Minimizing load shedding in electricity networks by preventive islanding

Waqquas Bukhsh, Andreas Grothey, Ken McKinnon,
Jacek Gondzio, Paul Trodden

School of Mathematics
University of Edinburgh

FERC
25th June 2012

Background

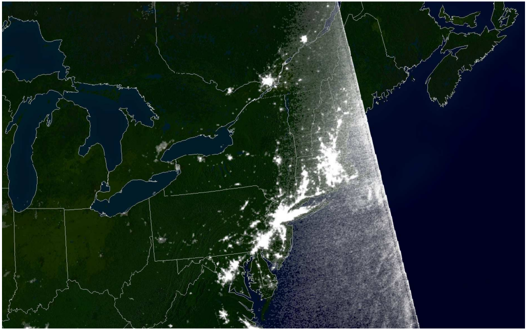
Project structure

- EPSRC funded project led by Janusz Bialek at Durham University with groups at Edinburgh and Southampton.

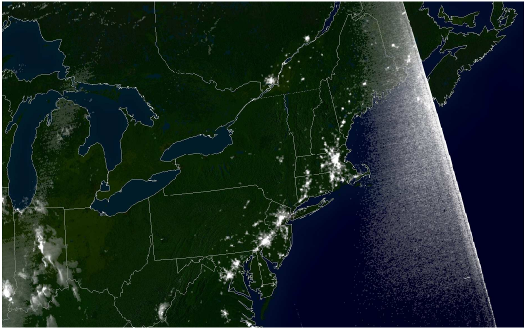
Group Responsibilities:

- Durham: Electrical Engineering: System dynamics
- Southampton: Mathematics: Laplacian based graph partitioning
- Edinburgh: OR and Optimization: Steady state optimization

USA Aug 2003



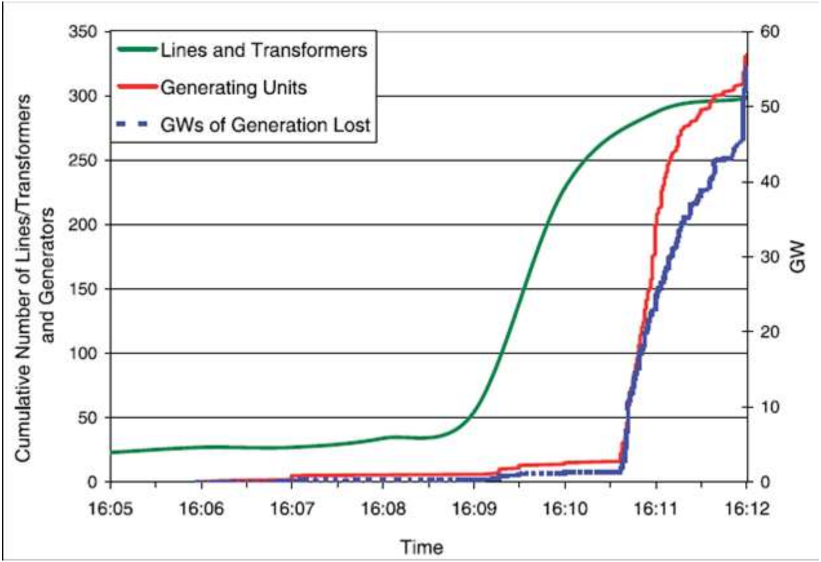
Happy customers



Oooops !

There goes
Long Island,
Detroit,
Ottowa,
Toronto ...

USA Aug 2003



Lines and Transformers
Generating Units
GWs of Generation Lost
Cumulative Number of Lines/Transformers and Generators
GW
Time
350
300
250
200
150
100
50
0
60
50
40
30
20
10
0
16:05
16:06
16:07
16:08
16:09
16:10
16:11
16:12

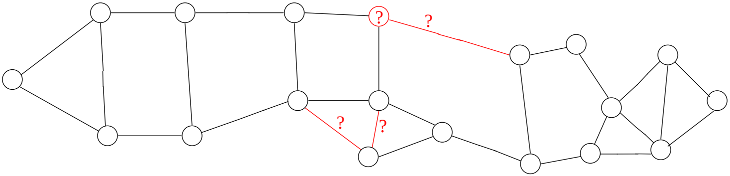
Background

Goal

- Develop a tool that can create a *Firewall* to isolate the troubled area from the rest of the network.
- Leave network in a stable steady state.
- Could be used for off line analysis to prepare responses to faults in different areas or, if fast enough, to react in real time.

Sectioning “uncertain” parts of network

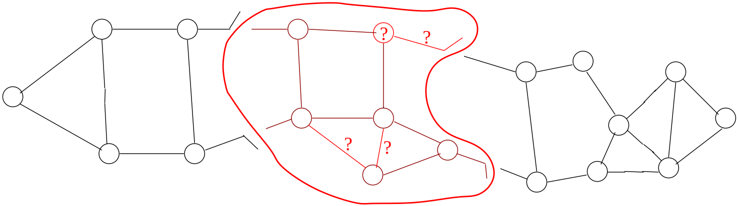
- Network has “uncertain” buses/lines/generators ? ? ?



Sectioning “uncertain” parts of network

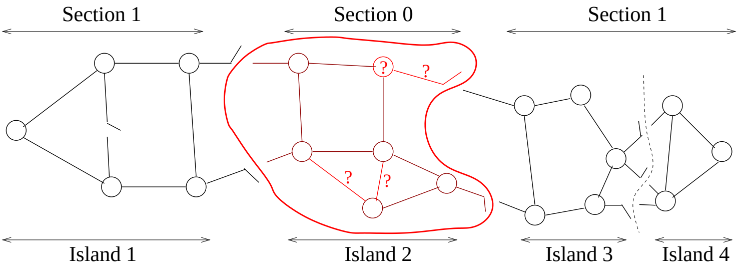
Form a Firewall

- to: Achieve a new “safe” steady state that
  isolates all “uncertain” buses/lines/generators
- by: Cutting lines & Shedding loads & Adjusting generation
- goal: Minimize the load shed



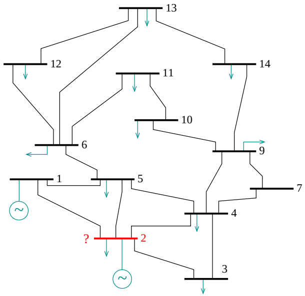
Sectioning "uncertain" parts of network

- Section 0 – uncertain parts, Section 1 – certain parts
- All load in section 0 is at risk, with chance β of surviving

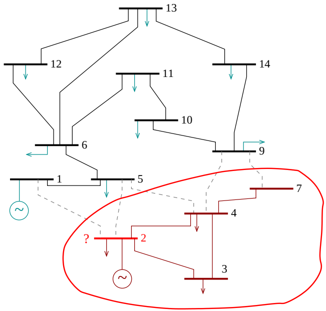


Sectioning with uncertainty: $\beta = 0.5$

Before



After

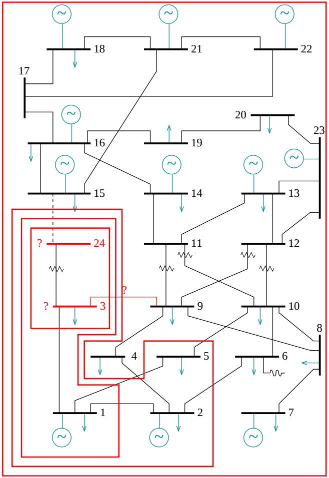


- 259 MW total load
- 0.0 MW load shed
- 129.5 expected loss

- 0.0 MW shed in Section 1
- 35.9 MW shed in Section 0
- 127.8 MW at risk in Section 0
- 99.8 MW expected loss

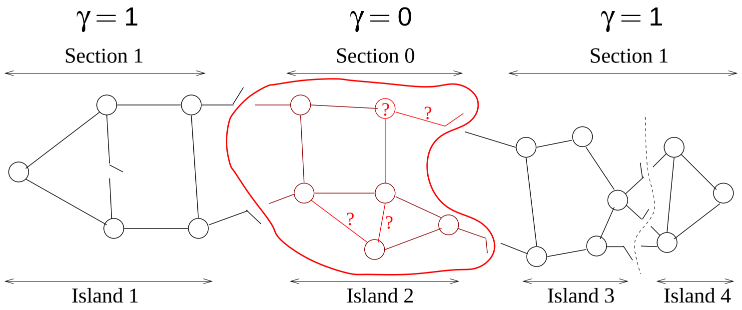
Growth of section 0 in 24 bus network as $\beta \to 1$

$0.00 \leq \beta \leq 1.00$



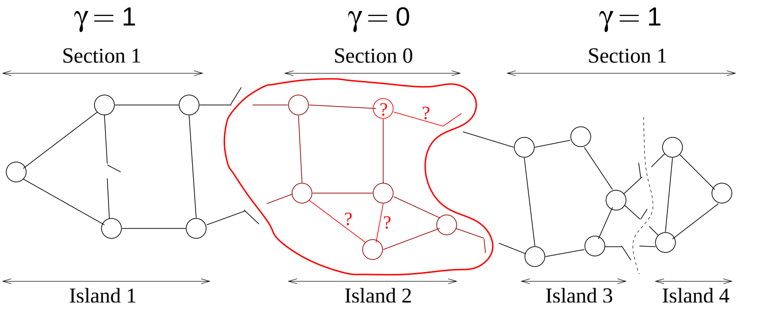
Sectioning Constraints

- Uncertain buses, $\mathcal{B}^0$, and lines, $\mathcal{L}^0$
- Optimization decides what else in 0 & 1 and which line to cut
- $\gamma_b =$ section for bus b
- $\rho_{bb'} = 0$ if line bb' is cut, and 1 otherwise

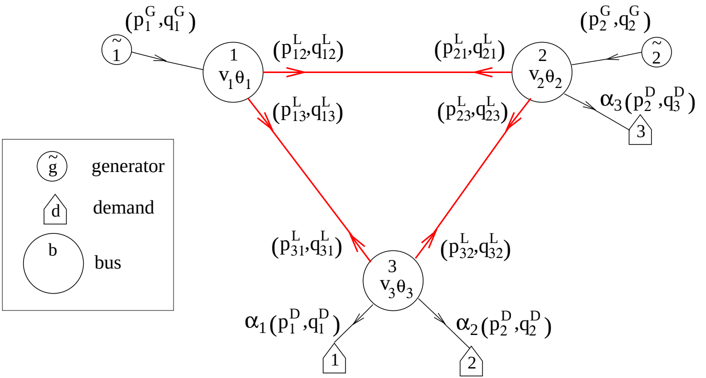


Sectioning Constraints

$\rho_{bb'}$	$\leq$	$1+\gamma_b-\gamma_{b'}$	$bb' \in \mathcal{L}\setminus\mathcal{L}^0$	No lines between sections
$\rho_{bb'}$	$\leq$	$1-\gamma_b+\gamma_{b'}$	$bb' \in \mathcal{L}\setminus\mathcal{L}^0$	No lines between sections
$\rho_{bb'}$	$\leq$	$1-\gamma_b$	$bb' \in \mathcal{L}^0$	No unsafe lines in Section 1
$\rho_{bb'}$	$\leq$	$1-\gamma_{b'}$	$bb' \in \mathcal{L}^0$	No unsafe lines in Section 1
γ_b	$=$	0	$b \in \mathcal{B}^0$	All unsafe buses in Section 0

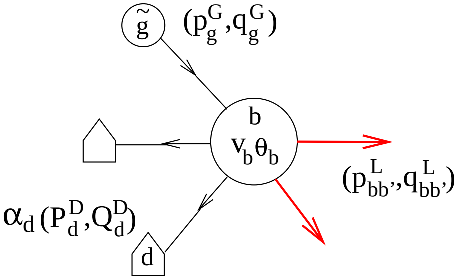


AC power flow model: Notation



$v_b,\ \theta_b$	voltage and phase angle at bus b
$p^L_{bb'}, q^L_{bb'}$	real and reactive power flows into line (b, b') from bus b
p^G_g, q^G_g	real and reactive power output of generator g
P^D_d, Q^D_d	real and reactive power demand at load d
α_d	proportion of load d connected — $\alpha_d \in [0, 1]$

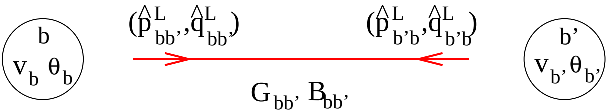
Kirchhoff Current Law (KCL): conservation of flow at buses



$$\sum_{g \in G_b} p_g^G = \sum_{d \in D_b} \alpha_d P_d^D + \sum_{b' \in B_b} p_{bb'}^L \quad \text{Real balance}$$

$$\sum_{g \in G_b} q_g^G = \sum_{d \in D_b} \alpha_d Q_d^D + \sum_{b' \in B_b} q_{bb'}^L \quad \text{Reactive balance}$$

Kirchhoff Voltage Law (KVL): flow-voltage relations on lines

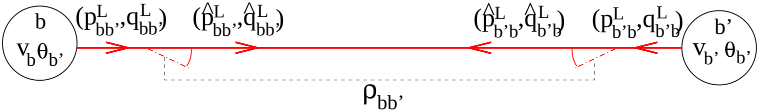


$$
\begin{aligned}
\delta_{bb'} &= \theta_b - \theta_{b'} \\
\hat{p}^L_{bb'} &= G_{bb'} v_b (v_b - v_{b'} \cos \delta_{bb'}) - B_{bb'} v_b v_{b'} \sin \delta_{bb'} \\
\hat{q}^L_{bb'} &= B_{bb'} v_b (v_{b'} \cos \delta_{bb'} - v_b) - G_{bb'} v_b v_{b'} \sin \delta_{bb'} - v_b^2 C_{bb'}
\end{aligned}
$$

- Because of power losses in the lines

$$\hat{p}^L_{bb'} \neq -\hat{p}^L_{b'b} \text{ and } \hat{q}^L_{bb'} \neq -\hat{q}^L_{b'b}.$$

Disconnecting lines ($p^L_{bb'}$ case)



When $\rho_{bb'} = 0$, line is not connected, so $p^L_{bb'} = 0$
When $\rho_{bb'} = 1$, line is connected, so $p^L_{bb'} = \hat{p}^L_{bb'}$

MILP Formulation

$$-\rho_{bb'} P^{L\max}_{bb'} \leq p^L_{bb'} \leq P^{L\max}_{bb'} \rho_{bb'},$$
$$-(1-\rho_{bb'}) \hat{P}^{L\max}_{bb'} \leq \hat{p}^L_{bb'} - p^L_{bb'} \leq \hat{P}^{L\max}_{bb'} (1-\rho_{bb'}).$$

$P^{L\max}_{bb'}$ is the maximum allowed real flow into the line from b
$\hat{P}^{L\max}_{bb'}$ is the maximum possible value of $\hat{p}^L_{bb'} - p^L_{bb'}$

Bounds on variables

$p_{bb'} + p_{b'b} \leq H_{bb'}$	Thermal limit on line
$V_b^- \leq v_b \leq V_b^+$	Voltage limit at bus e.g. $v_b \in [0.95, 1.05]$
$Q_g^{\mathrm{G}-} \leq q_g^{\mathrm{G}} \leq Q_g^{\mathrm{G}+}$	Reactive Power limits $Q_g^{\mathrm{G}-}, Q_g^{\mathrm{G}+}$ *independent* of current operating point
$\zeta_g P_g^{\mathrm{G}-} \leq p_g^{\mathrm{G}} \leq \zeta_g P_g^{\mathrm{G}+}$	Real Power limits $P_g^{\mathrm{G}-}, P_g^{\mathrm{G}+}$ *dependent* on current operating point $\zeta_g = 0$ if generator is switched off $\zeta_g = 1$ if generator remains on

Objective

Definition

- Planned supply: $P_d^D \alpha_d$
- Supply probability: β in Section 0
- Objective: Maximize expected load supplied

Objective: $$\max \ \beta \sum_{d \in S^0} P_d^D \alpha_d + \sum_{d \in S^1} P_d^D \alpha_d$$

IP Formulation

- Split each α_d into α_{0d} for section 0 and α_{1d} for Section 1

$$\alpha_d = \alpha_{0d} + \alpha_{1d}, \quad 0 \leq \alpha_{1d} \leq \gamma_b$$

Objective: $$\max \sum_d P_d^D (\beta \alpha_{0d} + \alpha_{1d})$$

Variables for average line flow and loss

- Define

$$p_{bb'}^{Av} = \frac{\hat{p}_{bb'} - \hat{p}_{b'b}}{2} \qquad q_{bb'}^{Av} = \frac{\hat{q}_{bb'} - \hat{q}_{b'b}}{2}$$

$$p_{bb'}^{Loss} = \hat{p}_{bb'} + \hat{p}_{b'b} \qquad q_{bb'}^{Loss} = \hat{q}_{bb'} + \hat{q}_{b'b}$$

$$\hat{p}_{bb'} = p_{bb'}^{Av} + \frac{p_{bb'}^{Loss}}{2} \qquad \hat{q}_{bb'} = q_{bb'}^{Av} + \frac{q_{bb'}^{Loss}}{2}$$

$$\hat{p}_{b'b} = -p_{bb'}^{Av} + \frac{p_{bb'}^{Loss}}{2} \qquad \hat{q}_{b'b} = -q_{bb'}^{Av} + \frac{q_{bb'}^{Loss}}{2}$$

Approximations of Kirchhoff's Laws

We consider 3 approximations to Kirchhoff's Laws

- "DC" Constant Loss:
  - Drop the reactive power constraints
  - Set all voltages to 1
  - Assume line loss $p_{bb'}^{Loss} =$ its pre-islanding value
- AC Linear:
  - Include reactive power constraints
  - Linearize KVL
- AC PWL:
  - Include reactive power constraints
  - Linearize KVL
  - Include PWL approximation of cosine terms

Case: $B = 5, G = 1, C = 0.5, \quad |\delta| <= 57^\circ, \quad 0.95 \leq v \leq 1.05.$

	max	"DC"	AC Lin	AC PWL
$p^{Av} = -B\delta$	2.500	✓	✓	✓
$+G(v_b - v_{b'})$	0.100		✓	✓
$+G/2\,((v_b-1)^2 - (v_{b'}-1)^2)$	0.001			
$+B(1 - v_b v_{b'})\sin\delta$	0.245			
$+B(\delta - \sin\delta)$	0.103			
$p^{Loss} = 2G(1-\cos\delta)$	0.245			✓
$+G(v_b - v_{b'})^2$	0.001			
$+2G(v_b v_{b'} - 1)(1-\cos\delta)$	0.023			
$q^{Av} = -G\delta$	0.500		✓	✓
$-(B+C)(v_b - v_{b'})$	0.495		✓	✓
$-(B+C)/2\,((v_b-1)^2 - (v_{b'}-1)^2)$	0.001			
$+G(\delta - \sin\delta))$	0.021			
$+G(1 - v_b v_{b'})\sin\delta)$	0.051			
$q^{Loss} = -C(v_b - v_{b'} - 1)$	0.550		✓	✓
$-2B(1-\cos\delta)$	1.224			✓
$-B(v_b - v_{b'})^2$	0.050			
$-C((v_b-1)^2 + (v_{b'}-1)^2)+$	0.000			
$+2B(v_b v_{b'} - 1)(1-\cos\delta)$	0.125			

Computational tests

Steps

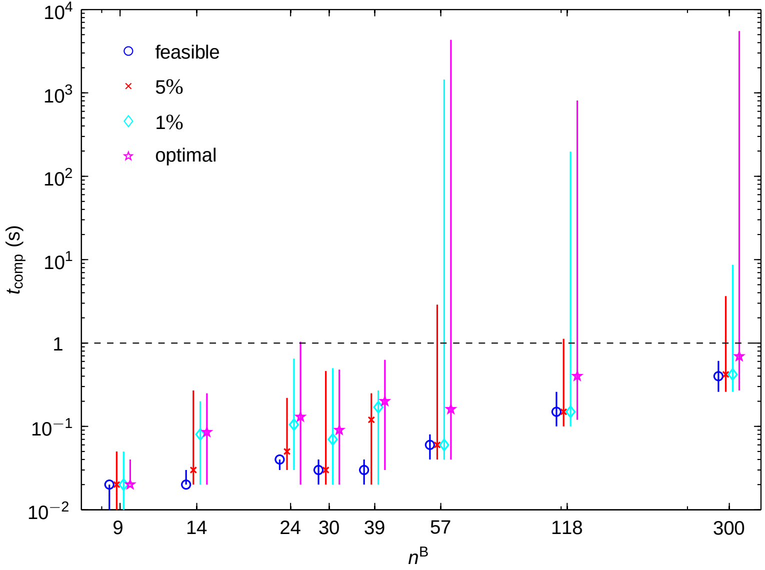
1. Solve OPF: for pre-islanded state using the full AC equations.
2. Based on this solution solve the MILP problem to decide the best islands and generator shut downs.
3. With the islands and generator shutdowns fixed, solve an optimal load shedding problem using the full AC equations.

Software and examples

- 1 and 3 use IPOPT (COIN-OR).
- 2 uses CPLEX 12.3 (using 8 cores)
- IEEE test cases up to 300 buses.

Computational Times (DC constant loss)

Min, Median and Max



MILP Suboptimality (DC constant loss)

How close is the MILP solution to the MILP optimal?

Much better than the bounds indicate:

Average amount above proven optimal solution

Feasible	5% gap	1% gap
9.12%	0.37%	0.04%

AC infeasibility (DC constant loss)

- Some AC solutions were infeasible because voltage limits were violated.
- Proportion where one or more methods gave an AC infeasibility:

n^{B}	9	14	24	30	39	57	118	300
AC infeasible %	0%	15%	50%	27%	35%	17%	14%	28%

What can be done?

- Relaxing voltage limits in the contingency?
  $0.9 \leq v \leq 1.1$ "cures" some cases.
- Alter the tap setting post-islanding?
- Use AC PWL in MILP for islanding

AC feasibility problems

DC constant loss MILP

Voltage problems

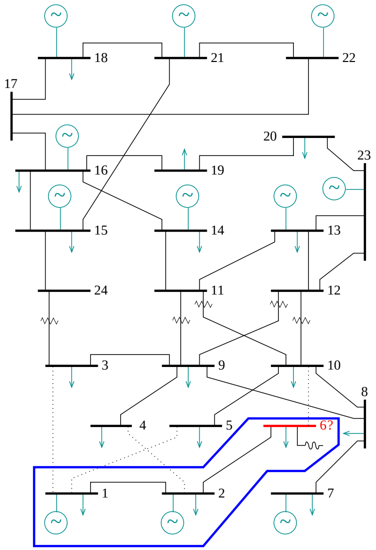
$v_2 = 1.1461$

$v_6 = 0.8452$

Disconnect static reactor at bus 6:

$v_2 = 1.0709$

$v_6 = 0.9291$



AC feasibility problem solved

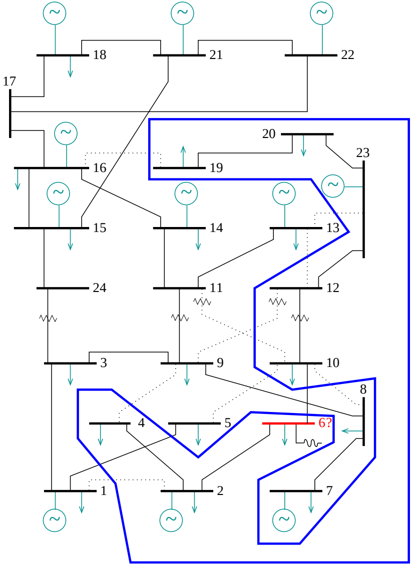
AC PWL MILP

No Voltage problems

$v_2 = 1.02$

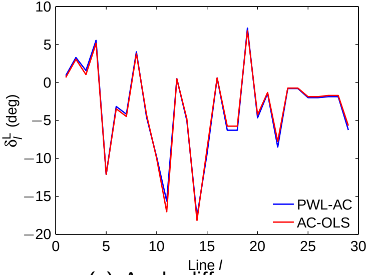
$v_6 = 0.9851$

$v_{10} = 0.9984$

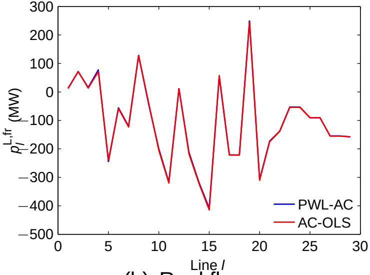


Comparison of AC PWL and AC solutions

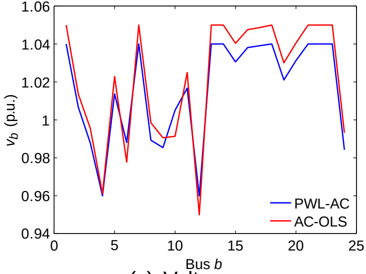
24-bus network: angles, voltages & flows



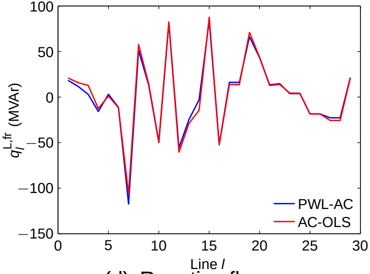
(a) Angle differences



(b) Real flows



(c) Voltages



(d) Reactive flows

AC Infeasible cases

24 Bus case. 24 single bus contingencies

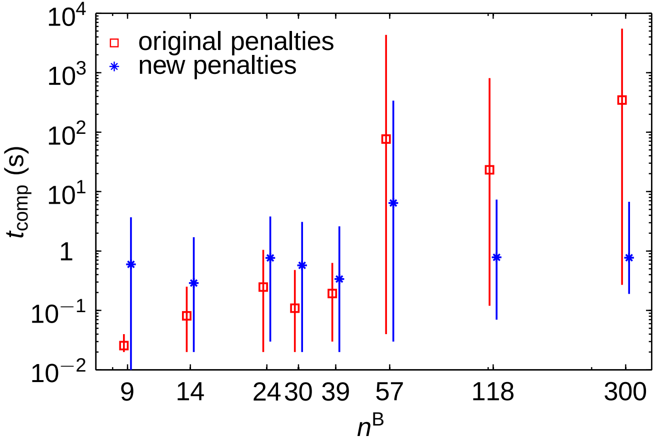
Method	Number AC infeasible	Average Elapsed Time (sec)
DC Constant	7	0.32
AC Linear	3	2.33
AC PWL	0	7.61

Results are similar for other IEEE cases

Dynamic instability

- Islanding creates a shock that could excite dynamic instability, and so prevent the system converging to the planned steady state.
- Tests of previous 452 islanding solutions using 2nd order models showed 14 to be transiently unstable.
- Penalise line and generator disconnections to reduce shock

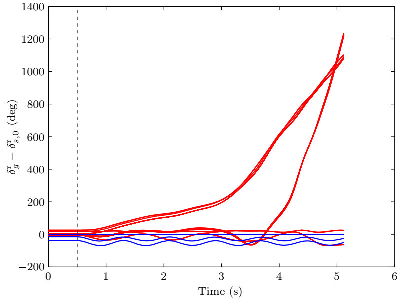
Times with and without line and generator penalties

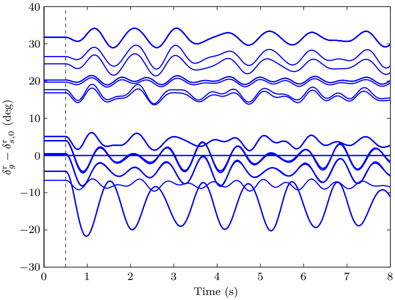


- All problems now transiently stable
- Average load supply decreased by 0.34%

Transient dynamics 24 Bus

Note different scales





Low penalty - Unstable
Predicted load supplied
= 2707 MW
25 generators lose synchrony
— all in “safe” section

High penalty - Stable
Predicted load supplied
= 2675 MW
No loss of synchrony

Current & Future

- Scale up: We can solve “2500” bus Polish networks with DC MILP
- Computation: Develop heuristics and aggregation methods
- Bus splitting: to give more flexibility in islanding.
- Dynamic stability: Incorporate more realistic indicators in MILP
  — using work being done at Durham.

Thank You

Questions?