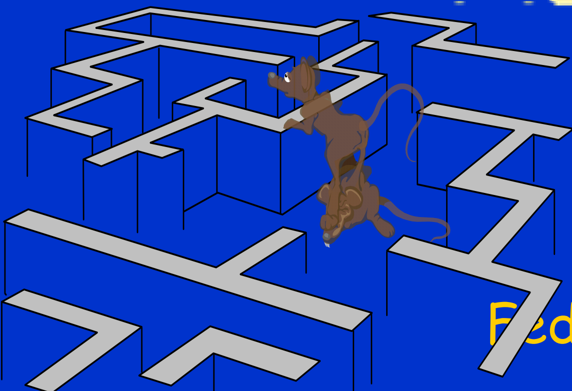
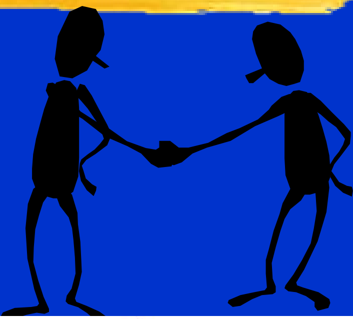
The IV Formulation of the ACOPF and its linearizations





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Views expressed are not necessarily those of the Commission

From real time dispatch to planning

Mixed Integer Nonconvex Program

$$
\begin{aligned}
\text{maximize} \quad & c(x) \\
\text{subject to} \quad & g(x) \le 0, \\
& Ax \le b \\
& l \le x \le u, \\
& \text{some } x \in \{0,1\}
\end{aligned}
$$

c(x), g(x) may be non-convex

I didn't know what I would find there

AC Optimal Flow Problem

"DC OPF" formulations linearize the nonlinearities .

'ACOPF' formulation is a continuous nonconvex optimization problem

Most nonlinear solvers find at best local optimal solutions

Linear IV approximation to ACOPF

If promising, it can be embedded in binary formulations:

unit commitment models, and optimal topology models.

allows the use of exceptionally fast and robust MIP algorithms

Power Flow Equations

Polar Power-Voltage: 2N nonlinear equality constraints

$$P_n = \sum_{mk} V_n V_m (G_{nmk} \cos\theta_{nm} + B_{nmk} \sin\theta_{nm})$$

$$Q_n = \sum_{mk} V_n V_m (G_{nmk} \sin\theta_{nm} - B_{nmk} \cos\theta_{nm})$$

Rectangular Power-Voltage: 2N quadratic equality constraints

$$S = P + jQ = \mathrm{diag}(V) I^* = \mathrm{diag}(V)[YV]^* = \mathrm{diag}(V) Y^* V^*$$

Rectangular Current-Voltage (IV) formulation.

Network-wide ***LINEAR*** constraints: 2N linear equality constraints

$$I = YV = (G + jB)(V^r + jV^j) = GV^r - BV^j + j(BV^r + GV^j)$$

where $I^r = GV^r - BV^j$ and $I^j = BV^r + GV^j$

Rectangular ACOPF-IV formulation.

Network-wide objective function: Min $c(P, Q, I, V)$ (50)

Network-wide constraint: $I = YV$ (51)

Bus-specific constraints:

$P = V^r \cdot I^r + V^j \cdot I^j \leq P^{max}$ (54) $P^{min} \leq P = V^r \cdot I^r + V^j \cdot I^j$ (55)

$Q = V^j \cdot I^r - V^r \cdot I^j \leq Q^{max}$ (56) $Q^{min} \leq Q = V^j \cdot I^r - V^r \cdot I^j$ (57)

$V^r \cdot V^r + V^j \cdot V^j \leq (V^{max})^2$ (58) $(V^{min})^2 \leq V^r \cdot V^r + V^j \cdot V^j$ (59)

$(i_{nmk})^2 \leq (i^{max}{}_k)^2$ for all k (60)

[$\theta^{min}{}_{nm} \leq \arctan(v^j{}_n / v^r{}_n) - \arctan(v^j{}_m / v^r{}_m) \leq \theta^{max}{}_{nm}$ (61)]

$V^r \geq 0$ (62)

(51) are 2N linear equality constraints that apply throughout the network,

(54) – (57) are quadratic and non-convex.

(58) are convex quadratic inequality constraints, but

(59) are non-convex quadratic inequality constraints.

(61) could be eliminated and the problem becomes quadratic with linear network equations.

Generator and Load Constraints.

The lower and upper bound constraints for generation and load are:

$$P^{min} \leq P \leq P^{max} \quad (24) \qquad Q^{min} \leq Q \leq Q^{max} \quad (26)$$

In terms of V and I,

$$V^r \cdot I^r + V^j \cdot I^j \leq P^{max} \quad (28) \qquad P^{min} \leq V^r \cdot I^r + V^j \cdot I^j \quad (29)$$

$$V^j \cdot I^r - V^r \cdot I^j \leq Q^{max} \quad (30) \qquad Q^{min} \leq V^j \cdot I^r - V^r \cdot I^j \quad (31)$$

(28)-(31) are non-convex constraints.

Voltage constraints.

in rectangular coordinates

$$(v^r_m)^2 + (v^j_m)^2 \leq (v^{max}_m)^2$$

$$(v^{min}_m)^2 \leq (v^r_m)^2 + (v^j_m)^2$$

voltage magnitude bounds are generally in the range, [.95, 1.05].

high voltages are often constrained by circuit breakers capabilities.

Low voltage constraints can be due operating requirements of motors or generators.

Line Flow Constraints

Power Line Flow Constraints.

$$(s^{r}_{nmk})^2 + (s^{j}_{nmk})^2 = |s_{nmk}|^2 \leq (s^{max}_{k})^2 \qquad (37)$$

Current Line Flow Limitations.

$$(i^{r}_{nmk})^2 + (i^{j}_{nmk})^2 \leq (i^{max}_{nmk})^2 \qquad (38)$$

convex quadratic and isolated to the complex current at the bus.

(38) appears to be the better choice

The Linear Approximations to the IV Formulation

We take three approaches to constraint formulation.

If the constraint is nonlinear,

use the first order Taylor series approximation

updated at each LP iteration

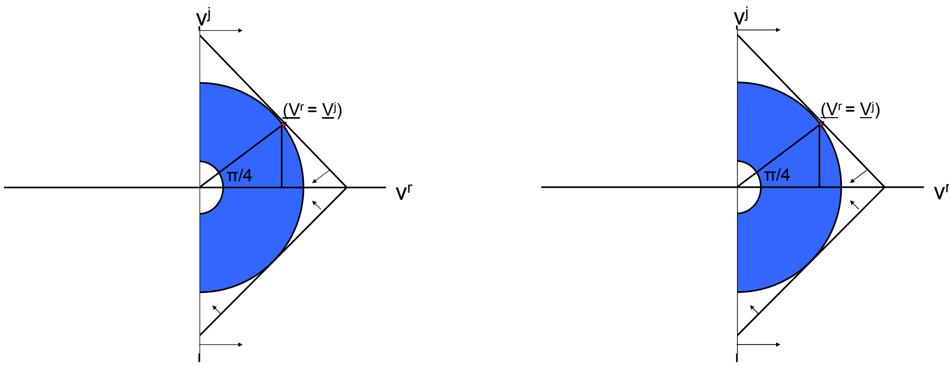
If the constraint is convex, use outer approximation

add linear cutting planes to remove infeasible points

Can we guarantee feasibility with this approach?

Preprocessed Linear Voltage and Current Constraints.

$$(v^r_m)^2 + (v^j_m)^2 \le (v^{max}_m)^2$$



Can add any number of preprocessed constraints before iterating

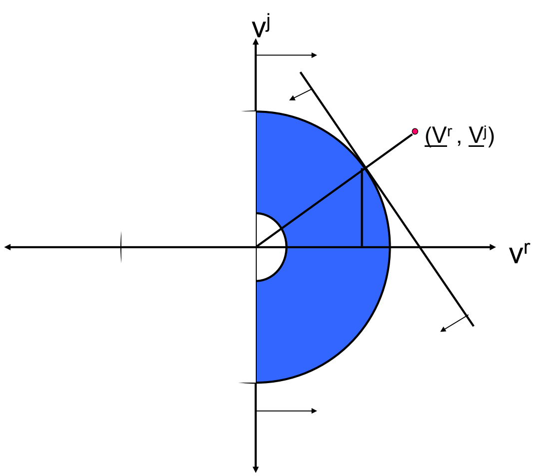
Current constraint set has no hole

Linear Voltage Approximations.

a first order Taylor's series approximation about $(\underline{v}^r_n, \underline{v}^j_n)$

$$v^r_n v^r_n + v^j_n v^j_n \approx 2\underline{v}^r v^r_n + 2\underline{v}^j_n v^j_n - \underline{v}^r_n \underline{v}^r_n - \underline{v}^j_n \underline{v}^j_n$$

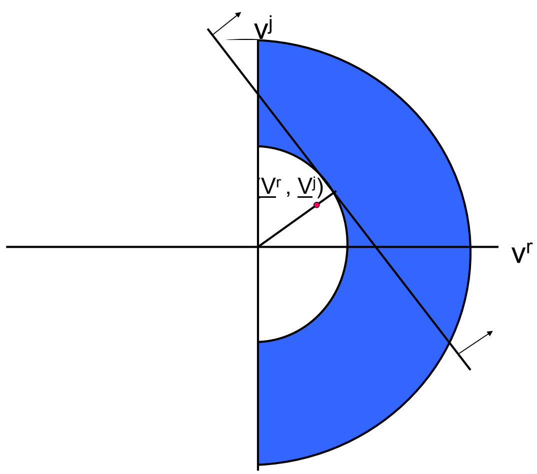
Since higher losses occur at lower voltages, the natural tendency of the optimization will be toward higher voltages.

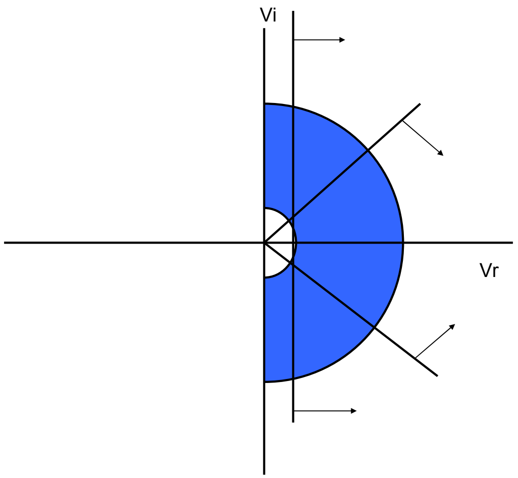


Non-Convex Minimum Voltage Constraints.

$$(v^{min}_m)^2 \leq (v^r_m)^2 + (v^j_m)^2$$

1. non-convex, the linear approximation is problematic.
2. approximation and eliminates parts of the feasible region





3.

This is probably not a good idea, but maybe

Real Power Constraints.

At each generator or load bus dropping the bus index

First order approximation around $\underline{v}^r$, $\underline{i}^r$, $\underline{v}^j$, $\underline{i}^j$

$$p^{\approx} = \underline{v}^r i^r + \underline{v}^j i^j + v^r \underline{i}^r + v^j \underline{i}^j - (\underline{v}^r \underline{i}^r + \underline{v}^j \underline{i}^j)$$

$$p^{\approx min} - p^{\approx minrelax} \leq p^{\approx} \leq p^{\approx max} + p^{\approx maxrelax}$$

Hessian has the off-diagonal identity matrices

0	0	1	0	vr
0	0	0	1	vj
1	0	0	0	ir
0	1	0	0	ij

Eigenvalues: 2 are 1 and 2 are -1

Reactive Power Constraints.

At each generator or load bus dropping the bus index

First order approximation around $\underline{v}^r, \underline{i}^r, \underline{v}^j, \underline{i}^j$

$$q^{\approx} = \underline{v}^j i^r - \underline{v}^r i^j - v^r \underline{i}^j + v^j \underline{i}^r - (\underline{v}^j \underline{i}^r - \underline{v}^r \underline{i}^j)$$

$$q^{\approx min} - q^{\approx minrelax} \leq q^{\approx} \leq q^{\approx max} + q^{\approx maxrelax}$$

The Hessian has positive and negative identity matrices

0	0	0	-1	vr
0	0	1	0	vj
0	1	0	0	ir
-1	0	0	0	ij

Eigenvalues: 2 are 1 and 2 are -1.

Computational experience

MINOS, CONOPT, IPOPT, KNITRO SNOPT

All nonlinear except Knitro find the 'optimal' solution

Ten random starting points, the average cpu time

- 14 bus: GUROBI < all nonlinear solvers
- 30 bus: GUROBI < 2 of 5 nonlinear solvers
- 57 bus: GUROBI < all nonlinear solvers
- 118 bus: CPLEX and GUROBI < all but one nonlinear solver
- 300 bus: CPLEX and GUROBI <all but two nonlinear solver

For the naïve approximation and implementation,

LP approach is **faster or competitive** with nonlinear solvers