Exploration of the ACOPF Feasible Region for the Standard IEEE Test Set

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1. Introduction and Motivation for Research
2. Convex Combination Feasibility
3. Variable Ranges and Elasticities
4. Variable-Cost Relationships
5. Conclusions

- The goal of our investigation is to gain some perspective on how non-convex the feasible region of the Alternating Current Optimal Power Flow (ACOPF) problem is.
- We use the IEEE standard test suite (14, 30, 57, 118, & 300 bus) as test problems because of their frequency of use in industry and academia

- First, we will develop a metric for comparing how infeasible convex combinations of different solutions are.
- Second, we would like to determine how elastic the area around the global optimum is.
- Finally, we will examine the two dimensional relationships between the optimization variables and the objective function value.

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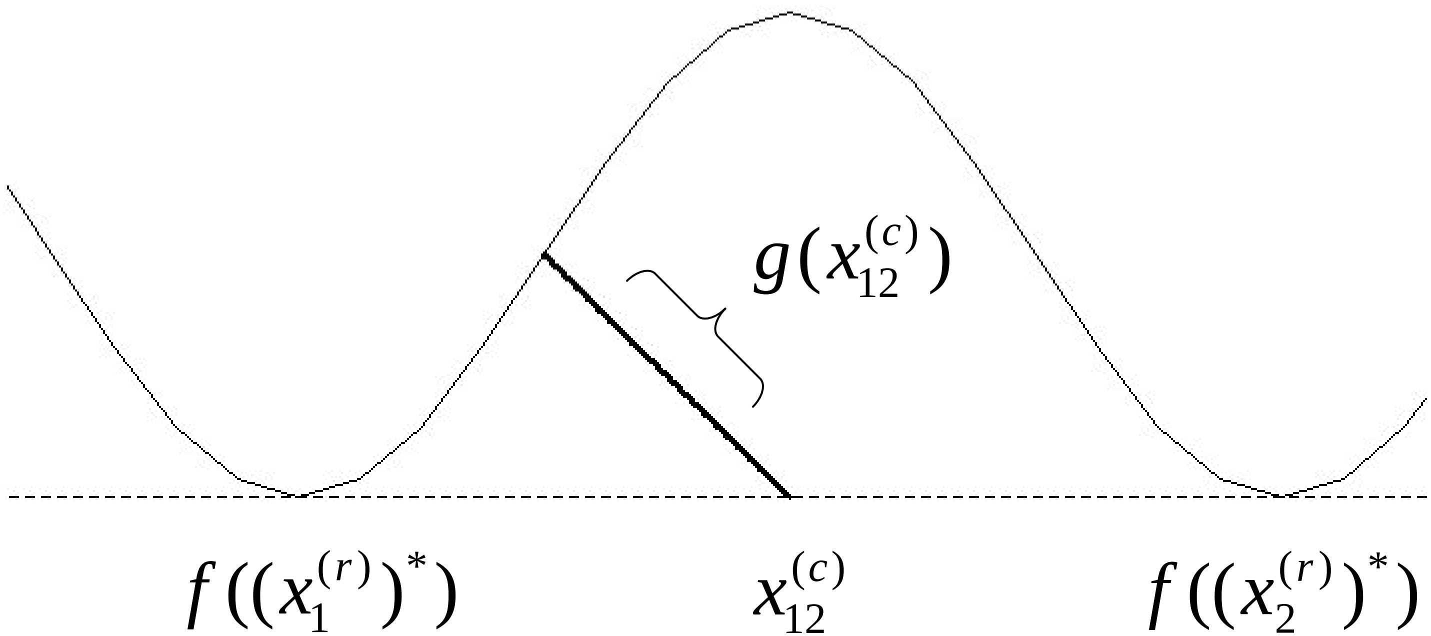
Illustration of Procedure



Procedure

1. Let (P) denote the optimization problem

   $\min f(x) = \sum_{g \in G} U c_g P_g$

   s. t. $x \in S$ and U is a random variable
2. Solve (P) 100 times
3. Let $x_i^{(r)}$ be the i^{th} solution
4. For each pair of solutions $i \neq j$

   $x_{ij}^{(c)} = 0.5 x_i^{(r)} + 0.5 x_j^{(r)}$
5. Solve the following for all convex combinations:

   $\min g(x) = \sum_{g \in G} \frac{(P_{g,ij}^{(c)} - P_g)^2}{|P^*|} + \sum_{g \in G} \frac{(Q_{g,ij}^{(c)} - Q_g)^2}{|Q^*|} + \sum_{n \in N} \frac{(v_{n,ij}^{(c)} - v_n)^2}{|v^*|}$

   s. t. $x \in S$

   Note: S is the standard feasible region of the ACOPF in the polar formulation

Results: *U* in [0,2]

Case	Mean	Median	Feasible Points	Minimum	Maximum
14	0.542	0.512	0	0.29	1.595
30	0.47	0.455	0	0.336	0.823
57	0.66	0.643	0	0.341	1.178
118	1.39	1.335	0	0.648	2.704
300	0.082	0.082	0	0.062	0.105

Values above are summary statistics for the objective function value *g(x)* from the procedure described in the last slide

Results: U in [0.99,1.01]

Case	Mean	Median	Feasible Points	Minimum	Maximum
14	0.551	0.552	0	0.292	1.535
30	0.468	0.453	0	0.329	0.806
57	0.686	0.673	0	0.424	1.071
118	1.469	1.466	0	0.501	2.473
300	0.077	0.078	0	0.050	0.103

Values above are summary statistics for the objective function value $g(x)$ from the procedure described previously

Observations

- None of the convex combinations are feasible points
- There is no obvious correlation between size of the problem and "how non-convex" the constraint set is
- Changing U does not have much effect on the mean and median values

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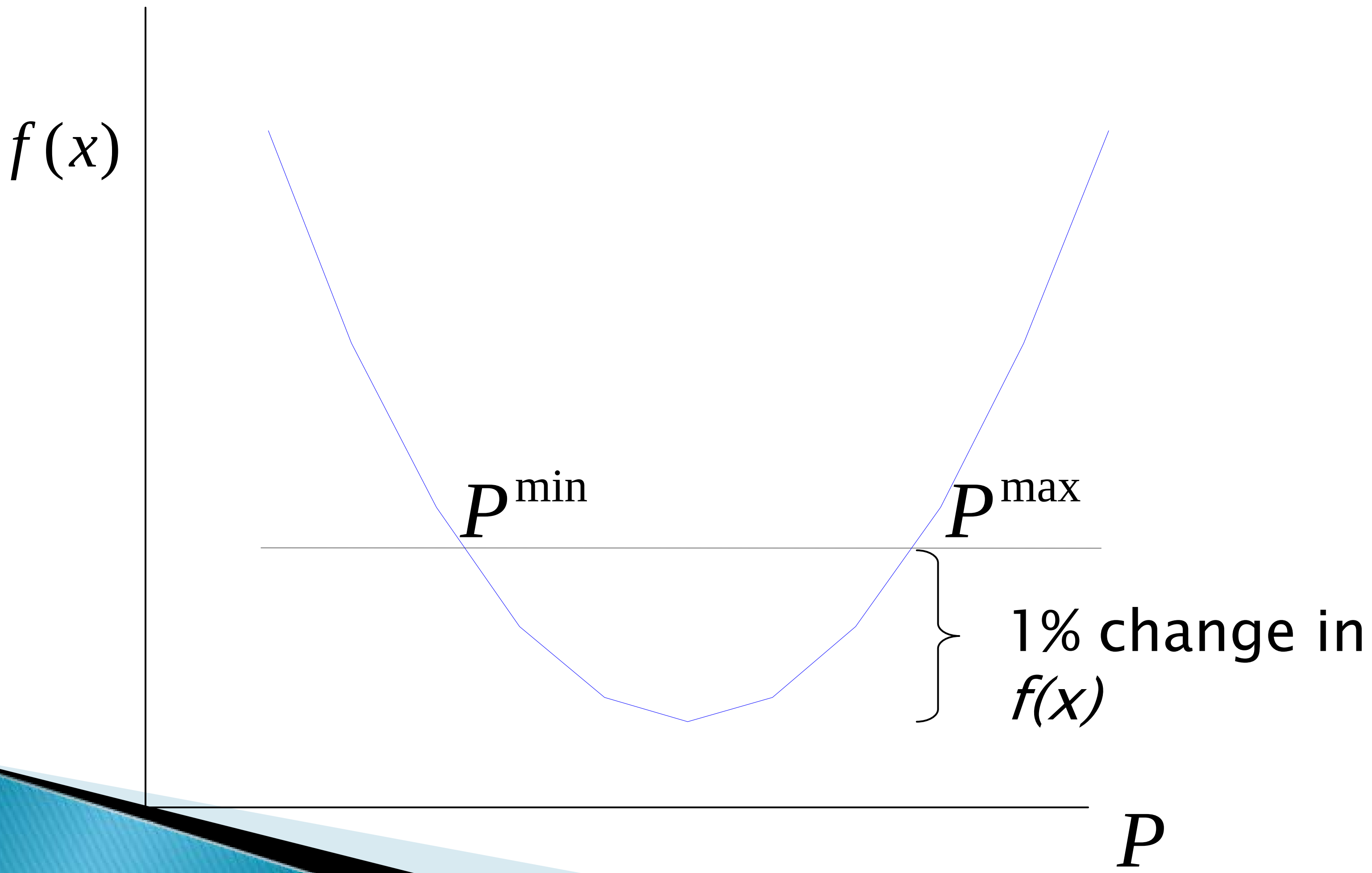
Elasticity

$$E_{y,x} = \left| \frac{\Delta y / y}{\Delta x / x} \right|$$

In our tests, we look at the elasticity of each optimization variable with respect to *f(x)*

Illustration of Procedure



Procedure

1. Solve for the optimal solution $x^* = [\theta^*, V^*, P^*, Q^*]$
2. Calculate $f(x^*) = \sum_{g \in G} c_{g2}(P_g^*)^2 + c_{g1}P_g^* + c_{g0}$
3. Add new constraint $f(x) \leq 1.01 f(x^*)$ to the constraint set S
4. Solve the new optimization problems

   min/max v_n

   subject to $x \in S$, for all $n \in N$

   min/max P_g

   subject to $x \in S$, for all $g \in G$

   min/max Q_g

   subject to $x \in S$, for all $g \in G$
5. Compute the elasticities

   $$E_{v_n,f} = \frac{v_n^{max} - v_n^{min}}{v_n^*} / 0.01$$

   $$E_{P_g,f} = \frac{P_g^{max} - P_g^{min}}{P_g^*} / 0.01$$

   $$E_{Q_g,f} = \frac{Q_g^{max} - Q_g^{min}}{Q_g^*} / 0.01$$

Results (all units are *100%)

Case		Voltage	Power	Reactive Power
14	Mean	9.51	237.85	336.26
	Median	9.40	70340.16	8771.62
30	Mean	8.58	104.48	468.38
	Median	9.12	125.33	1391.32
57	Mean	2.88	102.49	243.15
	Median	3.79	101.87	360.35
118	Mean	11.02	499.33	676.37
	Median	10.88	57053.86	4228.16
300	Mean	7.80	137.19	229.32
	Median	7.46	10712.25	3403.35

Observations

- Each variable has an elasticity much higher than 1 (implying a "flat" region around the solution).
- Reactive power is especially elastic, and is consistently more elastic than the other optimization variables.
- There is not a consistent trend between size of the problem and elasticity of the variables, similar to our findings regarding our infeasibility metric.

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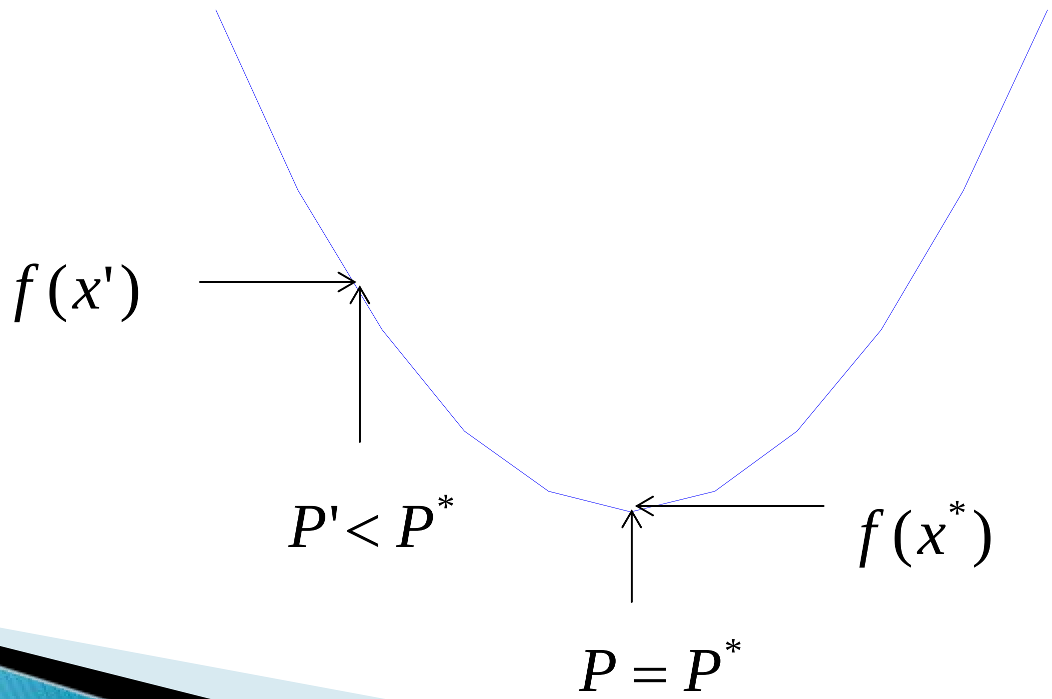
Illustration of Procedure

- Determine the 2-Dimensional relationship between each optimization variable (Vm, P, Q) and cost *f(x)*



Procedure

1. For each optimization variable v_n, P_g, or Q_g ($\forall n \in N$ and $\forall g \in G$), determine the range of feasible values from the problem data
2. Let $v_n^{(i)} = v_n^{min} + c^{(i)}(v_n^{max} - v_n^{min})$ where $c^{(i)} = \frac{i}{100}$, for $i = 1, \ldots, 100$
3. Add this constraint to the constraint set S, and solve the ACOPF problem with the remaining variables. The objective function value is $f^{(i)}(x^*)$
4. Plot $f^{(i)}(x^*)$ against $v_n^{(i)}$ for each i

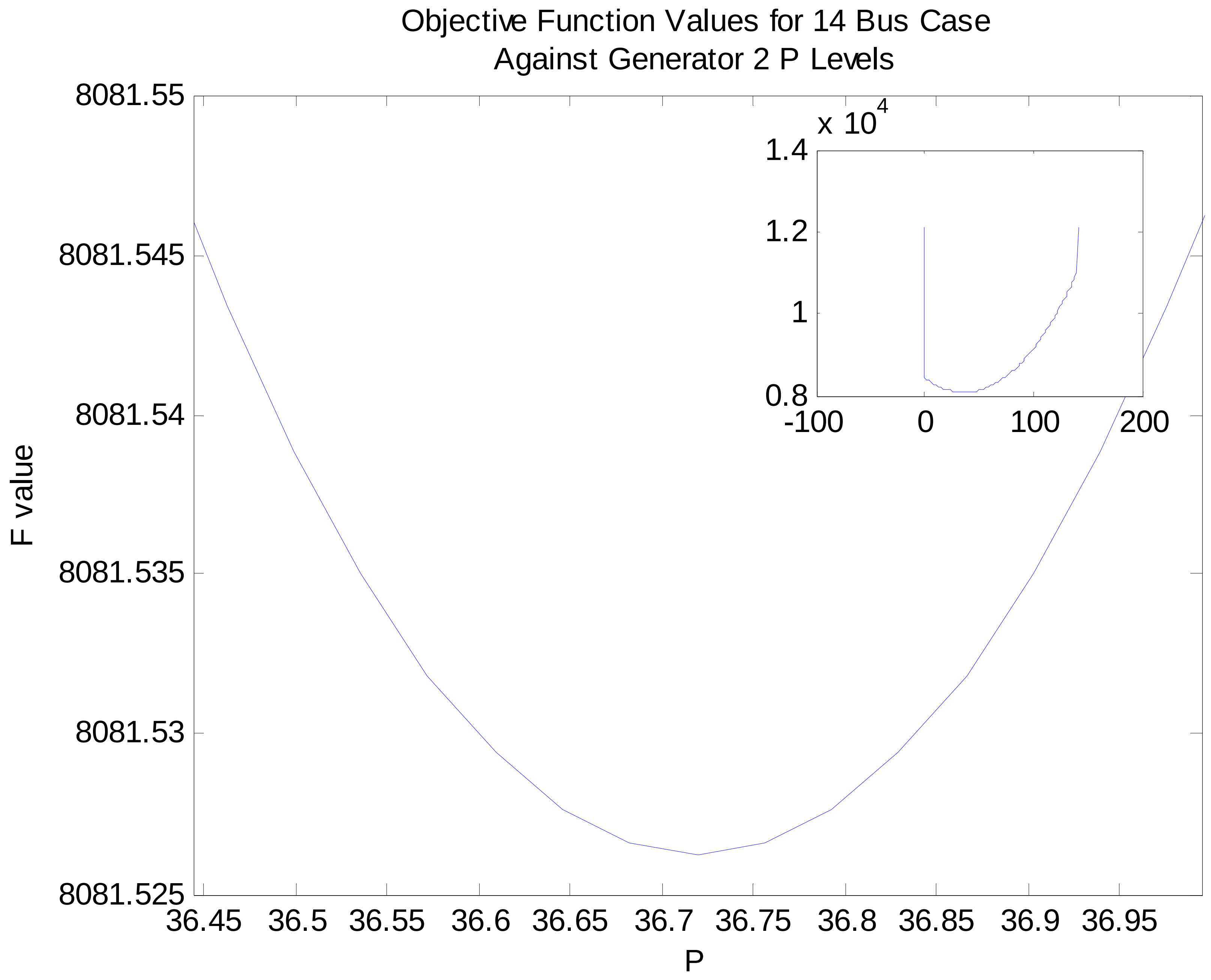
$v_n^{(i)}$

Results

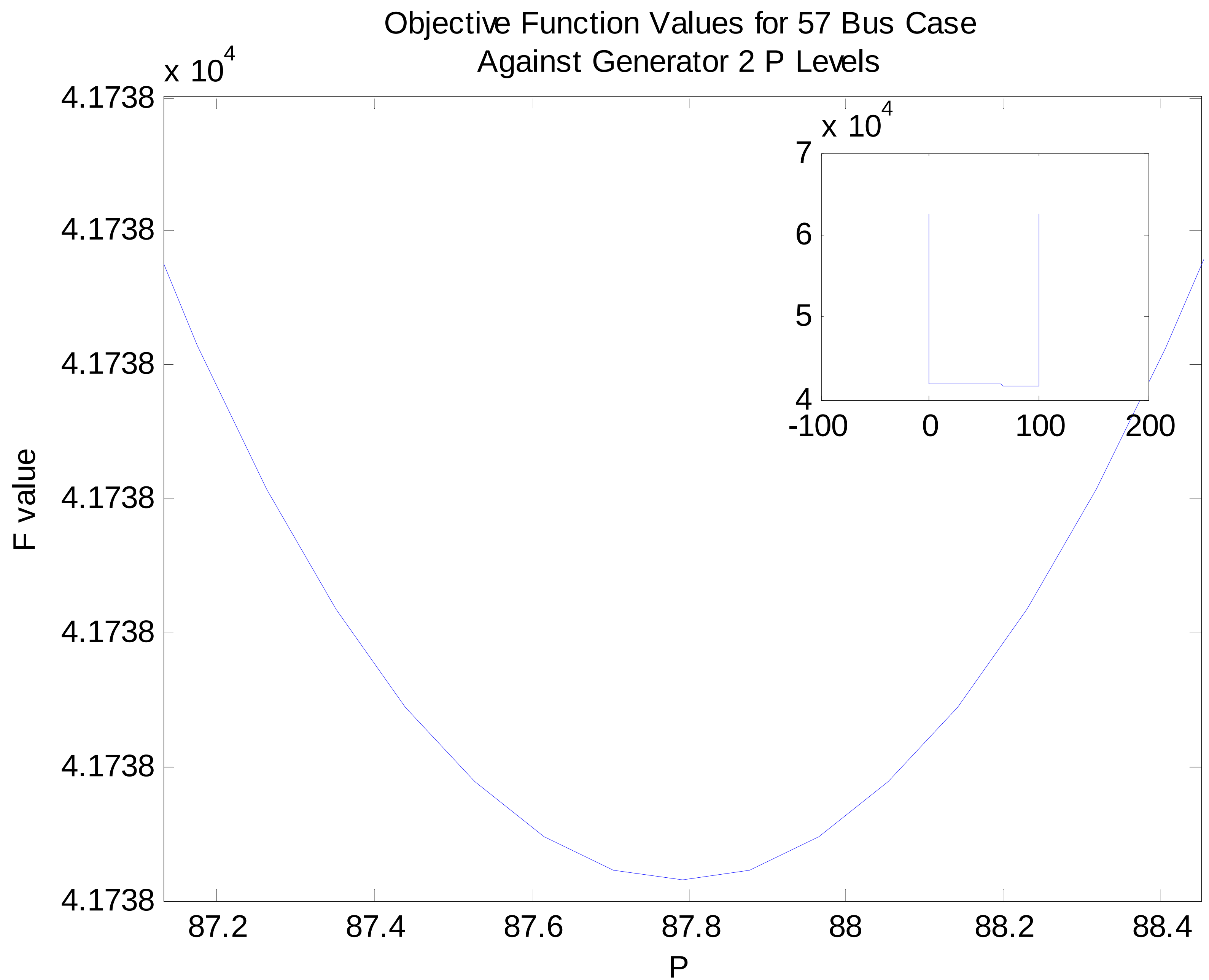
- We will present only a portion of the graphs, and in particular those with unique variable–cost relationships.
- Note that the main graph is zoomed in to the area around the global optimum (1% in each direction) and the inset represents the full range of feasible values.
- If the solver failed or if the problem was infeasible, the cost was set at 1.5 *f(x*)* for visualization purposes.

Real Power Examples

- Power generally has a locally convex relationship with overall cost
- Some generators have significantly less impact on overall cost (product of the cost terms), but nevertheless have the same shape in the neighborhood of the optimum



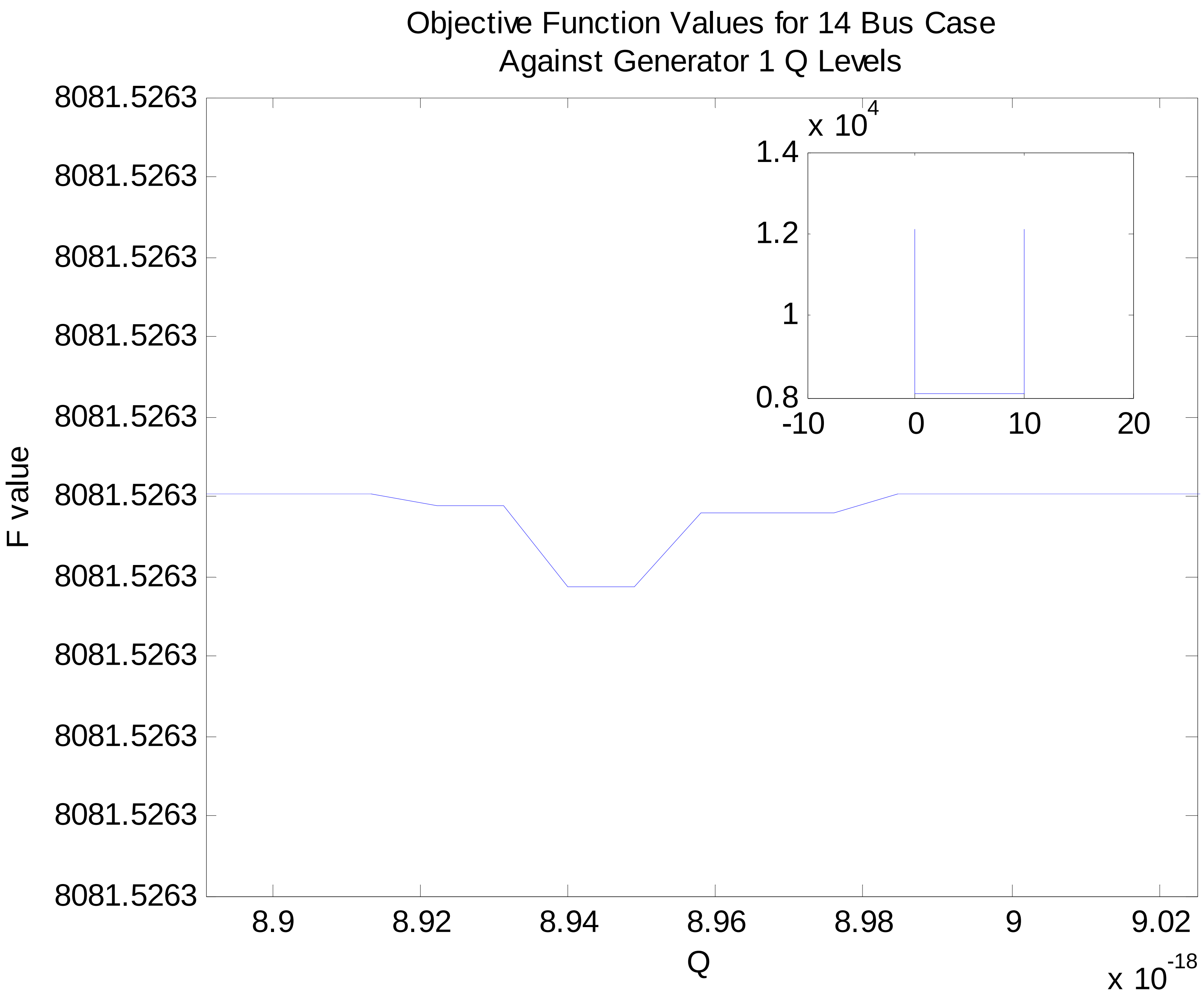
Objective Function Values for 14 Bus Case
Against Generator 2 P Levels
8081.55
8081.545
8081.54
8081.535
8081.53
8081.525
F value
36.45
36.5
36.55
36.6
36.65
36.7
36.75
36.8
36.85
36.9
36.95
P
x 10^4
1.4
1.2
1
0.8
-100
0
100
200

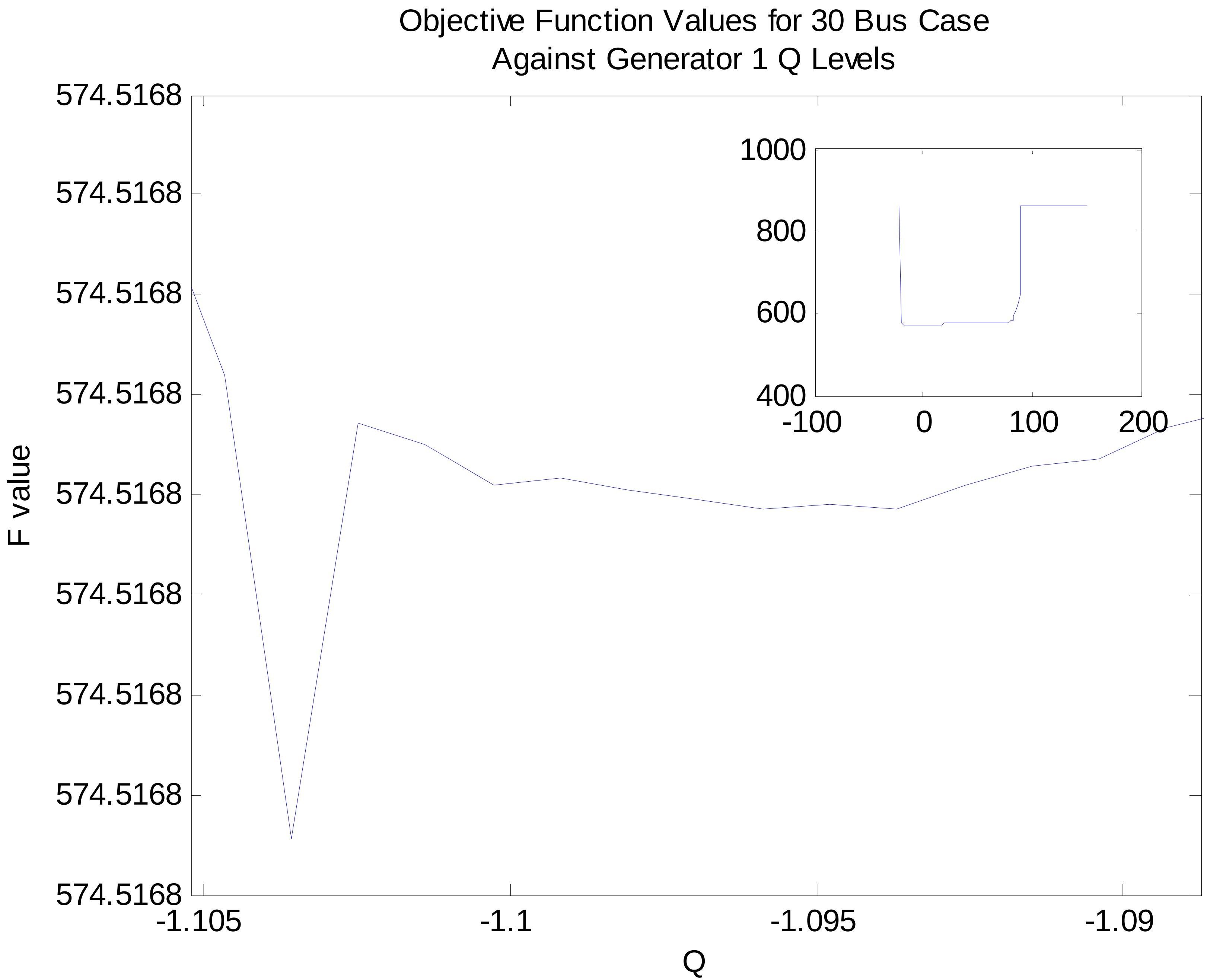


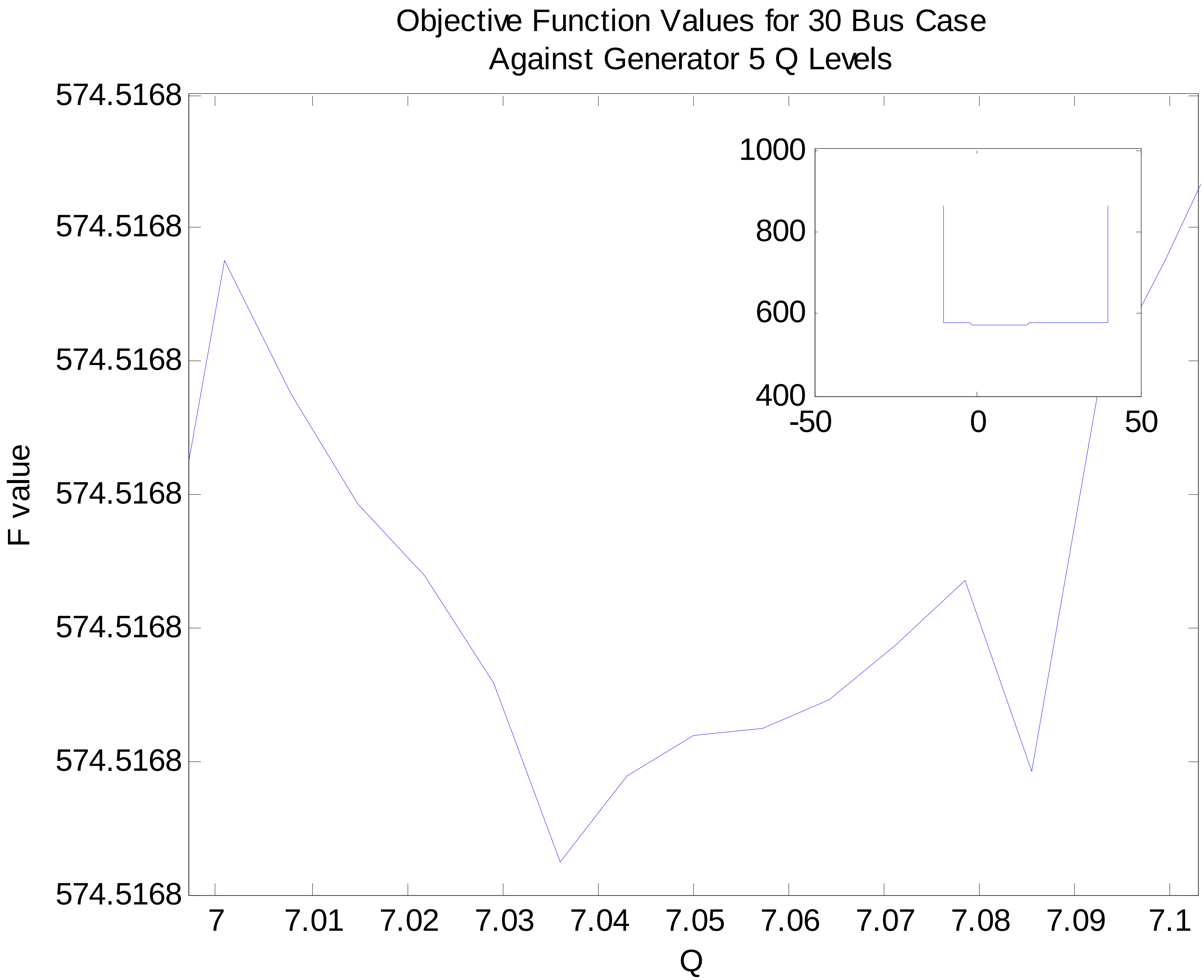
Objective Function Values for 57 Bus Case
Against Generator 2 P Levels
x 10^4
4.1738
4.1738
4.1738
4.1738
4.1738
4.1738
4.1738
F value
87.2
87.4
87.6
87.8
88
88.2
88.4
P
x 10^4
7
6
5
4
-100
0
100
200

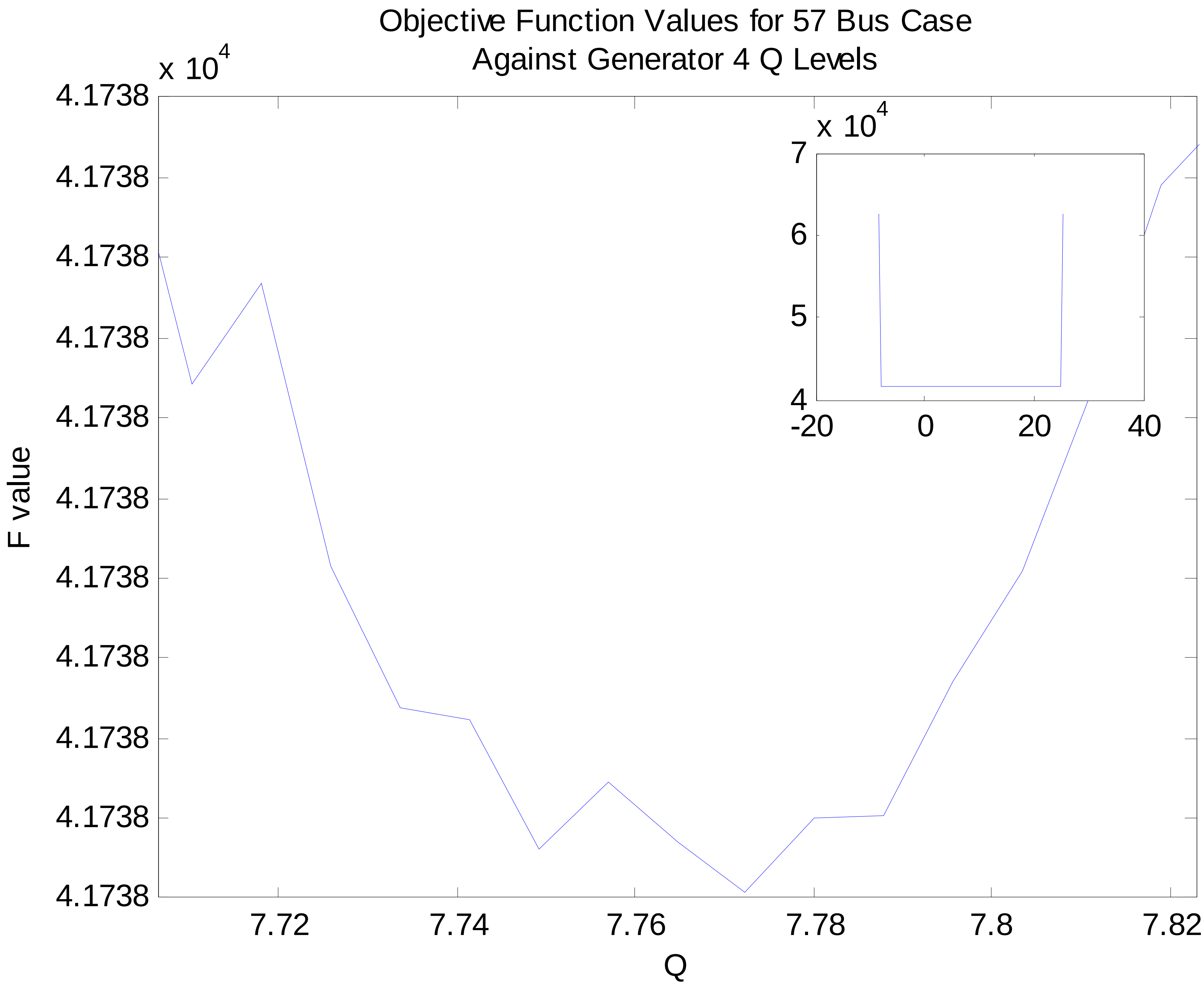
Reactive Power Examples

- Reactive power sometimes has a convex relationship with cost, but in general is much less predictable than power
- Several local optima are evident in a number of these graphs
- Overall, reactive power can range greatly without impacting cost significantly


Objective Function Values for 14 Bus Case
Against Generator 1 Q Levels
8081.5263
8081.5263
8081.5263
8081.5263
8081.5263
8081.5263
8081.5263
8081.5263
8081.5263
8081.5263
8081.5263
F value
8.9
8.92
8.94
8.96
8.98
9
9.02
Q
x 10^-18
x 10^4
1.4
1.2
1
0.8
-10
0
10
20


Objective Function Values for 30 Bus Case
Against Generator 1 Q Levels
574.5168
574.5168
574.5168
574.5168
574.5168
574.5168
574.5168
574.5168
574.5168
F value
-1.105
-1.1
-1.095
-1.09
Q
1000
800
600
400
-100
0
100
200

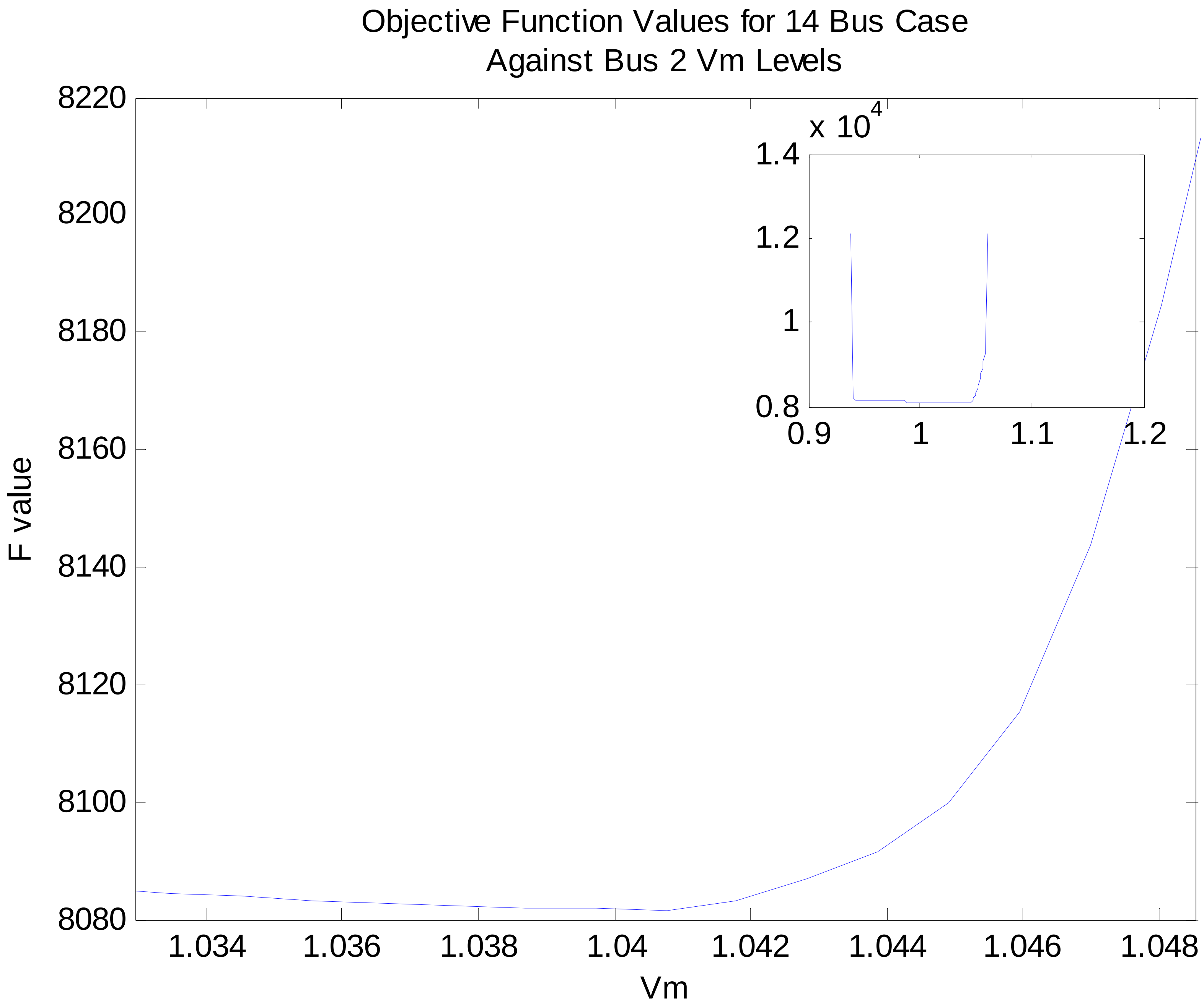

Objective Function Values for 30 Bus Case
Against Generator 5 Q Levels
574.5168
574.5168
574.5168
574.5168
574.5168
574.5168
574.5168
F value
7
7.01
7.02
7.03
7.04
7.05
7.06
7.07
7.08
7.09
7.1
Q
1000
800
600
400
-50
0
50



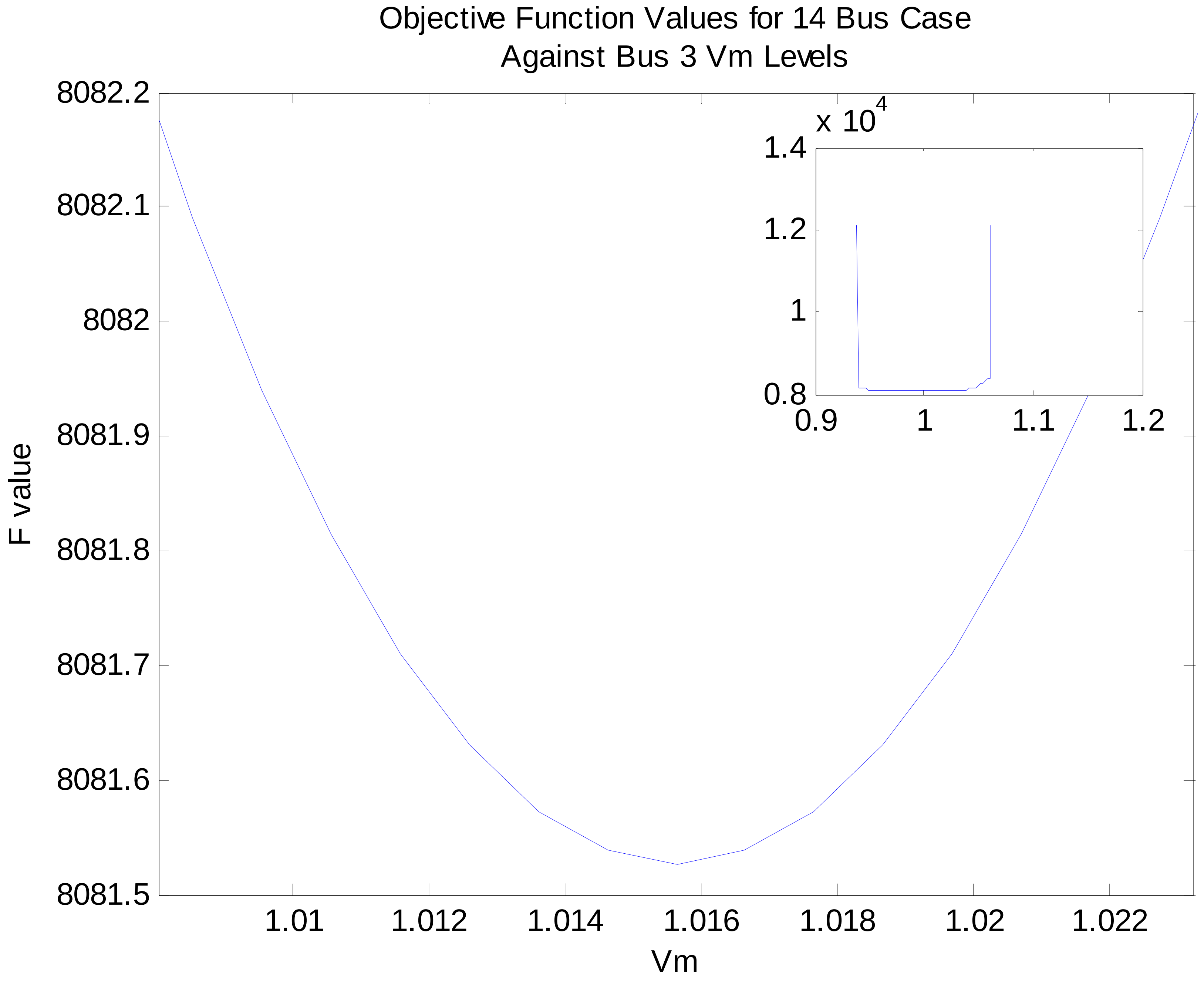
Objective Function Values for 57 Bus Case
Against Generator 4 Q Levels
x 10^4
4.1738
4.1738
4.1738
4.1738
4.1738
4.1738
4.1738
4.1738
4.1738
4.1738
4.1738
F value
7.72
7.74
7.76
7.78
7.8
7.82
Q
x 10^4
7
6
5
4
-20
0
20
40

Voltage Magnitude Examples

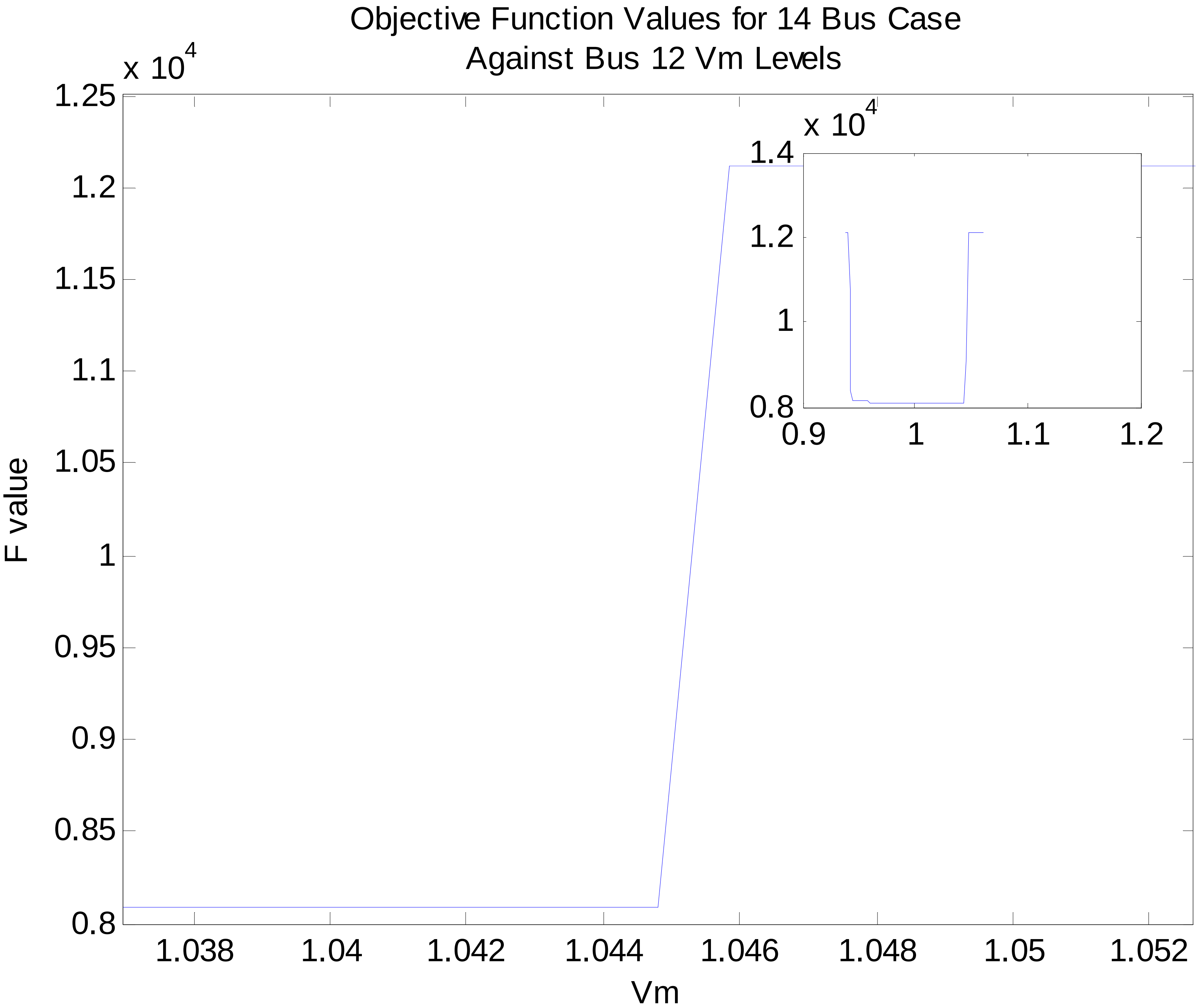
- Voltage magnitude generally has a convex relationship with cost, but the global optimum is generally at the higher end of the feasible range
- The ACOPF problem can become infeasible with only a small upward change in voltage magnitude; conversely, not much additional cost is incurred by a lower-than-optimal voltage value
- In the 57 bus case, there is only a small range of voltage values where the problem is feasible



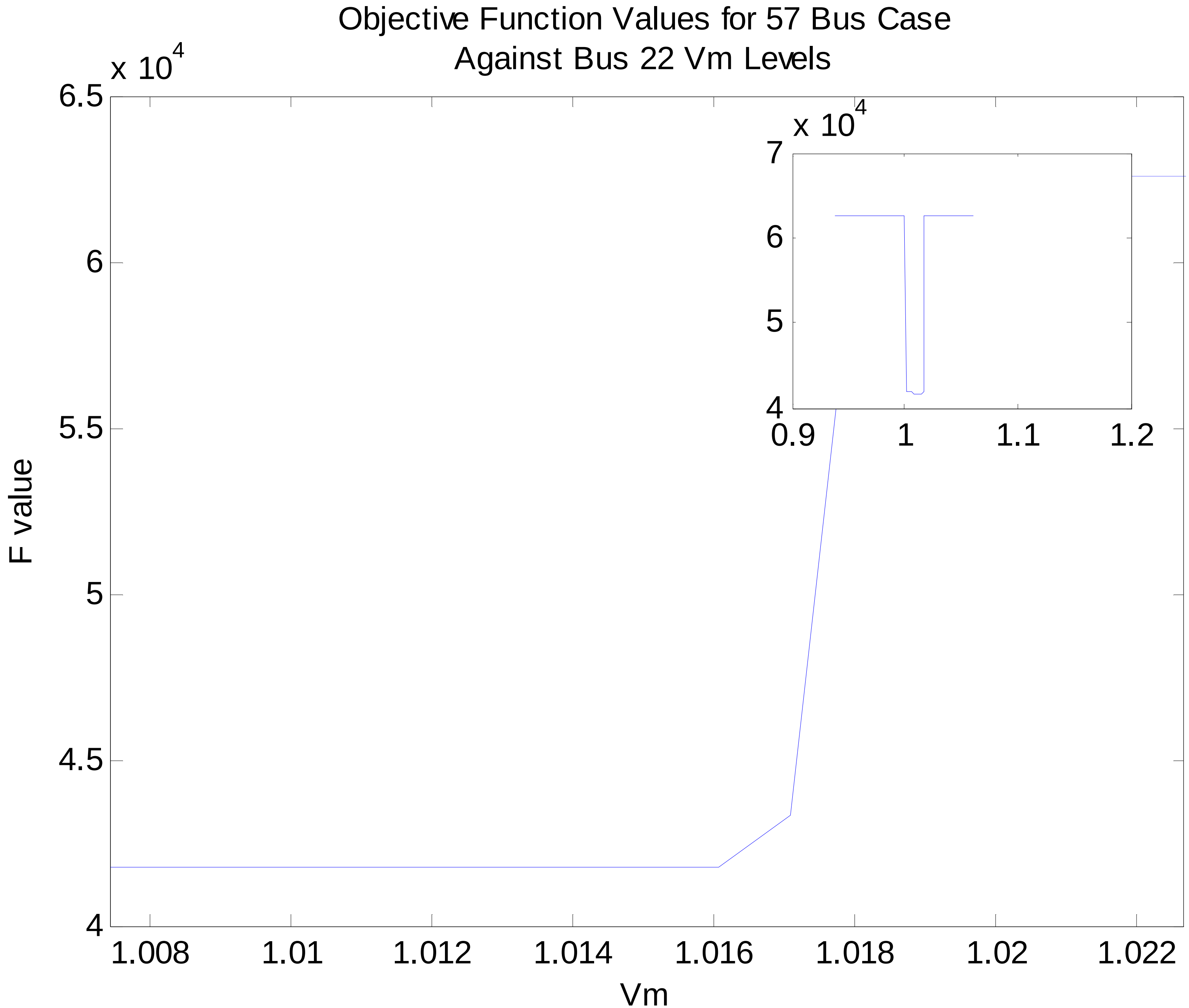
Objective Function Values for 14 Bus Case
Against Bus 2 Vm Levels
F value
Vm
8220
8200
8180
8160
8140
8120
8100
8080
1.034
1.036
1.038
1.04
1.042
1.044
1.046
1.048
x 10^4
1.4
1.2
1
0.8
0.9
1
1.1
1.2



Objective Function Values for 14 Bus Case
Against Bus 3 Vm Levels
8082.2
8082.1
8082
8081.9
8081.8
8081.7
8081.6
8081.5
F value
1.01
1.012
1.014
1.016
1.018
1.02
1.022
Vm
x 10^4
1.4
1.2
1
0.8
0.9
1
1.1
1.2



Objective Function Values for 14 Bus Case
Against Bus 12 Vm Levels
x 10^4
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
F value
1.038
1.04
1.042
1.044
1.046
1.048
1.05
1.052
Vm
x 10^4
1.4
1.2
1
0.8
0.9
1
1.1
1.2



Objective Function Values for 57 Bus Case
Against Bus 22 Vm Levels
x 10^4
6.5
6
5.5
5
4.5
4
F value
1.008
1.01
1.012
1.014
1.016
1.018
1.02
1.022
Vm
x 10^4
7
6
5
4
0.9
1
1.1
1.2

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Conclusions

- We can conclude that the region is globally flat but locally very dynamic for any constricted region.
- Our results lend support to the frequent observation of local optima; because the area around the true optimum is flat, solvers may quickly converge to values close to the optimum.
- The local topology would make it very easy for the solver to find a feasible, sub-optimal point and declare optimality.