Two-stage Robust Optimization for N-k Contingency-constrained Unit Commitment

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FERC 2012, Washington, DC

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Outline

Reliability and N-k Rule

- Reliability is among one of the most primary concerns in power system operations
- The unexpected outages of power grid elements, such as generators and transmission lines, probably result in a dramatic electricity shortage or even large scale blackout
- N-k rule: N components can always work in order, whenever any k components suffer contingency

Literature Review

N-k criterion for the bus and transmission network

- Contingency screening [1]
- Interdiction analysis ([2] and [3])
- Vulnerability evaluation [4]

Their common objective is to identify the critical contingency on the bus and transmission network under the N-k criterion.

Two-stage Stochastic CCUC

The traditional approach relies on identification of a credible contingency set of generators and transmission lines. Different solution approaches and objectives have been proposed.

- All possible contingency scenarios [5]
- Benders decomposition [6]
- Minimize the total expected cost [5]
- Minimize the sum of pre-contingency dispatch cost and the cost of spinning and non-spinning operating reserves ([7] and [6])
- Generation unit contingencies ([7] and [5])
- Both generation unit and transmission line contingencies [6]

Related Work-Robust Optimization

- Two-stage robust unit commitment problem [8]
- Two-stage robust power grid optimization problem [9]
- Two-stage robust unit commitment with wind power and demand response [10]
- Price-taker producer offering strategy through robust optimization [13]
- Robust unit commitment with wind power uncertainty [12]
- Robust planning: integrating PHEVs into the electric grid [14]
- Adaptive robust optimization for the security constrained unit commitment problem [11]

Related Work

- In [15], contingencies for both generators and transmission lines are considered.
  - Focusing on the co-optimization of unit commitment and transmission switching problem under *N*-1 reliability.
  - A full credible set for the *N*-1 reliability is provided.
- In [16], the first robust optimization approach is introduced to solve the CCUC with *N*-*k* security criterion.
  - Applying bilevel programming to address the unit commitment and robust contingency analysis.
  - Allowing the system operator to consider all possible contingency combinations of *k* out of *N* generators (transmission contingencies are not considered).

Our Contribution

1. We extend the work in [15] to consider the *N*-*k* security criterion.
2. We apply a decomposition algorithm with a separation algorithm embedded to detect the most critical *k* components. This allows for smaller sub-problems, allowing us to solve larger problems to optimality.
3. We extend the study in [16] to include transmission capacity constraints and to consider transmission contingencies. That is, we consider both generator and transmission contingencies.
4. In [16], spinning and nonspinning reserves are adjusted to ensure system reliability under contingency for the single bus case. In this paper, due to inclusion of transmission constraints for a multi-bus system, we consider economic redispatch to satisfy post-contingency physical constraints.

Notation-Sets and Parameters

I, E, T	Index sets of buses, transmission lines, and time
Λ	Set of all generators
Λ_i	Set of generators at bus i
$\hat{Z}$	Set of all possible contingencies
N	Number of components (e.g., generators, transmission lines) in the power system
R	Number of points selected in power generation cost curve for piecewise linear approximation
F''_g / F'_g	Start-up/Shut-down cost of generator g
H''_g / H'_g	Minimum up/down time for generator g
R''_g	Ramp-up rate limit for generator g
R'_g	Ramp-down rate limit for generator g
L_g	Lower limit of generator g's power output
U_g	Upper limit of generator g's power output
θ''_i	Maximum value of the phase angle at bus i
θ'_i	Minimum value of the phase angle at bus i
f''_{ij}	Maximum power flow on transmission line (i, j)
f'_{ij}	Minimum power flow on transmission line (i, j)
x_{ij}	The reactance of the transmission line (i, j)
$G_g(q)$	Fuel cost for generator g when its power output is q
D_{it}	Demand at bus i at time t
q^r_g	r^{th} point of piecewise linear approximation of power output by generator g
O_t	Spinning reserve requirement for the power system at time t

Notation-Decision Variables

q_{gt}	Power output by generator g at time t
y_{gt}	Binary decision variable: "1" if generator g is on at time t; "0" otherwise
u_{gt}	Binary decision variable: "1" if generator g is started up at time t; "0" otherwise
v_{gt}	Binary decision variable: "1" if generator g is shut down at time t; "0" otherwise
z_g	Binary decision variable: "1" if generator g is under contingency; "0" otherwise
z_{ij}	Binary decision variable: "1" if transmission line (i, j) is under contingency; "0" otherwise
θ_{it}	Phase angle for bus i at time t
f_{ij}^t	Power flow on transmission line (i, j) at time t
λ_{gt}^r	Weight associated with the r^{th} point q_g^r in piecewise linear approximation at time t
d_{it}	Load imbalance amount for bus i at time t
$\hat{\mathcal{Q}}$	Auxiliary variables to represent the optimal objective value for the subproblem

The Robust Optimization Formulation

$$
\begin{aligned}
\min_{\{u,v,y\}} \quad & \sum_{t\in T}\sum_{g\in\Lambda}(F''_g u_{gt} + F'_g v_{gt}) + \max_{z\in\hat{Z}} Q(z) && (1)\\
& s.t. \\
& -y_{g(t-1)} + y_{gt} - y_{gk} \leq 0, && (2)\\
& \qquad \forall g\in\Lambda, \forall t\in T, \forall k: 1 \leq k-(t-1) \leq H''_g \\
& y_{g(t-1)} - y_{gt} + y_{gk} \leq 1, && (3)\\
& \qquad \forall g\in\Lambda, \forall t\in T, \forall k: 1 \leq k-(t-1) \leq H'_g \\
& -y_{g(t-1)} + y_{gt} - u_{gt} \leq 0, \quad \forall g\in\Lambda, \forall t\in T && (4)\\
& y_{g(t-1)} - y_{gt} - v_{gt} \leq 0, \quad \forall g\in\Lambda, \forall t\in T && (5)\\
& \sum_{g\in\Lambda} U_g y_{gt} \geq O_t + \sum_{i\in I} D_{it}, \quad \forall t\in T && (6)\\
& y_{gt}, u_{gt}, v_{gt} \in \{0,1\}, \quad \forall g\in\Lambda, \forall t\in T, && (7)
\end{aligned}
$$

The Uncertainty Set

$$\hat{Z} = \left\{ (z_g, z_{ij}) : \sum_{g \in \Lambda} z_g + \sum_{(i,j) \in E} z_{ij} \geq N - k \right\}, \tag{8}$$

The Formulation

$$
\begin{aligned}
Q(z) \quad & = \min \sum_{t\in T}\sum_{g\in\Lambda}\sum_{r=1}^{R} \lambda_{gt}^{r} G_g(q_g^r) + \sum_{t\in T}\sum_{i\in I} M d_{it} && (9)\\
& z_g y_{gt} L_g \le q_{gt} \le z_g y_{gt} U_g, \quad \forall g\in\Lambda, \forall t\in T && (10)\\
& z_{ij} f_{ij}' \le f_{ij}^t \le z_{ij} f_{ij}'', \quad \forall (i,j)\in E, \forall t\in T && (11)\\
& \theta_i' \le \theta_{it} \le \theta_i'', \quad \forall i\in I, \forall t\in T && (12)\\
& (\theta_{it}-\theta_{jt})/x_{ij} - f_{ij}^t + (1-z_{ij})M \ge 0, \forall (i,j)\in E, \forall t\in T && (13)\\
& (\theta_{it}-\theta_{jt})/x_{ij} - f_{ij}^t - (1-z_{ij})M \le 0, \forall (i,j)\in E, \forall t\in T && (14)\\
& q_{gt} - q_{g(t-1)} \le (2 - y_{g(t-1)} - y_{gt})L_g + (1 + y_{g(t-1)} - y_{gt})R_g'', \quad \forall g\in\Lambda, \forall t\in T && (15)\\
& q_{g(t-1)} - q_{gt} \le (2 - y_{g(t-1)} - y_{gt})L_g + (1 - y_{g(t-1)} + y_{gt})R_g', \quad \forall g\in\Lambda, \forall t\in T && (16)\\
& -d_{it} \le \sum_{\forall j\in E_{(\cdot,i)}} f_{ji}^t - \sum_{\forall j\in E_{(i,\cdot)}} f_{ij}^t + \sum_{g\in\Lambda_i} q_{gt} - D_{it} \le d_{it}, \quad \forall i\in I, \forall t\in T && (17)\\
& \sum_{r=1}^{R} \lambda_{gt}^r = y_{gt}, \quad \forall g\in\Lambda, \forall t\in T && (18)\\
& \sum_{r=1}^{R} \lambda_{gt}^r q_g^r = q_{gt}, \quad \forall g\in\Lambda, \forall t\in T && (19)\\
& q_{gt}, \theta_{it}, \lambda_{gt}^r, d_{it} \ge 0, f_{ij}^t \text{ urs}, z_g, z_{ij} \in \{0,1\}, \\
& \qquad \forall i\in I, \forall (i,j)\in E, \forall g\in\Lambda, \forall t\in T. && (20)
\end{aligned}
$$

The Pre-Contingency Formulation

$$
\begin{aligned}
X^0 = \{ \quad & (q^0, f^0, \theta^0, \lambda^0): \\
& y_{gt} L_g \leq q^0_{gt} \leq y_{gt} U_g, \ \forall g \in \Lambda, \forall t \in T && (21) \\
& f'_{ij} \leq f^{t0}_{ij} \leq f''_{ij}, \quad \forall (i,j) \in E, \forall t \in T && (22) \\
& \theta'_i \leq \theta^0_{it} \leq \theta''_i, \quad \forall i \in I, \forall t \in T && (23) \\
& (\theta^0_{it} - \theta^0_{jt})/x_{ij} = f^{t0}_{ij}, \ \forall (i,j) \in E, \forall t \in T && (24) \\
& q^0_{gt} - q^0_{g(t-1)} \leq (2 - y_{g(t-1)} - y_{gt}) L_g + (1 + y_{g(t-1)} - y_{gt}) R''_g, \ \forall g \in \Lambda, \forall t \in T && (25) \\
& q^0_{g(t-1)} - q^0_{gt} \leq (2 - y_{g(t-1)} - y_{gt}) L_g + (1 - y_{g(t-1)} + y_{gt}) R'_g, \ \forall g \in \Lambda, \forall t \in T && (26) \\
& \sum_{\forall j \in E_{(\cdot,i)}} f^{t0}_{ji} - \sum_{\forall j \in E_{(i,\cdot)}} f^{t0}_{ij} + \sum_{g \in \Lambda_i} q^0_{gt} = D_{it}, \quad \forall i \in I, \forall t \in T && (27) \\
& \sum_{r=1}^{R} \lambda^{r0}_{gt} = y_{gt}, \ \forall g \in \Lambda, \forall t \in T && (28) \\
& \sum_{r=1}^{R} \lambda^{r0}_{gt} q^r_g = q^0_{gt}, \ \forall g \in \Lambda, \forall t \in T && (29) \\
& q^0_{gt}, \theta^0_{it}, \lambda^{r0}_{gt} \geq 0, \ f^{t0}_{ij} \text{ urs}, \ \forall i \in I, \forall g \in \Lambda, \\
& \qquad \forall (i,j) \in E, \forall t \in T && (30) \\
\}&,
\end{aligned}
$$

where X^0 represents the pre-contingency feasibility set.

The Pre-Contingency Formulation

With the objective of minimizing the total pre-contingency cost, the corresponding model can be updated as follows:

$$
\begin{aligned}
\min_{\{u,v,y\}} \quad & \sum_{t\in T}\sum_{g\in\Lambda}\left(F''_g u_{gt} + F'_g v_{gt} + \sum_{r=1}^{R}\lambda_{gt}^{r0} G_g(q_g^r)\right) \\
& + \max_{z\in\hat{Z}} Q(z) \\
s.t. \quad & \text{constraints } (2) - (8) \\
& (q_{gt}^0, f_{ij}^{t0}, \theta_{it}^0, \lambda_{gt}^{r0}) \in X^0 \\
& Q(z) = \min \sum_{t\in T}\sum_{i\in I} M d_{it} \\
& \text{constraints } (10) - (20).
\end{aligned}
$$

An Extended Reformulation

$$\min_{\{u,v,y\}} \quad \sum_{t\in T}\sum_{g\in\Lambda}(F''_g u_{gt} + F'_g v_{gt}) + \hat{\mathcal{Q}} \tag{31}$$

$s.t.$

constraints $(2)-(7)$

$$\hat{\mathcal{Q}} \geq \sum_{t\in T}\sum_{i\in I}\left(\sum_{g\in\Lambda_i}\sum_{r=1}^{R}\lambda_{gt}^{rm}G_g(q_g^r) + Md_{it}^m\right)$$

$$1 \leq m \leq K,$$

$$(q^m, f^m, \theta^m, \lambda^m, d^m) \in X^m,\ 1 \leq m \leq K$$

An Extended Reformulation

$$
\begin{aligned}
X^m = \quad & \{(q^m, f^m, \theta^m, \lambda^m, d^m) : \\
& z_g(m) y_{gt} L_g \leq q^m_{gt} \leq z_g(m) y_{gt} U_g, \forall g \in \Lambda, \forall t \in T && (32) \\
& z_{ij}(m) f'_{ij} \leq f^{tm}_{ij} \leq z_{ij}(m) f''_{ij}, \forall (i,j) \in E, \forall t \in T \\
& \theta'_i \leq \theta^m_{it} \leq \theta''_i, \forall i \in I, \forall t \in T && (33) \\
& (\theta^m_{it} - \theta^m_{jt}) z_{ij}(m)/x_{ij} = f^{tm}_{ij}, \forall (i,j) \in E, \forall t \in T && (34) \\
& q^m_{gt} - q^m_{g(t-1)} \leq (2 - y_{g(t-1)} - y_{gt}) L_g + (1 + y_{g(t-1)} - y_{gt}) R''_g, \ \forall g \in \Lambda, \forall t \in T && (35) \\
& q^m_{g(t-1)} - q^m_{gt} \leq (2 - y_{g(t-1)} - y_{gt}) L_g + (1 - y_{g(t-1)} + y_{gt}) R'_g, \ \forall g \in \Lambda, \forall t \in T && (36) \\
& -d^m_{it} \leq \sum_{\forall j \in E_{(\cdot, i)}} f^{tm}_{ji} - \sum_{\forall j \in E_{(i, \cdot)}} f^{tm}_{ij} + \sum_{g \in \Lambda_i} q^m_{gt} - D_{it} \leq d^m_{it}, \ \forall i \in I, \forall t \in T && (37) \\
& \sum_{r=1}^{R} \lambda^{rm}_{gt} = y_{gt}, \ \forall g \in \Lambda, \forall t \in T && (38) \\
& \sum_{r=1}^{R} \lambda^{rm}_{gt} q^r_g = q^m_{gt}, \ \forall g \in \Lambda, \forall t \in T && (39) \\
& q^m_{gt}, \theta^m_{it}, \lambda^{rm}_{gt}, d^m_{it} \geq 0, f^{tm}_{ij} \text{ urs}, \forall i \in I, \forall g \in \Lambda, \forall (i,j) \in E, \forall t \in T && (40) \\
\}. &
\end{aligned}
$$

Note here X^0 can be considered as a special case of X^m with $m = 0$ and $z_g(0) = z_{ij}(0) = 1, \forall g \in \Lambda, \forall (i,j) \in E$.

Decomposition Framework-Master problem

For the master problem, we consider the pre-contingency case as the start point. For instance, the initial master problem can be described as follows:

$$
\begin{aligned}
\min_{\{u,v,y\}} \quad & \sum_{t\in T}\sum_{i\in I}\sum_{g\in\Lambda_i}(F''_g u_{gt} + F'_g v_{gt}) + \hat{\mathcal{Q}} \\
s.t. \quad & \text{constraints } (2)-(7) \\
& \hat{\mathcal{Q}} \geq \sum_{t\in T}\sum_{i\in I}\left(\sum_{g\in\Lambda_i}\sum_{r=1}^{R}\lambda_{gt}^{r0} G_g(q_g^r)\right) \\
& (q^0, f^0, \theta^0, \lambda^0) \in X^0.
\end{aligned}
$$

Decomposition Framework-Subproblem

After solving each master problem, we can obtain the solution $(\bar{u}, \bar{v}, \bar{y})$. The subproblem is $\max_z Q(z)$. We expect to solve this problem with $(u, v, y) = (\bar{u}, \bar{v}, \bar{y})$ to identify the worst contingency scenario and add the most violated inequalities into the master problem. The subproblem can be described as follows:

$$\max_z Q(z) = \max_z \min \sum_{t \in T} \sum_{i \in I} \left(\sum_{g \in \Lambda_i} \sum_{r=1}^{R} \lambda_{gt}^r G_g(q_g^r) + Md_{it} \right)$$
$$s.t. \;\; \text{constraints } (8), (10) - (20).$$

Subproblem Reformulation by Duality

$$
\begin{aligned}
& \max \mathcal{G}(\gamma, \delta, \eta, \kappa, \varsigma, \pi, \varphi, \tau, z) \\
s.t. \quad & \text{constraint (8)} \\
& \gamma^+_{gt} - \gamma^-_{gt} - \varsigma^+_{gt} + \varsigma^+_{g(t+1)} + \varsigma^-_{gt} - \varsigma^-_{g(t+1)} + \pi^+_{it} - \pi^-_{it} - \tau_{gt} \le 0, \qquad (41) \\
& \quad \forall i \in I, \forall g \in \Lambda_i, \forall t \in T \\
& \delta^+_{ij,t} - \delta^-_{ij,t} - \kappa^+_{ij,t} + \kappa^-_{ij,t} - \pi^+_{it} + \pi^+_{jt} + \pi^-_{it} - \pi^-_{jt} \le 0, \forall (i,j) \in E, \forall t \in T \\
& \eta^+_{it} - \eta^-_{it} + \sum_{j \in E_{(i,\cdot)}} \frac{1}{x_{ij}} \kappa^+_{ij,t} - \sum_{j \in E_{(\cdot,i)}} \frac{1}{x_{ji}} \kappa^+_{ji,t} - \sum_{j \in E_{(i,\cdot)}} \frac{1}{x_{ij}} \kappa^-_{ij,t} + \sum_{j \in E_{(\cdot,i)}} \frac{1}{x_{ji}} \kappa^-_{ji,t} = 0, \qquad (42) \\
& \quad \forall i \in I, \forall t \in T \\
& -G_g(q^r_g) + \varphi_{gt} + q^r_g \tau_{gt} \le 0, \forall g \in \Lambda, \forall t \in T, \forall r : 1 \le r \le R \\
& \pi^+_{it} + \pi^-_{it} - M \le 0, \forall i \in I, \forall t \in T \\
& \gamma, \delta, \eta, \kappa, \varsigma, \pi \ge 0, \varphi, \tau, \text{unrestricted}, \qquad (43)
\end{aligned}
$$

where $\gamma, \delta, \eta, \kappa, \varsigma, \pi, \varphi$, and τ are dual variables for constraints (10), (11), (12), (13)-(14), (15)-(16), (17), (18), and (19), respectively.

Nonlinear Terms in the Objective Function

The nonlinear terms in the objective function with the following format

$$\max z\mu, \text{ subject to } z \in \{0, 1\} \text{ and } \mu \in R^+$$

can be reformulated as

$$\begin{aligned} & \max \mu_z \\ s.t. \quad & \mu_z \leq (1 - z)M + \mu, \ \mu_z \leq zM \\ & \mu, \mu_z \in R^+ \end{aligned}$$

to finally make the subprogram an MILP.

Primal Cuts

After solving the separation problem $\max \mathcal{G}$, we obtain the optimal solution $z(m^*)$ which indicates the worst contingency scenario. If $\hat{\mathcal{Q}} < \max \mathcal{G}$, the corresponding most violated inequalities

$$\hat{\mathcal{Q}} \geq \sum_{t \in T} \sum_{i \in I} \left(\sum_{g \in \Lambda_i} \sum_{r=1}^{R} \lambda_{gt}^{rm^*} G_g(q_g^r) + M d_{it}^{m^*} \right)$$
$$(q^{m^*}, f^{m^*}, \theta^{m^*}, \lambda^{m^*}, d^{m^*}) \in X^{m^*}$$

will be added into the master problem.

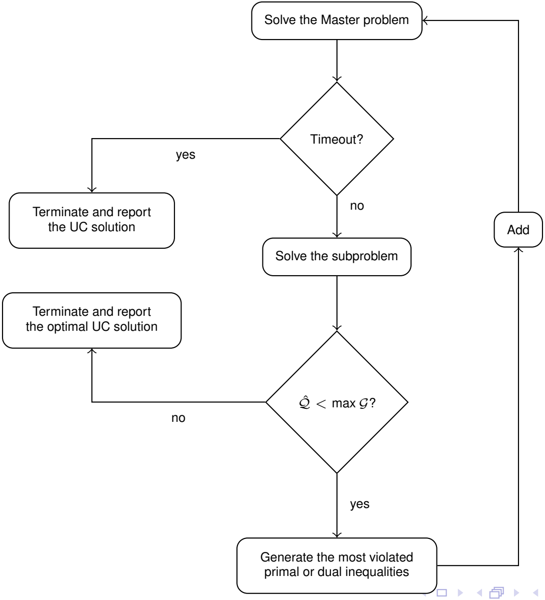
Dual Cuts

After solving the subprogram $\max \mathcal{G}$, if $\hat{\mathcal{Q}} < \max \mathcal{G}$, the following inequality

$$\hat{\mathcal{Q}} \geq \mathcal{G}(\gamma^*, \delta^*, \eta^*, \kappa^*, \varsigma^*, \pi^*, \varphi^*, \tau^*, z^*), \tag{44}$$

where $(\gamma^*, \delta^*, \eta^*, \kappa^*, \varsigma^*, \pi^*, \varphi^*, \tau^*, z^*)$ is the optimal solution for the subproblem, is added into the master problem.

Algorithm Framework



A Six-Bus Example

A six-bus system composed of three generators, six loads, and eight transmission lines, with the *N*-2 security criterion.

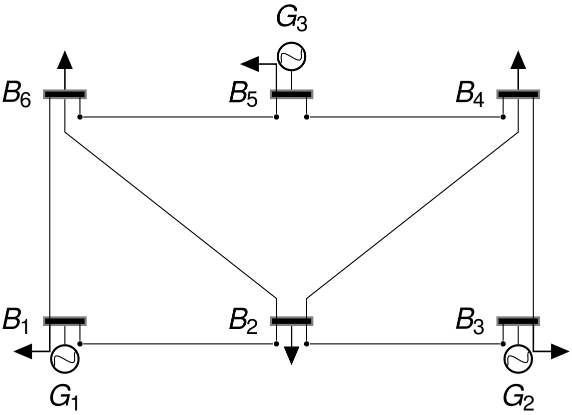


Figure: Six-bus System

A Six-Bus Example

The primal approach is performed in the decomposition framework for the six-bus system. The algorithm terminates in seven iterations.

1. only G_1 is committed: worst-case contingency scenario G_1 and L_1
2. G_2 is committed: worst-case contingency scenario G_2 and L_3
3. G_3 is committed
4. both G_1 and G_2 are committed: worst-case contingency scenario G_1 and G_2
5. both G_1 and G_3 are committed: worst-case contingency scenario G_1 and G_3
6. both G_2 and G_3 are committed: worst-case contingency scenario G_2 and G_3
7. all G_1, G_2, and G_3 are committed

An IEEE 118-bus System - N-1 Case

The objective is to minimize the total cost under the worst-case contingency scenario.

Table: Computational Results for the 118-bus System: Primal Cuts

Iteration	Type	Obj.	Time (sec)
1	master	754507	2.27
	subproblem	1309680	328.5
2	master	764379	6.45
	subproblem	762264	724.9
3	master	782815	19.2
	subproblem	785137	697.5
4	master	783606	39.06
	subproblem	762264	724.6

Terminate in four iterations to obtain the optimal solution!

An IEEE 118-bus System - *N*-1 Case

Terminate due to one hour time limit. The ultimate objective 755134 is much smaller than the optimal objective 783606.

Table: Computational Results for the 118-bus System: Dual Cuts

Iteration	Type	Obj.	Time (sec)
1	master	754507	3.12
	subproblem	1309680	464.4
2	master	754830	7.01
	subproblem	1218430	597.2
3	master	754948	10.16
	subproblem	1218370	708.07
4	master	755047	15.6
	subproblem	1218450	687.2
5	master	755134	12.3
	subproblem	1218430	700.5

An IEEE 118-bus System - *N*-1 Case

Comparison of different objectives:

- The objective of minimizing the total pre-contingency cost: 764379
- The objective of minimizing the total worst-case cost: 783606.

However, both approaches provide the same unit commitment decision. Therefore, for this instance, either approach will provide the same robust unit commitment decision, even though the conclusion is not in general true.

An IEEE 118-bus System - *N*-2 Case

Thirty-transmission lines and two generators are added to make $N-2$ feasible.

Table: Computational Results for the 118-bus System: *N*-2 Case

Iteration	Type	Obj.	Time (sec)
1	master	749727	3.74
	subproblem	1700200	2378.29
2	master	766615	13.02
	subproblem	1018590	3600.09
3	master	775288	33.05
	subproblem	1214330	3600.06
4	master	784406	273.18
	subproblem	801187	3600.02
5	master	805186	265.30
	subproblem	-	Timeout

An IEEE 118-bus System - *N*-2 Case

Terminate due to five hour time limit. The final objective is 805186 as compared to 774488 for the *N*-1 case, which is finished in three iterations.

Remark: As compared to the solution for the *N*-1 security criterion case, one more expensive generator is turned on in the final unit commitment solution. With the pre-contingency cost as the objective, the cost for *N*-2 (760179) is 10452 larger than that for *N*-1 (749727).

Conclusions and Future Research

- We proposed a two-stage robust optimization model to solve the CCUC problem with the *N*-*k* security criterion.
- A decomposition algorithm with both primal and dual approaches is studied to solve the problem
- For the *N*-*k*, $k \geq 2$ cases, the subproblem is hard to be solved to optimality, which is mainly due to the big-M formulation in the subproblem (e.g., the big-M formulation leads to a big optimality gap) (the similar issue happened in [15])
- In future research, we will explore alternative formulations and other methods to solve larger size problems

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