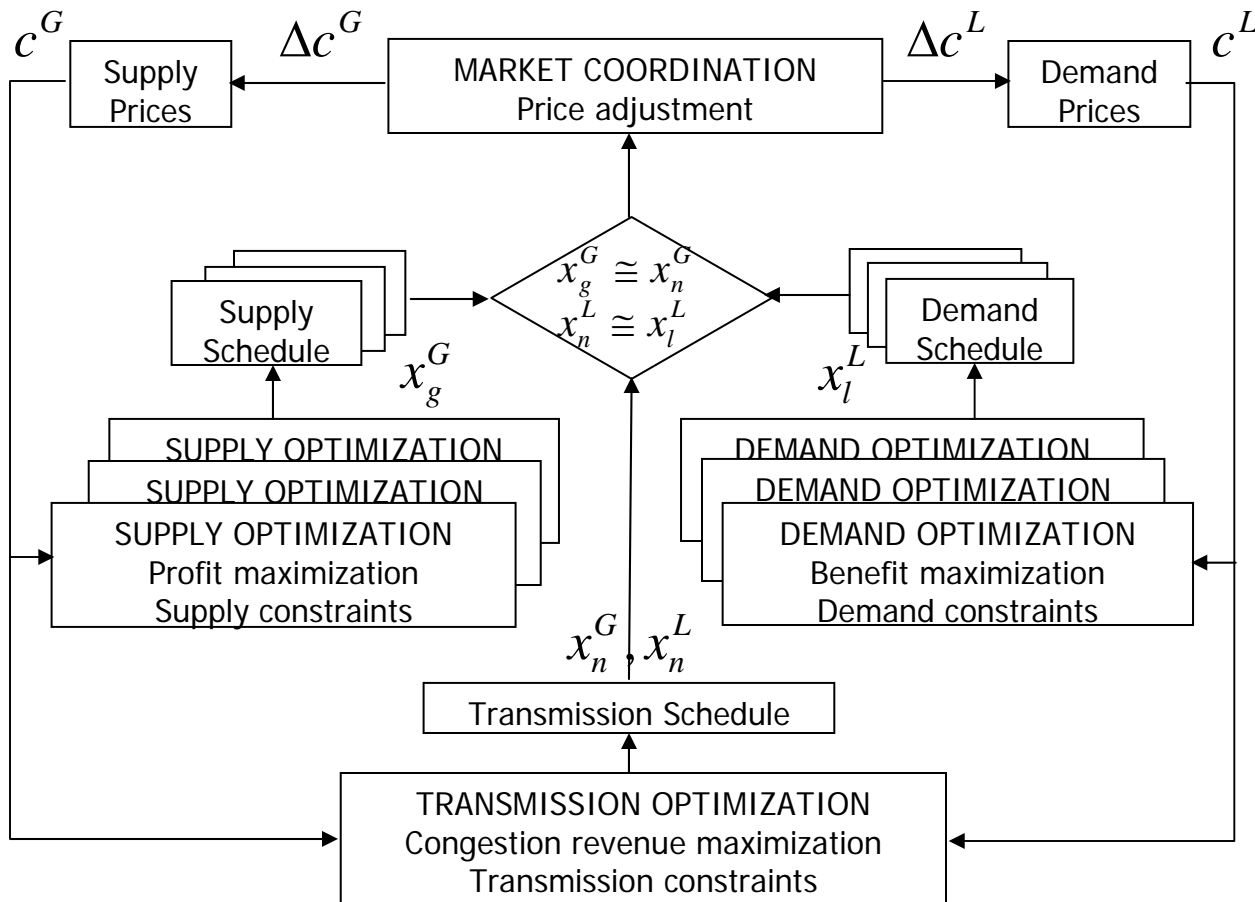
$$\varphi^L(c^L) = \max_{x^L} \left\{ f^L(x^L) - c^L \cdot x^L \mid x^L \in X^L \right\}$$

***Supply Profit + Network Profit + Demand Profit = Overall Market Profit***

- *Invisible Hand*

*If individual profit for each supply, demand and transmission entity is maximal then the overall market profit is maximal. (A. Smith)*

# Market Commodity Clearing ... and Coordination



# Market Commodity Clearing

## *Market Equilibrium*

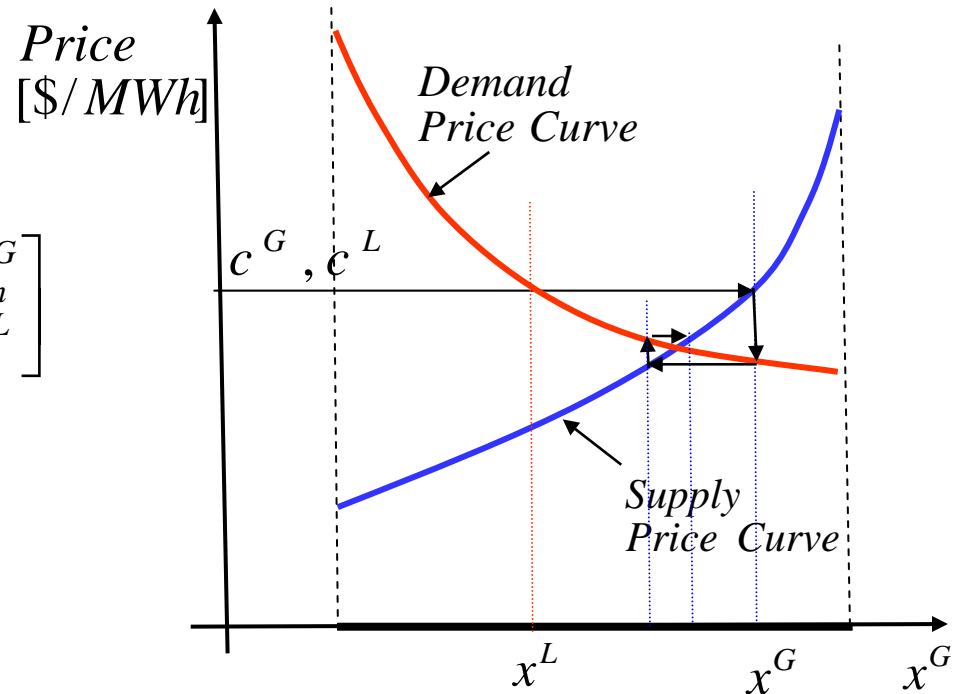
- Optimality Condition

$$\begin{bmatrix} x_g^G - x_n^G \\ x_n^L - x_l^L \end{bmatrix} = 0 \in \partial \varphi(c^G, c^L) = \partial \left\{ \sum_G \varphi^G(c^G) + \sum_{G,L} \varphi^N(c^G, c^L) + \sum_L \varphi^L(c^L) \right\}$$

- Price Adjustment

$$\begin{bmatrix} \Delta c^G \\ \Delta c^L \end{bmatrix}_{k+1} = \begin{bmatrix} c^G \\ c^L \end{bmatrix}_{k+1} - \begin{bmatrix} c^G \\ c^L \end{bmatrix}_k = - \begin{bmatrix} \alpha^G \\ \alpha^L \end{bmatrix} \cdot \begin{bmatrix} x_g^G - x_n^G \\ x_n^L - x_l^L \end{bmatrix}$$

- Supply/demand low



# Market Commodity Clearing

## *Supply Optimization*

- Profit Maximization Objective

$$\varphi^G(c^G) = \max_{x^G} \{c^G \cdot x^G - f^G(x^G) \mid x^G \in X^G\}$$

- Supply Technology Set  $X^G$

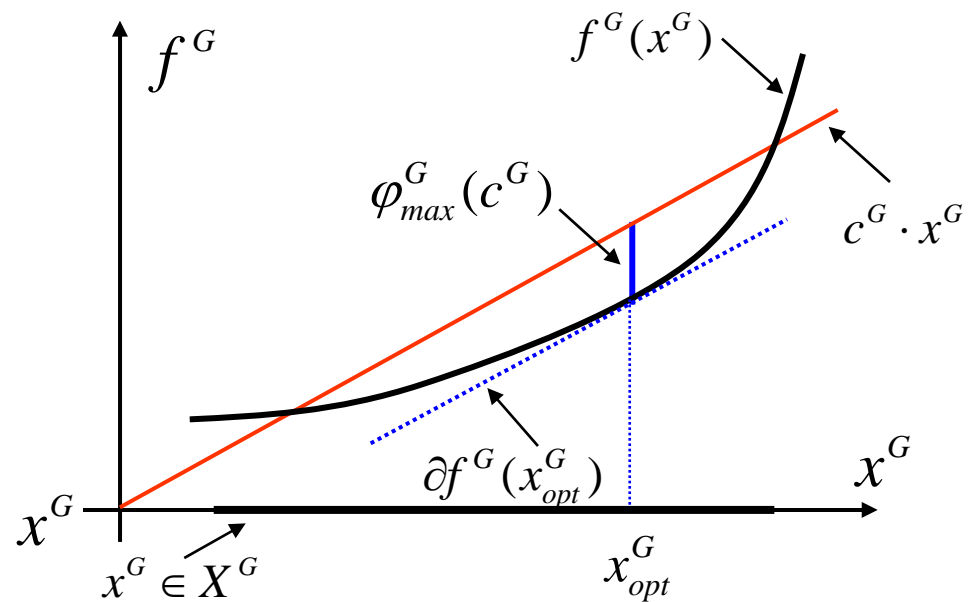
*Power output limits*

*Ramping limits*

*Inter-temporal constraints*

- Optimality Condition

$$c^G \in \partial f^G(x^G)$$



# Market Commodity Clearing

## *Demand Optimization*

- Benefit Maximization Objective

$$\varphi^L(c^L) = \max_{x^L} \{ f^L(x^L) - c^L \cdot x^L \mid x^L \in X^L \}$$

- Demand Technology Set  $X^L$

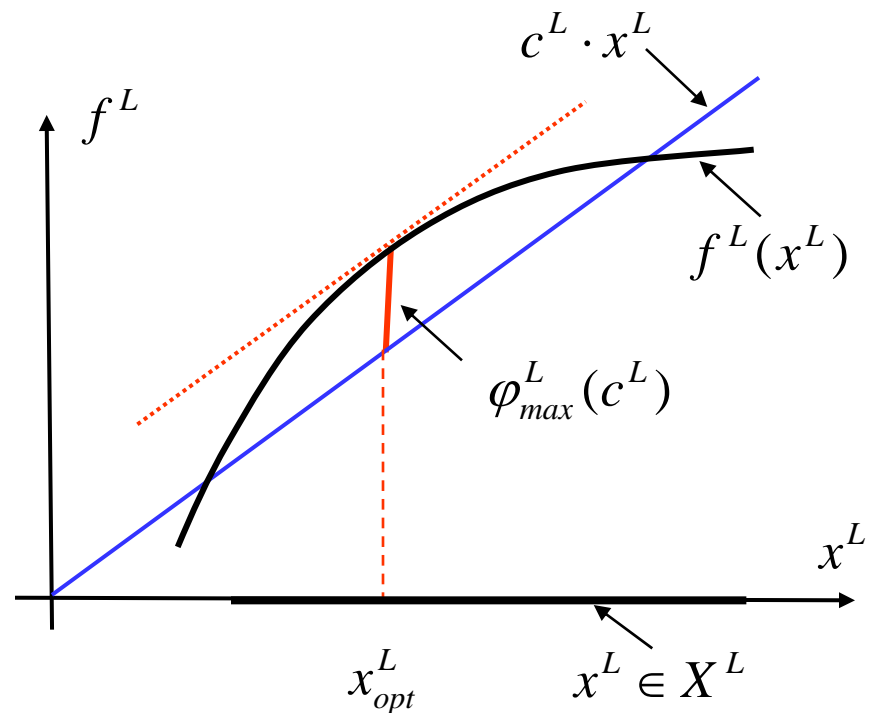
*Consumption limits*

*Ramping limits*

*Inter-temporal constraints*

- Optimality Condition

$$-c^L \in \partial(-f^L(x^L))$$



# Market Commodity Clearing

## *Transmission Network Optimization*

- Maximization of Congestion Revenue

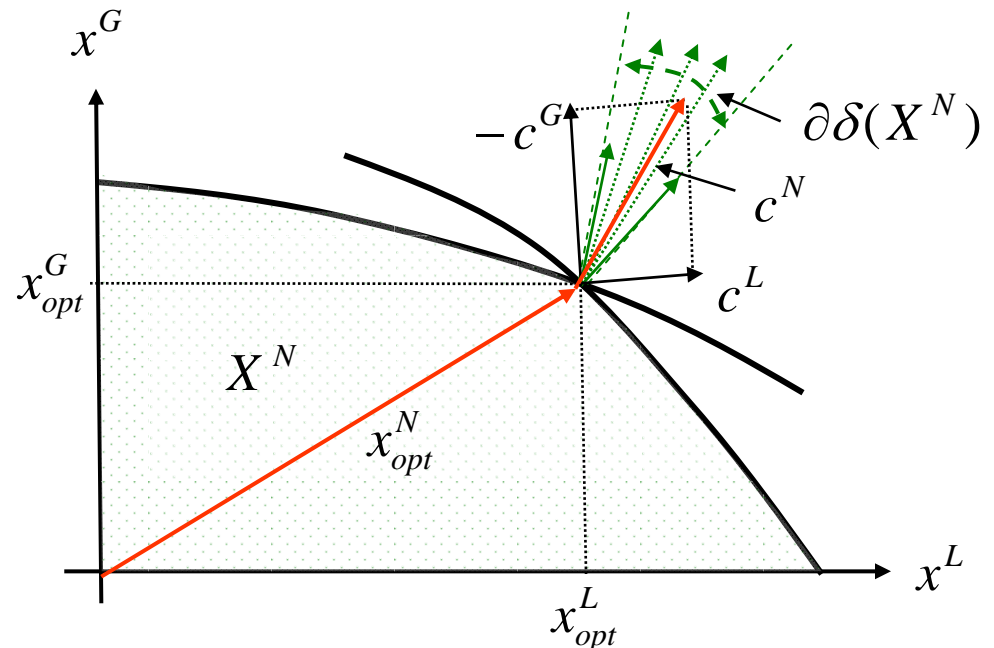
$$\begin{aligned} \varphi^N(c^G, c^L) &= \max_{x^G, x^L} \{c^L \cdot x^L - c^G \cdot x^G \mid (x^G, x^L) \in X^N\} \\ &= \max_{x^G, x^L} \{c^L \cdot x^L - c^G \cdot x^G - \delta(X^N)\} \quad \delta(X^N) = \begin{cases} 0 & (x^G, x^L) \in X^N \\ \infty & (x^G, x^L) \notin X^N \end{cases} \end{aligned}$$

- Optimality Condition

$$(-c^G, c^L) \in \partial\delta(X^N)$$

- Congestion Revenue

$$\varphi_{\max}^N = c^L \cdot x_{opt}^L - c^G \cdot x_{opt}^G \geq 0$$



# Market Commodity Pricing

## *Market Power Existence*

### ■ Supply Monopoly

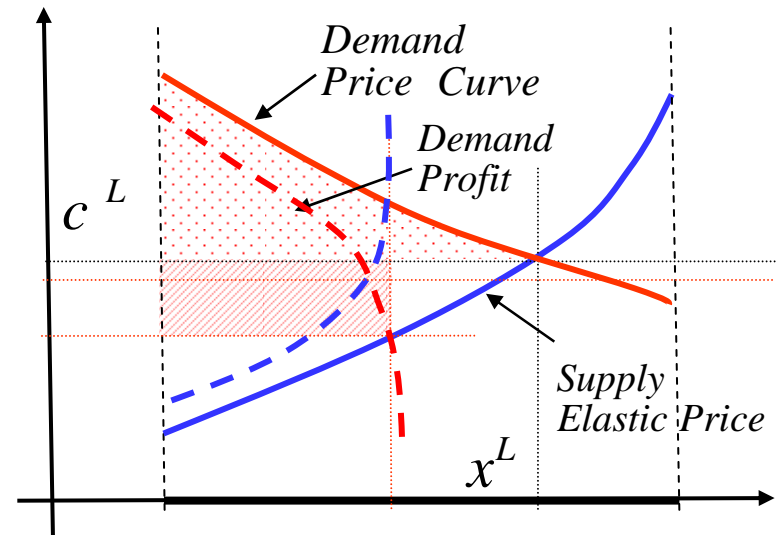
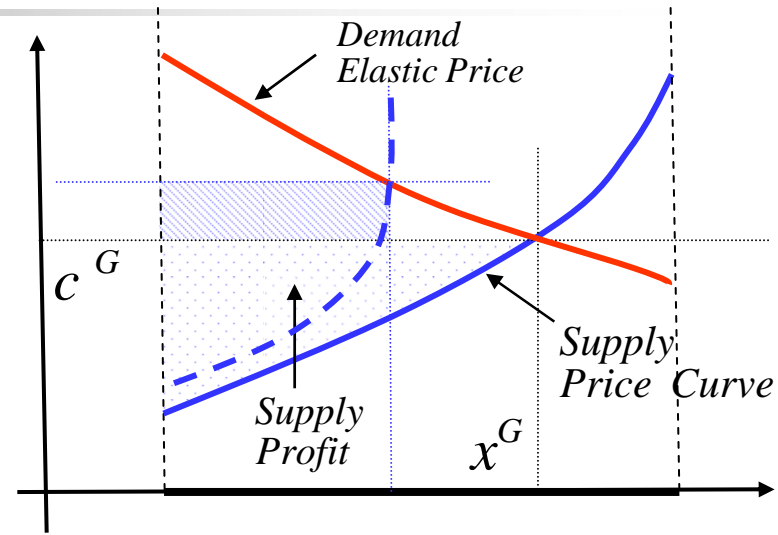
$$\max_{x^G} \{c^G(x^G) \cdot x^G - f^G(x^G) \mid x^G \in X^G\}$$

$$c^G(x^G) + x^G \cdot \frac{dc^G(x^G)}{dx^G} = \frac{df^G(x^G)}{dx^G}$$

### ■ Demand Monopsony

$$\max_{x^L} \{f^L(x^L) - c^L(x^L) \cdot x^L \mid x^L \in X^L\}$$

$$\frac{df^L(x^L)}{dx^L} = c^L(x^L) + x^L \cdot \frac{dc^L(x^L)}{dx^L}$$

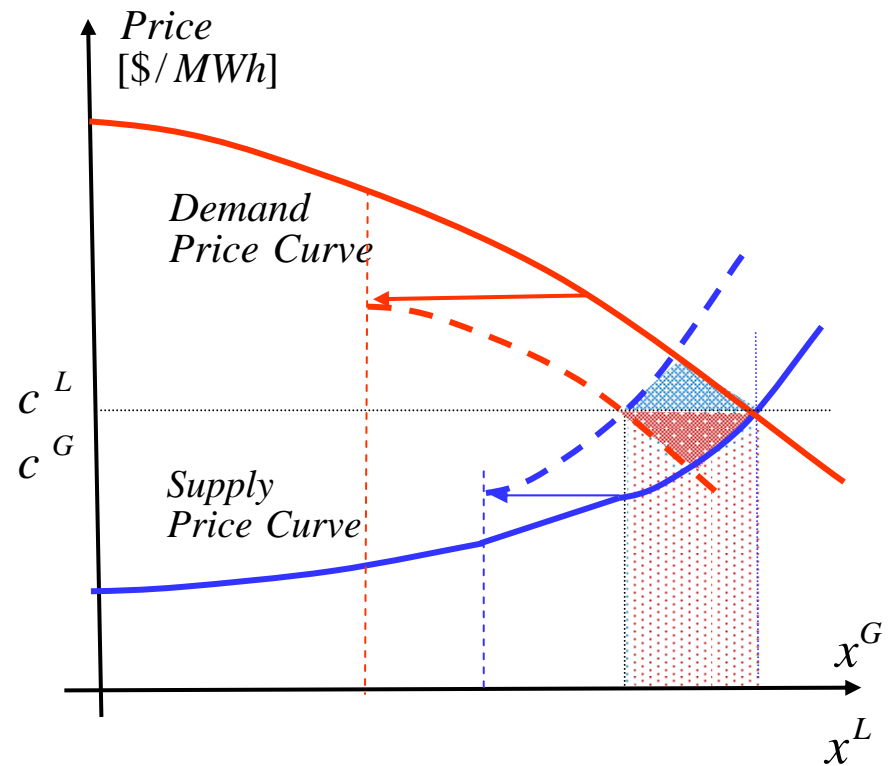




# Market Commodity Pricing

## *Vickrey Payments and Charges*

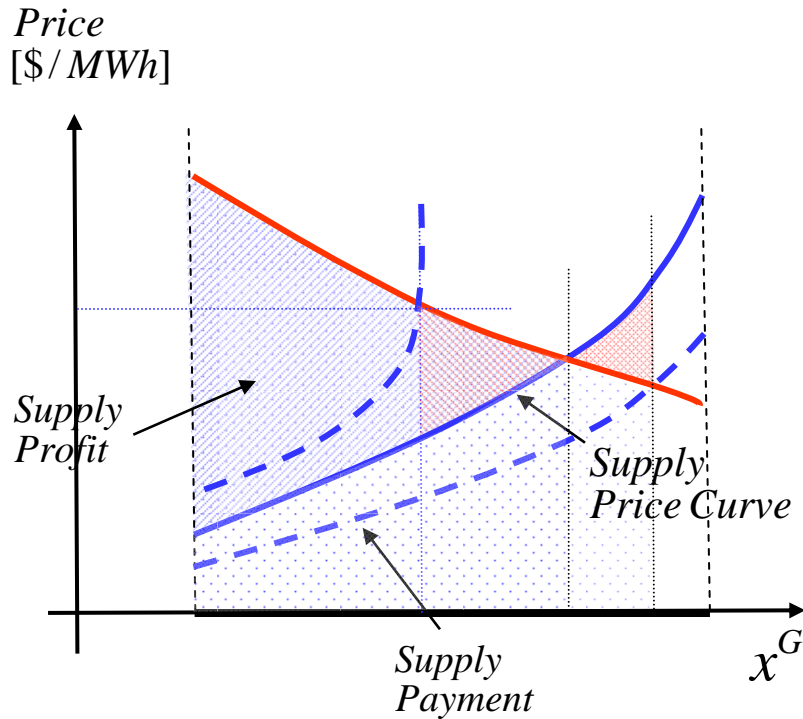
- Vickrey Payments
  - Suppliers paid all created profit
- Vickrey Charges
  - Consumers charges only for incurred costs
- Budget Deficit
  - Both supply and demand entities contribute to budget deficit



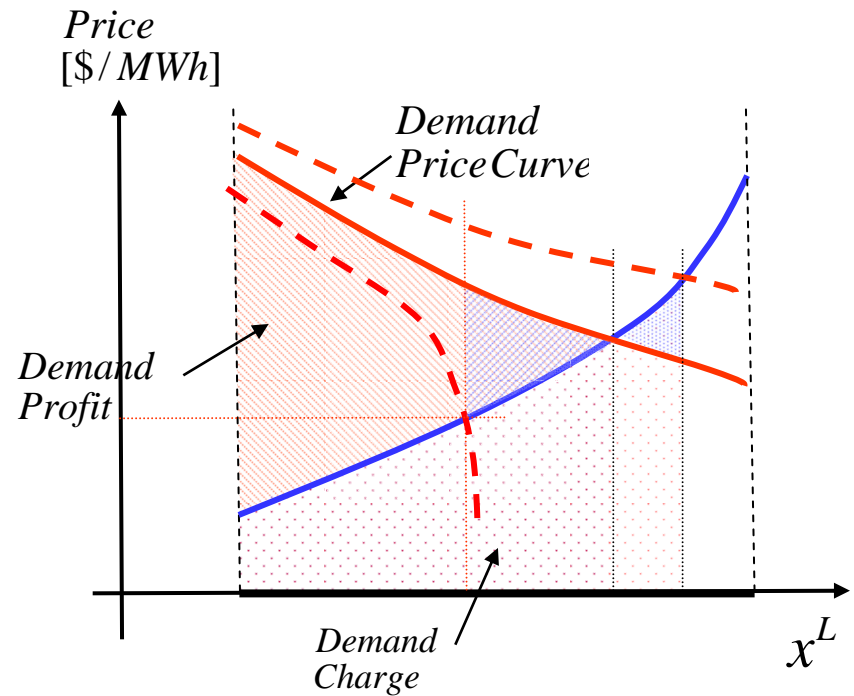
# Market Commodity Pricing

## *Incentive Compatibility*

- Supply Incentive Compatibility



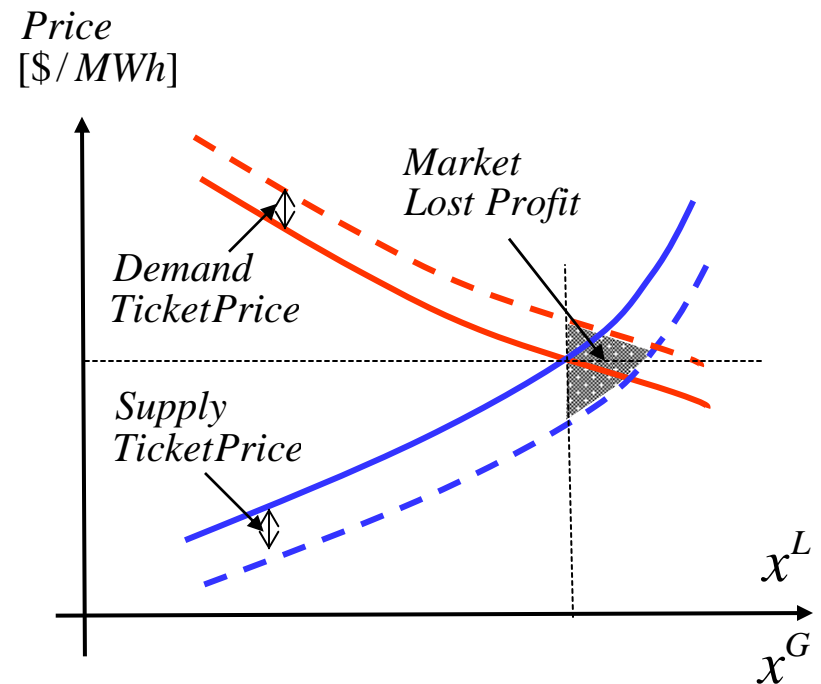
- Demand Incentive Compatibility



# Market Commodity Pricing

## *Participation Tickets*

- Supply Ticket Prices
  - Added to offers to sell
- Demand Ticket Prices
  - Subtracted from bids to buy
- Budget Balance
  - Deficit reduced
  - Remaining deficit distributed to price takers
- Economic Efficiency
  - Slightly degraded





# Market Commodity Pricing

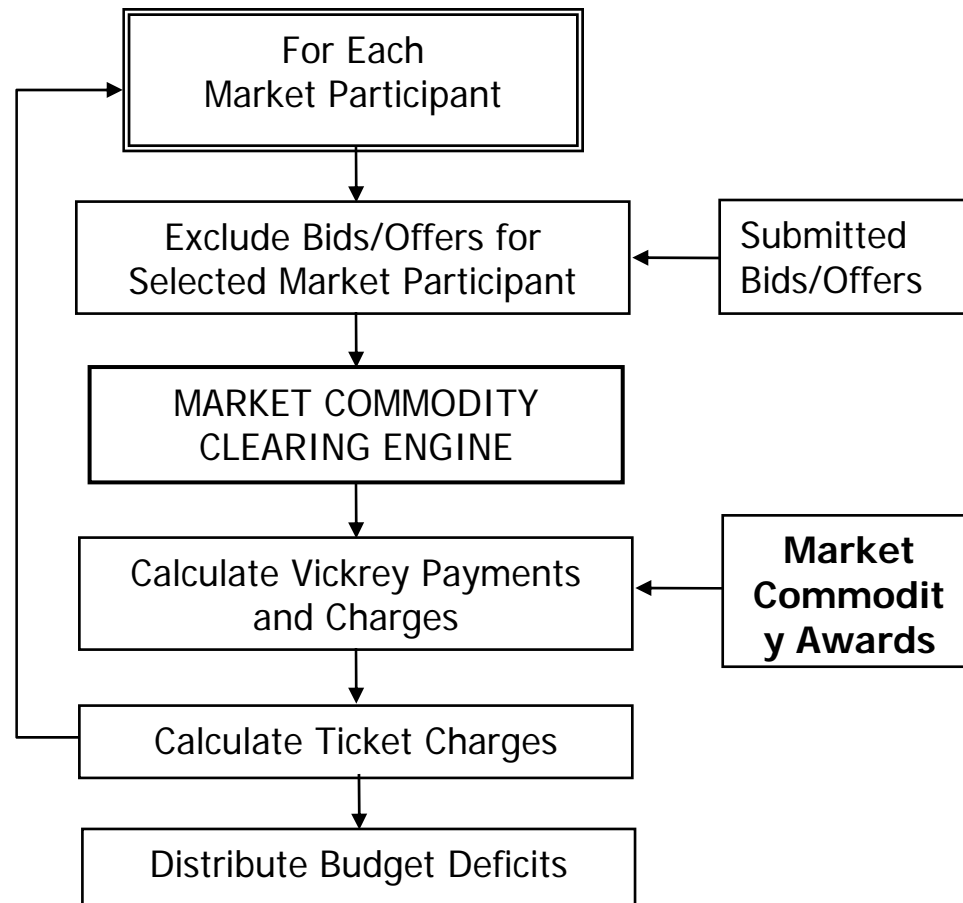
## *Budget Deficit Distribution*

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- Supply and Demand Price Takers
  - Do not submit bid and offer prices; no market power
  - Self-committed, self-scheduled and self-dispatched market commodities
  - Price non-sensitive market participants
  - Out-of-market entities
- Price Taker Charges
  - Pro-rata distribution of market budget deficit after each market run; always balanced budget
  - If price takers market share is significant than participation tickets are not needed

# Market Commodity Pricing

## *Pricing Engine*





# Electricity Market Design

## *Summary of Results*

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- Efficient Auction Design

- Economic efficiency for overall electricity market using competitive trading platform
- Separate market commodity clearing and market commodity pricing

- Market Commodity Clearing

- Decomposition into atomic supply/demand/transmission optimization sub-problems
- Supply and demand optimizations are convexified MIP models
- Transmission optimization is convexified MIP model with linear objective and AC power flow equations
- Optimization software can be utilized on multi-processor platforms

# Electricity Market Design

## *Summary of Results cont...*

- Market Commodity Pricing
  - Continuous and discrete commodity pricing
  - Incentive compatible strategies
  - Voluntary participation with free market exit
  - Budget balance at every market run

### Market profit reduction

- Due to market participation tickets



### Market power does not exist

- Due to Vickrey payments/charges



### Guaranteed profitability

- Due to Vickrey payments/charges



### Market budget balanced

- Due to deficit distribution



### 1. Economic Efficiency

- Maximize total market profit

### 2. Incentive Compatibility

- Truthful submissions

### 3. Volunteer Participation

- Non-negative market outcomes

### 4. Market Balance

- Commodity and budget balance



# Electricity Market Design

## *Next Steps*

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- Prototype MIP Decomposition Engine
  - Specify into details MIP models for supply, demand and transmission optimization problems
  - Develop Fenchel coordination mechanism
  - Evaluate performances and solution quality
- Prototype Pricing Engine
  - Calculate Vickrey payments and charges
  - Distribute budget deficit
  - Evaluate data volumes and calculation performances
- Proof the Concept of Auction Design





## Electricity Market Design *References*

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# Electricity Market Design *Discussion*

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**THANK  
YOU!**