Computational Approaches to the AC Optimal Power Flow (OPF) Problem

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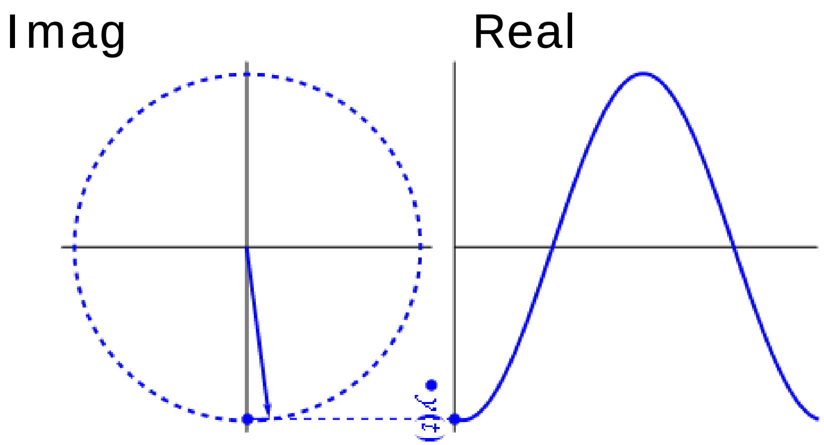
Overview

- ACOPF Problem Formulation
- Current Issues
- Model Description
- Simulation Results
- Toy Model Anecdote
- Discussion

The ACOPF Problem: 'Fictitious' Form

"These Imaginary Quantities (as they are commonly called) arising from the Supposed Root of a Negative Square (when they happen) are reputed to imply that the Case proposed is Impossible."

D. Wells, *The Penguin Dictionary of Curious and Interesting Numbers.*



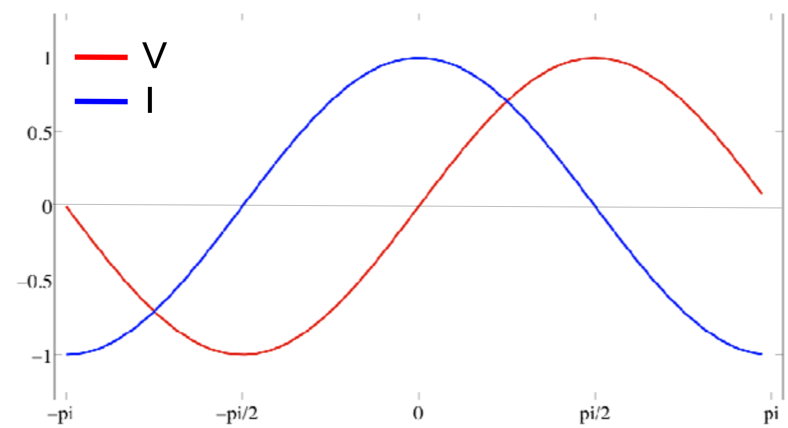
Net Complex Power Injection:
$S = P + jQ = diag(V)I^* = diag(V)\,[YV]^*$

where

$P = P_G - P_D$	Real Power Component
$Q = Q_G - Q_D$	Reactive (Imaginary) Power Component
$Y = G + jB$	Branch Admittance Matrix
$V = \lvert V\rvert \angle\theta = V_{re} + jV_{im}$	

Rectanglar-to-Polar Phasor Voltage Conversion:
$V_{re} = \lvert V\rvert \cos\theta$
$V_{im} = \lvert V\rvert \sin\theta$

90° out-of-phase: Zero Losses



Power Losses determined by the difference between the voltage phasor (V) and current phasor (I)

The ACOPF Problem: Polar Formulation

$\min\limits_{\theta,\lvert V\rvert,P,Q} \sum_{i\in N} f_i\left(P_{Gi}, Q_{Gi}\right)$	Cost Function
$\boxed{\lvert V_i\rvert \sum_{n\in N} \lvert V_n\rvert \left(G_{in}\cos\theta_{in} + B_{in}\sin\theta_{in}\right) - P_{Gi} + P_{Di} = 0}$	(1) Real Power Flow, $\forall i \in Nodes$
$\boxed{\lvert V_i\rvert \sum_{n\in N} \lvert V_n\rvert \left(G_{in}\sin\theta_{in} - B_{in}\cos\theta_{in}\right) - Q_{Gi} + Q_{Di} = 0}$	(2) Reactive Power Flow, $\forall i \in Nodes$
$P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max}$	(3) Generator Real Power Limits, $\forall i \in Nodes$
$Q_{Gi}^{min} \leq Q_{Gi} \leq Q_{Gi}^{max}$	(4) Generator Reactive Power Limits, $\forall i \in Nodes$
$V_i^{\min} \leq \lvert V_i\rvert \leq V_i^{\max}$	(5) Node Voltage Magnitude Limits, $\forall i \in Nodes$
$\theta_{in}^{min} \leq \theta_i - \theta_n \leq \theta_{in}^{max}$	(6) Maximum Phase Angle Difference, $\forall in \in Interconnections$
$\sqrt{P_{inl}^2 + Q_{inl}^2} \leq s_l^{max}$	(7) Line Thermal Limits, $\forall l \in Lines$

Flows satisfy Kirchhoff's 1st (KCL) and 2nd (KVL) Laws:

$$I_1 + \ldots + I_N = 0 \qquad \text{KCL}$$

$$V_1 + \ldots + V_N = 0 \qquad \text{KVL}$$

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Nonlinear function of state variables

May include other operational constraints (i.e. due to security considerations).

The ACOPF Problem: Rectangular Formulation

$\min\limits_{V_{im},V_{re},P,Q} \sum_{i\in N} f_i\left(P_{Gi}, Q_{Gi}\right)$	Cost Function
$V_{re,i}\sum_{n\in N}\left(G_{in}V_{re,n} - B_{in}V_{im,n}\right) + V_{im,i}\sum_{n\in N}\left(G_{in}V_{im,n} + B_{in}V_{re,n}\right) - P_{Gi} + P_{Di} = 0$	(1) Real Power Flow, $\forall i \in Nodes$
$V_{im,i}\sum_{n\in N}\left(G_{in}V_{re,n} - B_{in}V_{im,n}\right) - V_{re,i}\sum_{n\in N}\left(G_{in}V_{im,n} + B_{in}V_{re,n}\right) - Q_{Gi} + Q_{Di} = 0$	(2) Reactive Power Flow, $\forall i \in Nodes$
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$V_i^{min} \le \sqrt{V_{re,i}^2 + V_{im,i}^2} \le V_i^{max}$	(5) Node Voltage Magnitude Limits, $\forall i \in Nodes$
$\theta_{in}^{min} \le \arctan\left(V_{im,i}/V_{re,i}\right) - \arctan\left(V_{im,n}/V_{re,n}\right) \le \theta_{in}^{max}$	(6) Maximum Phase Angle Difference $\forall in \in Interconnections$
$\sqrt{P_{inl}^2 + Q_{inl}^2} \le s_l^{max}$	(7) Line Thermal Limits $\forall l \in Lines$

Polar Equivalent:

$$|V_i|\sum_{n\in N}|V_n|\left(G_{in}\cos\theta_{in} + B_{in}\sin\theta_{in}\right) - P_{Gi} + P_{Di} = 0$$

$$|V_i|\sum_{n\in N}|V_n|\left(G_{in}\sin\theta_{in} - B_{in}\cos\theta_{in}\right) - Q_{Gi} + Q_{Di} = 0$$

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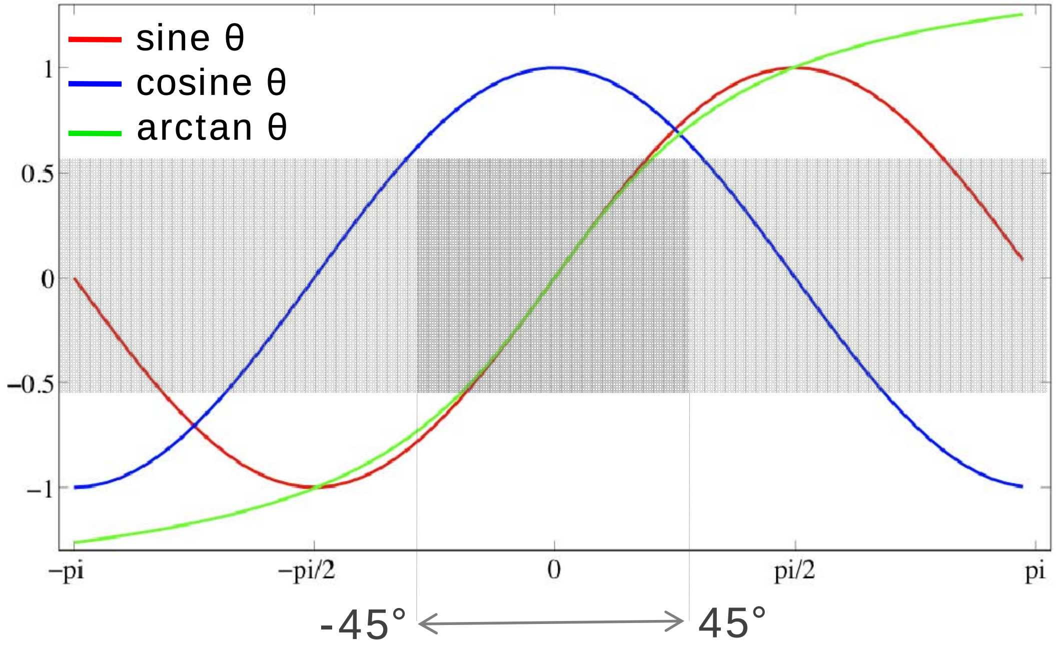
Polar Equivalent: $V_i^{\min} \leq |V_i| \leq V_i^{\max}$

Transcendental function
Polar Equivalent: $\theta_{in}^{min} \leq \theta_i - \theta_n \leq \theta_{in}^{max}$

Nonlinear function of state variables

The ACOPF Problem: Tricky Trig Approximations



angle	radians	sin	cos	arctan
0	0.000	0.000	1.000	0.000
$\pi/32$	0.098	0.098	0.995	0.098
$\pi/16$	0.196	0.195	0.981	0.194
$\pi/8$	0.393	0.383	0.924	0.374
$\pi/4$	0.785	0.707	0.707	0.666
$\pi/2$	1.571	1.000	0.000	1.000

10% error for sine, 15% error for arctan

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The Potential Impact...

- OECD Gross Production (2009): 10,295 TWh
- Non-OECD Gross Production (2009): 9,524 TWh
- United States Gross Production (2009): 4,184 TWh

Source: IEA Electricity Information, 2010.

Image Source: NASA, 2010.

... Is Promising and Problematic.

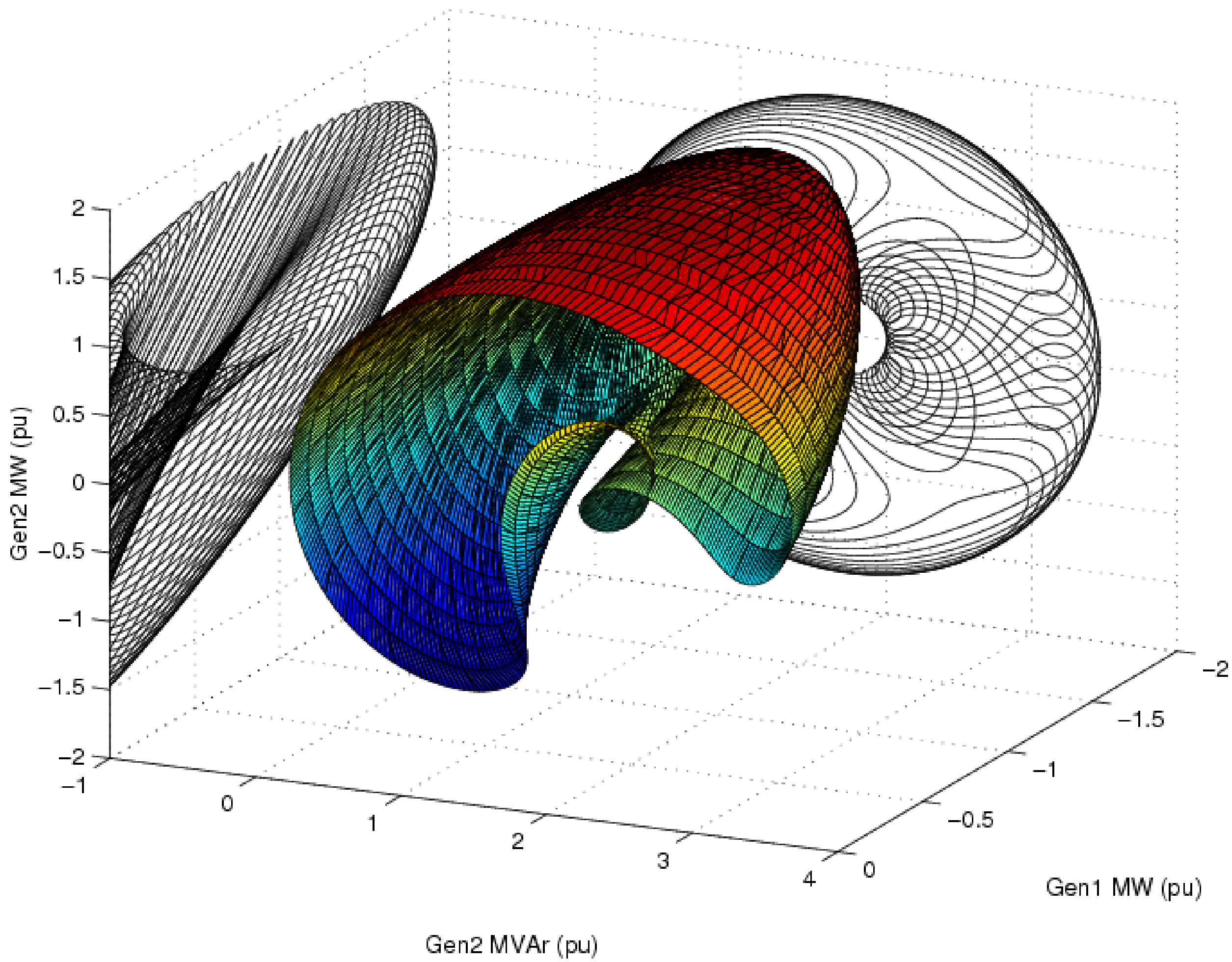

Image Source: Hiskens, 2001.

Problematic:

- Nonlinear, Nonconvex Solution Space
- Feasibility Conditions
- Locally Optimal, Not Globally Optimal Solutions

Promising:

- Highly Structured Network
- Physical Limitations

Current Literature

- Convex Relaxation
  - Javaei & Low, 2010. *Although NP-hard, a subset of OPF problems have zero duality gap.*
  - Jabr, 2008. *The extended conic quadratic OPF has certain computational advantages over the classical OPF.*
- Nonuniqueness and Nonconvexity
  - Hiskens & Davy, 2001. *Solution boundary behavior and robustness of operating points for security assessment.*
  - Klos & Wojcicka, 1991. *There is a multiplicity of load flow solutions, even so-called 'not right' solutions.*
- Structural Characteristics
  - Zhou & Ohsawa, 2006. *Convex analysis of static structural characteristics of power systems and nodal voltage stability.*

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Problem Set

Bus Test Cases					
Test Case	Buses	Branches	No. of Generators	No. of Loads	Sources
6 Bus	6	11	3	3	Wood & Wollenberg, 1996.
9 Bus	9	9	3	6	Chow, Matpower 2010.
14 Bus	14	20	5	9	IEEE (AEP 1960s).
30 Bus	30	41	6	24	IEEE (AEP 1960s).
39 Bus	39	46	10	29	IEEE (New England 1960s).
57 Bus	57	80	7	50	IEEE (AEP 1960s).
118 Bus	118	186	54	64	IEEE (AEP 1960s).
300 Bus	300	411	69	231	IEEE (Adibi, 1993).

Simulation Testbed

- Hardware Specs
  - Intel Xeon X3440, 8MB Cache, 2.53GHz
  - 8GB (4x2GB) RAM, 1333MHz
- Operating System
  - Ubuntu 10.10 Maverick Meerkat
- Model
  - Implemented polar and rectangular formulations (included semidefinite program of the rectangular formulation) in Matlab
  - User-defined first-order derivatives of objective and constraints (Jacobian matrix)
  - Solver approximates second-order derivatives (Lagrangian Hessian matrix)
- Solvers
  - Matlab 7.10 64-bit (Fmincon Interior Point)
  - Ziena Artelys Knitro 7.0 (Knitro Interior Point Direct)
  - COIN-OR Ipopt 3.9.3 (Interior Point)

Model Assumptions

- *B-θ Warm Start*
  - Solved to optimality for the *B-θ* linear approximation method
  - Instead of assuming |V|=1, randomized |V| as a parameter

$$\min_{\theta,P} \sum_{i\in N} c_i \cdot P_i$$

$$P_i - |\widetilde{V}_i| \sum_{n\in N} |\widetilde{V}_n| B_{in}\theta_{in} = 0$$

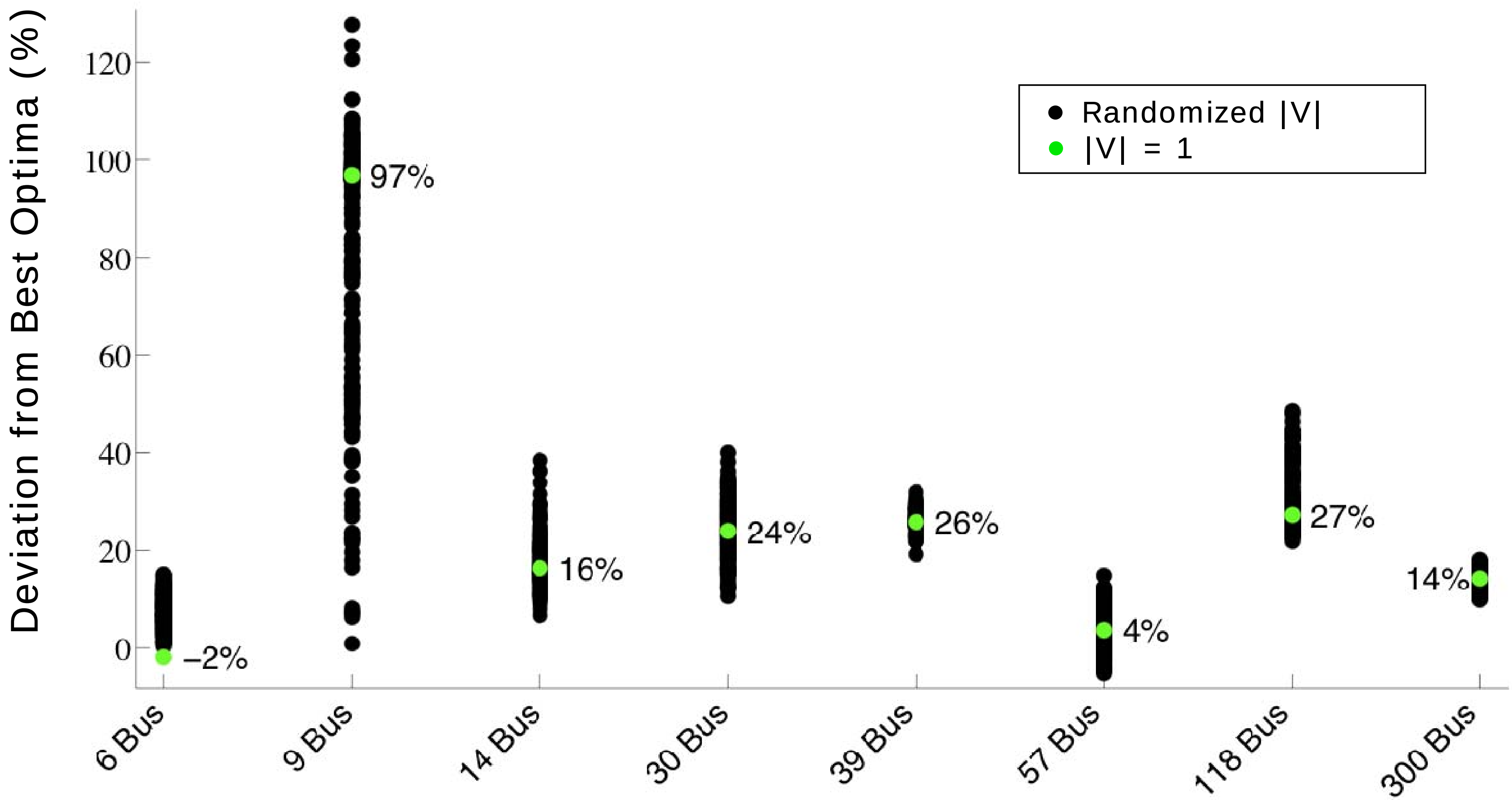
$$P_{i,min} \leq P_i \leq P_{i,max}$$

$$\theta_{ij,min} \leq \theta_i - \theta_j \leq \theta_{ij,max}$$

$$|\widetilde{V}_i| \in U\left(V_{i,min}, V_{i,max}\right)$$

- *ACOPF Runtime*
  - CPU Time capped at 15 minutes (~900 seconds)
  - Optimality (KKT) Tolerance (1e-5 relative)

B-θ Relaxation Gap



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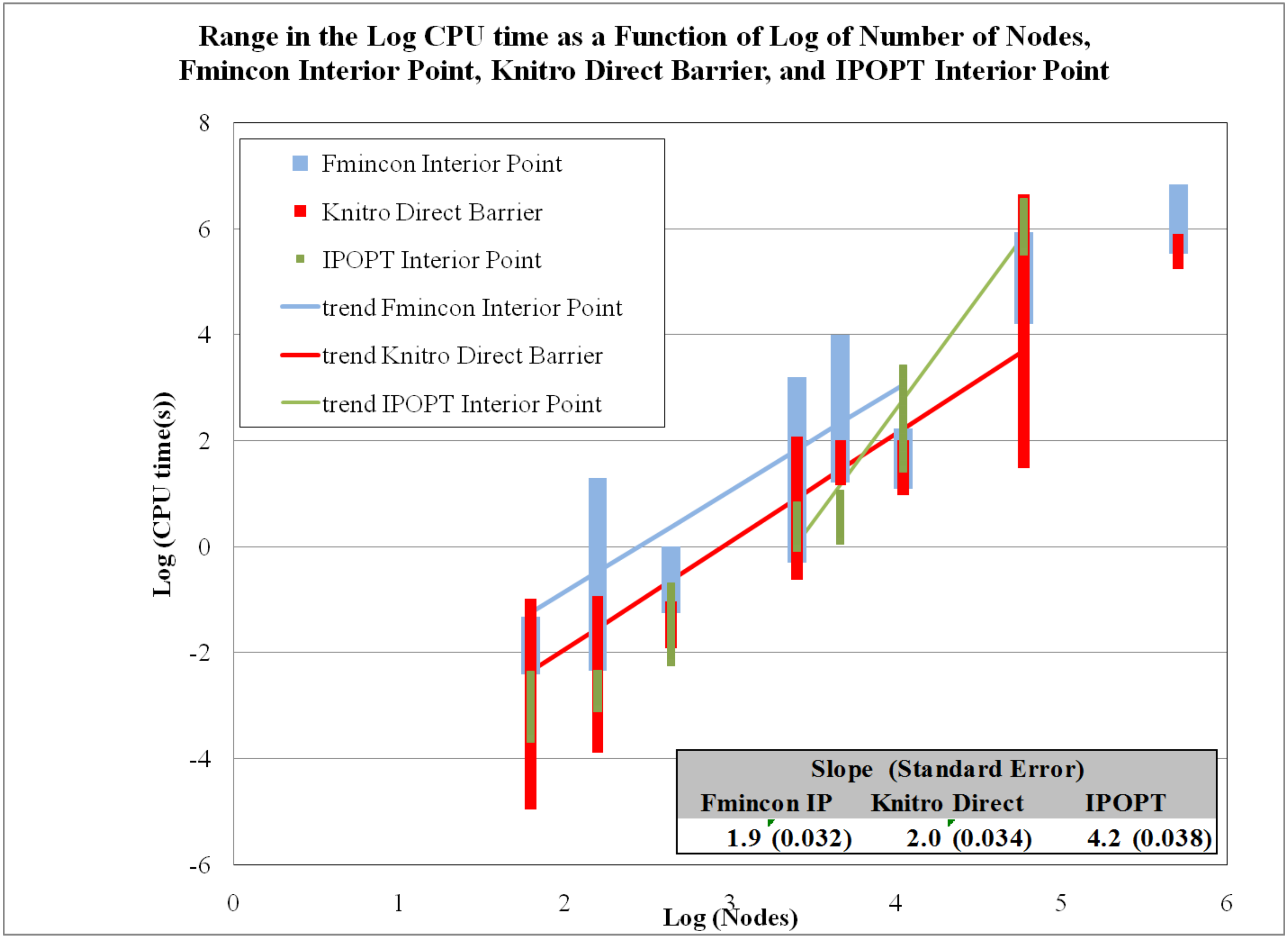
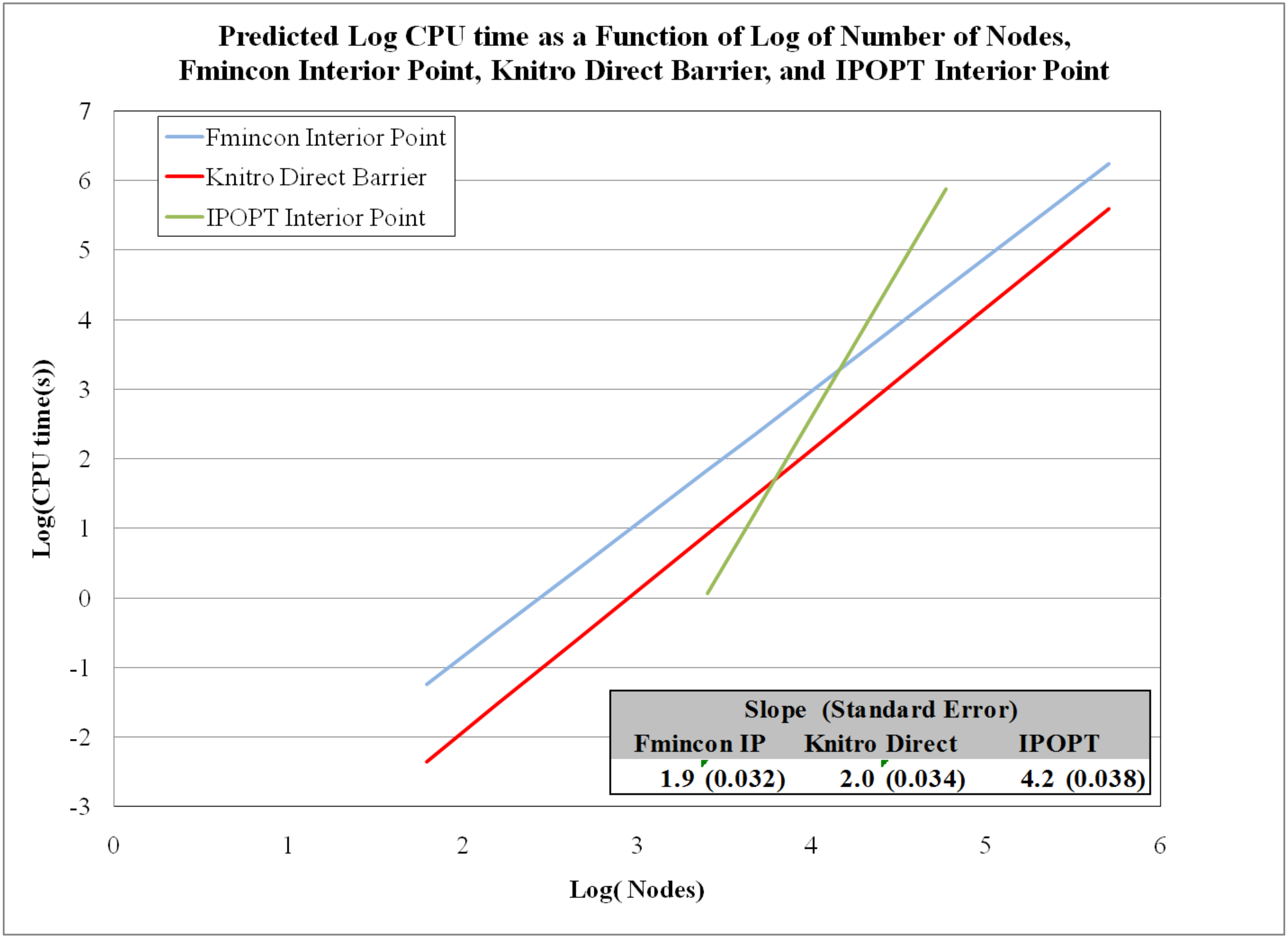


Range in the Log CPU time as a Function of Log of Number of Nodes,
Fmincon Interior Point, Knitro Direct Barrier, and IPOPT Interior Point
Fmincon Interior Point
Knitro Direct Barrier
IPOPT Interior Point
trend Fmincon Interior Point
trend Knitro Direct Barrier
trend IPOPT Interior Point
Slope (Standard Error)
Fmincon IP
Knitro Direct
IPOPT
1.9 (0.032)
2.0 (0.034)
4.2 (0.038)
Log (CPU time(s))
Log (Nodes)
8
6
4
2
0
-2
-4
-6
0
1
2
3
4
5
6



Predicted Log CPU time as a Function of Log of Number of Nodes,
Fmincon Interior Point, Knitro Direct Barrier, and IPOPT Interior Point
Fmincon Interior Point
Knitro Direct Barrier
IPOPT Interior Point
Log(CPU time(s))
7
6
5
4
3
2
1
0
-1
-2
-3
0
1
2
3
4
5
6
Log(Nodes)
Slope (Standard Error)
Fmincon IP
Knitro Direct
IPOPT
1.9 (0.032)
2.0 (0.034)
4.2 (0.038)

Comparing Formulations and Solvers

		Percent of runs that converged		
		57 Bus (57x80)	118 Bus (118x186)	300 Bus (300x411)
Fmincon Interior Point	Polar	97.2%	100%	85.6%
	Rectangular	100%	100%	96.0%
Knitro Interior Point (Direct)	Polar	100%	100%	99.1%
	Rectangular	100%	86.1%	0.0%
IPOPT Interior Point	Polar	100%	72.5%	0.0%
	Rectangular	72.4%	27.3%	0.0%

		Log(CPU time (s))		
		57 Bus (57x80)	118 Bus (118x186)	300 Bus (300x411)
Fmincon Interior Point	Polar	1.68	5.12	6.21
	Rectangular	1.56	5.11	6.09
Knitro Interior Point (Direct)	Polar	1.51	4.80	5.58
	Rectangular	1.89	5.28	N/A
IPOPT Interior Point	Polar	2.53	6.06	N/A
	Rectangular	2.24	6.01	N/A
Semidefinite Program	Rectangular	3.92	5.09	7.61

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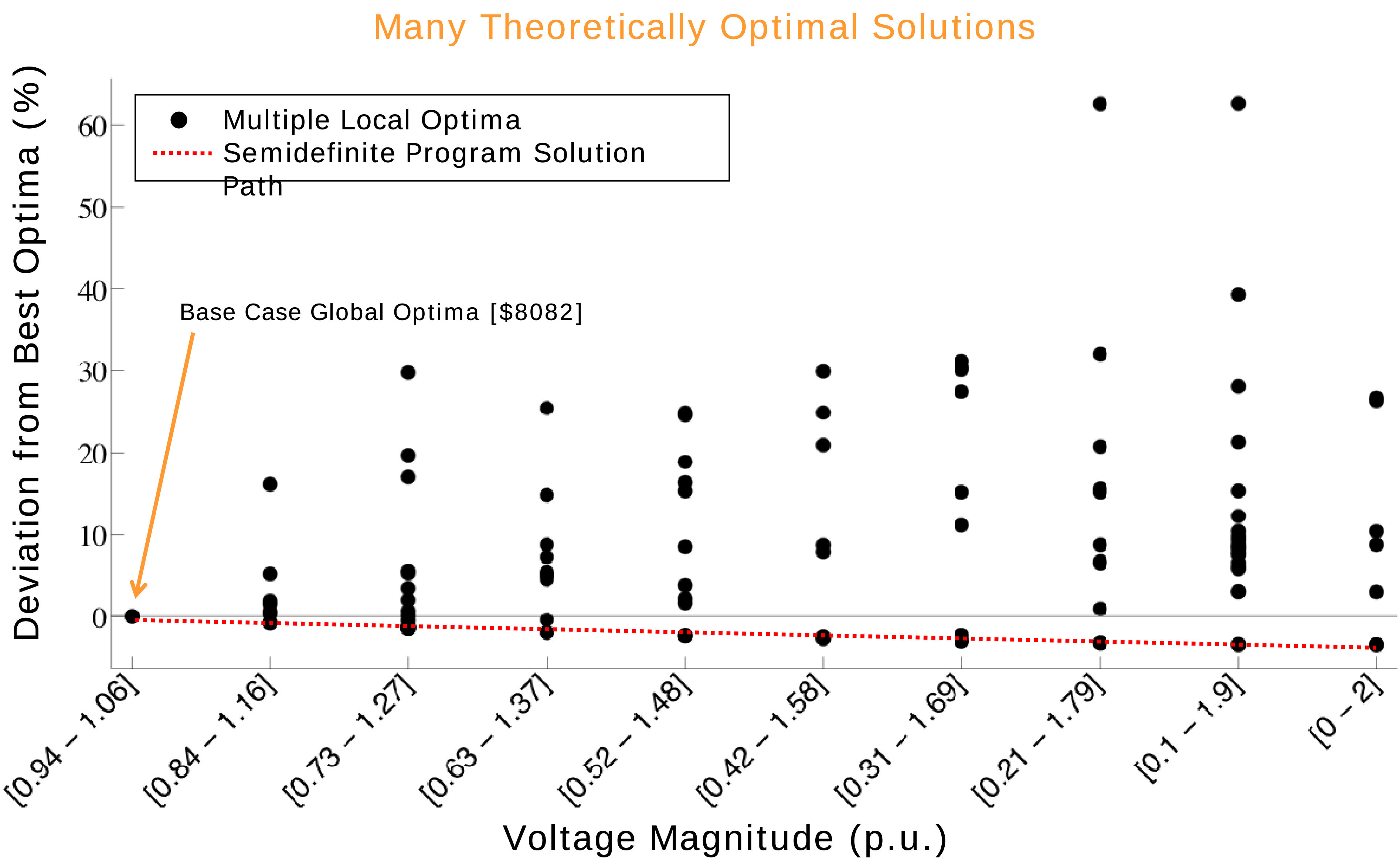
14 Bus Anecdote: Multiple Local Optima



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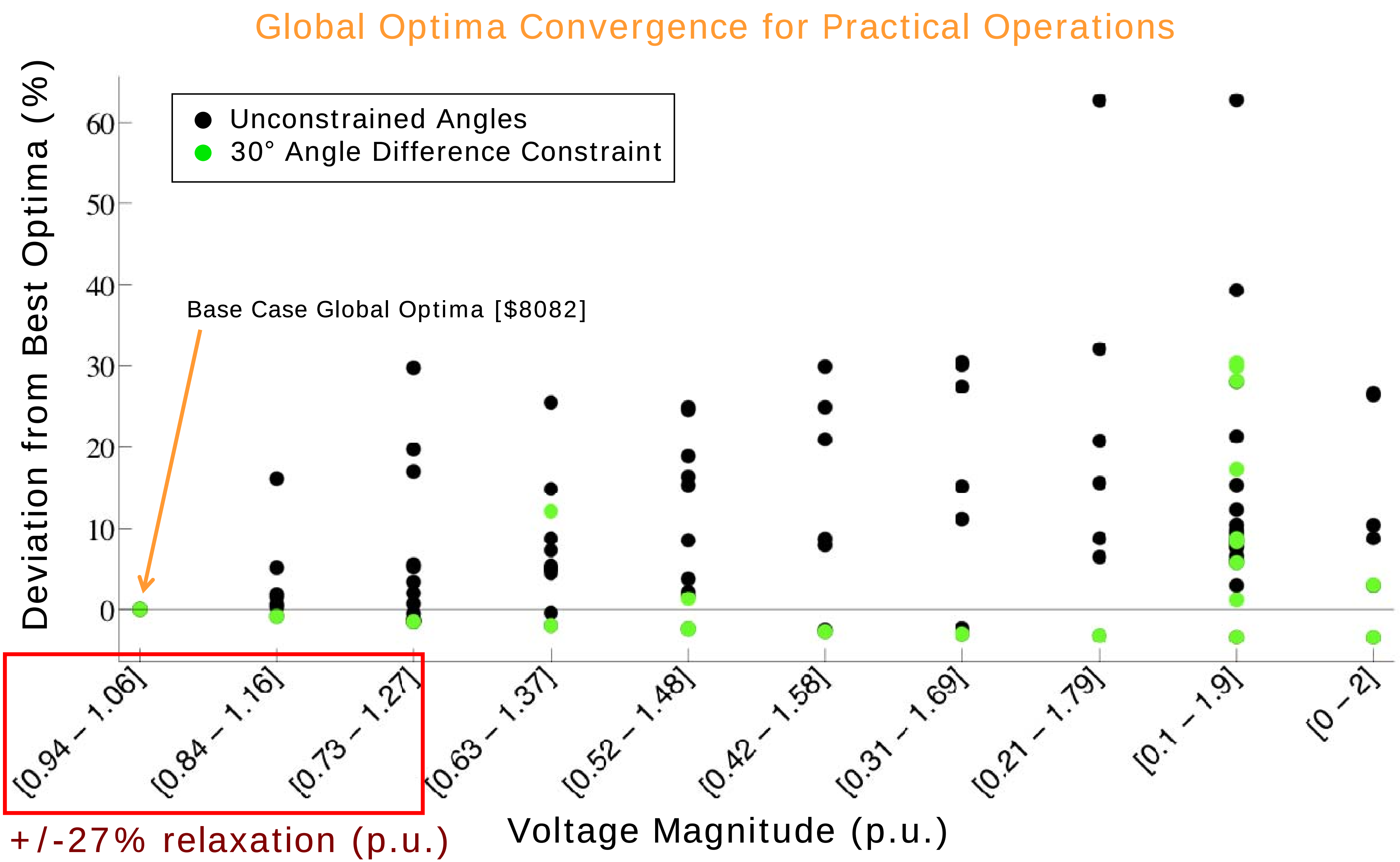

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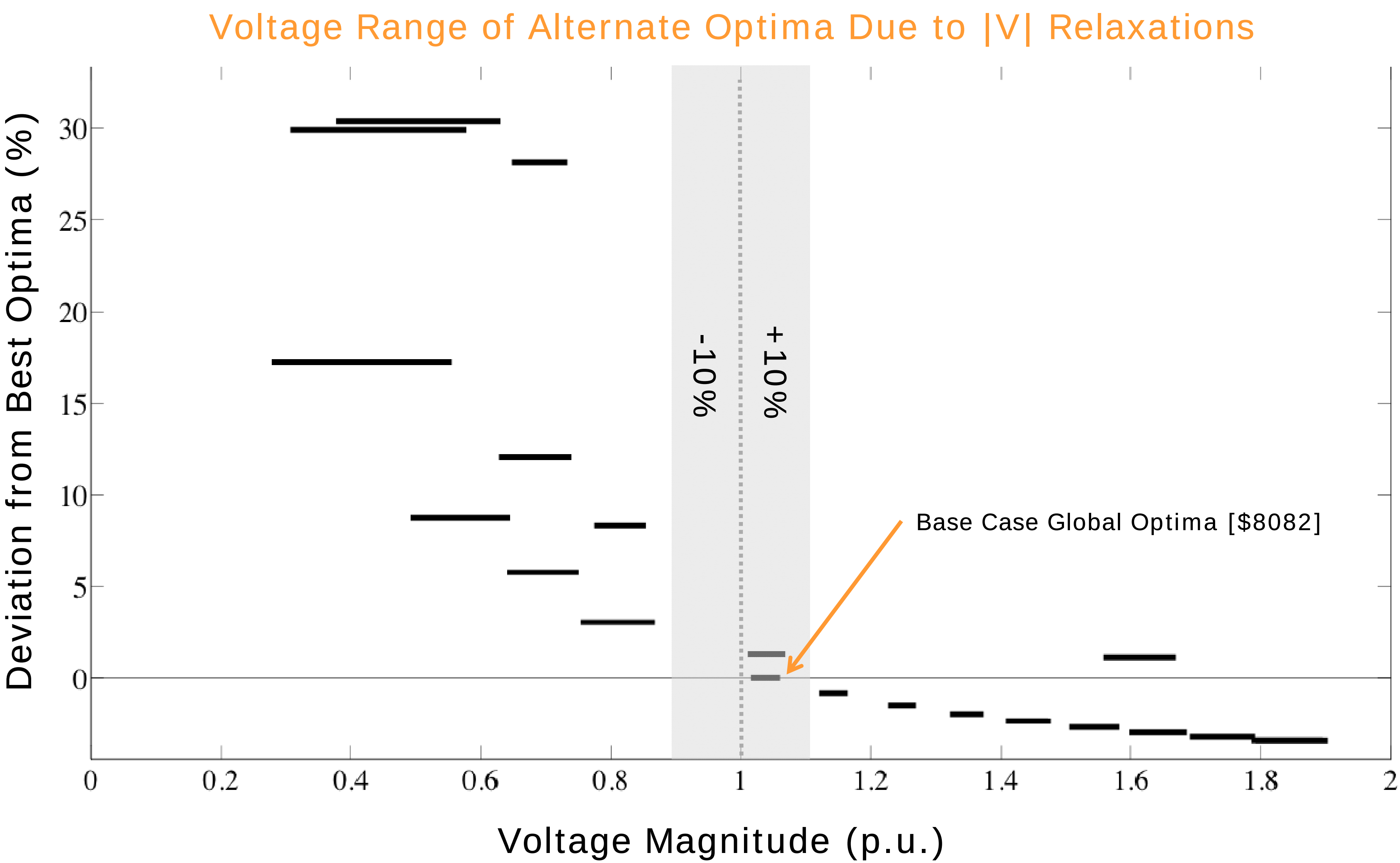

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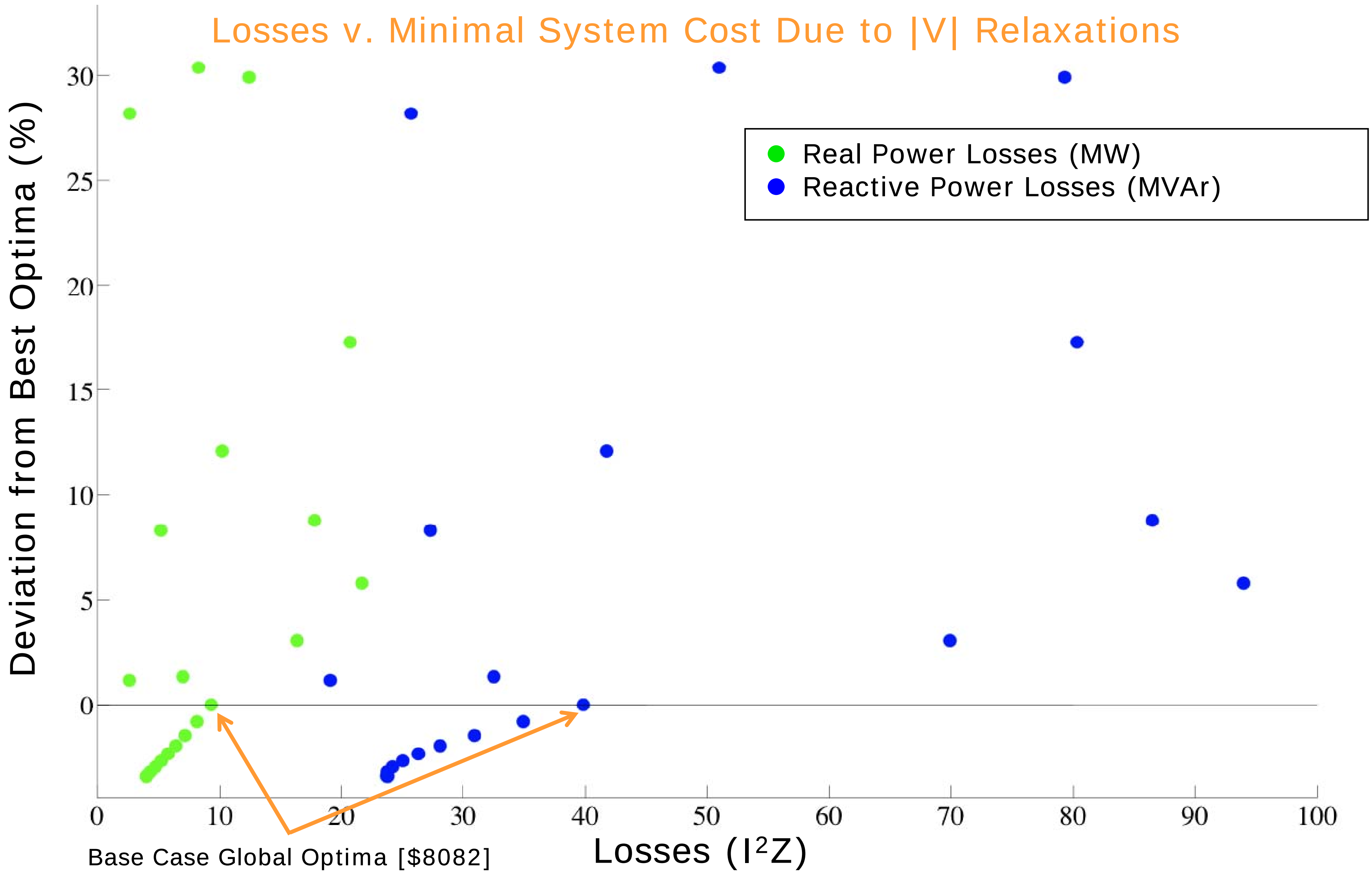

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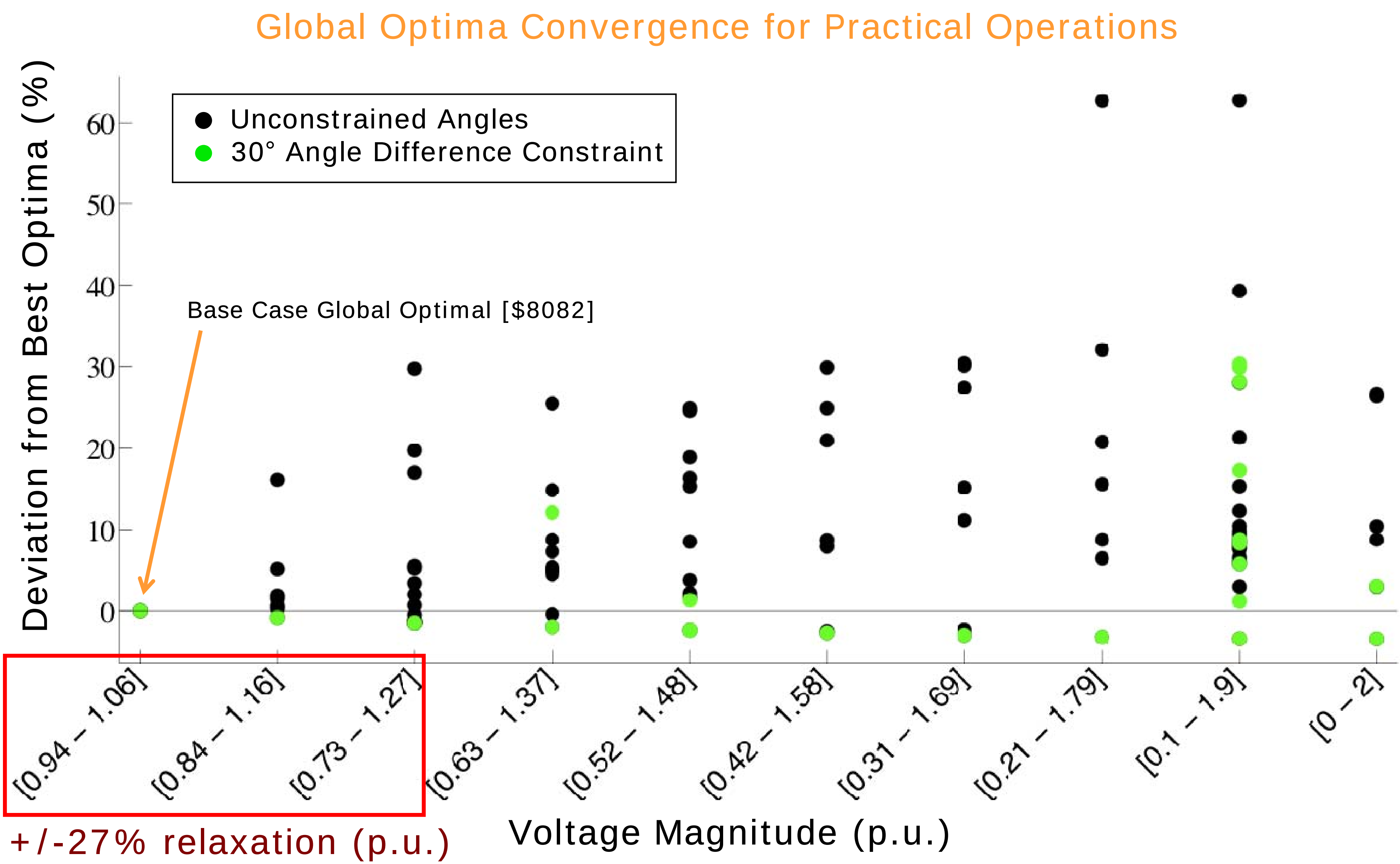

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Takeaways and Next Steps

- Rectangular formulation surprisingly more difficult to handle.
- Semidefinite program empirically always found the global optimal solution, even with constraint relaxations.
- Local optima can lead the power system to highly undesirable (or unrealistic) outcomes.
- For a certain regime of practical and realistic operations, is this solution space convex?
- Next Steps:
  - Measuring nonconvexities for the toy models
  - Examining the robustness of optima
  - Theoretical study
  - Better approaches and formulations to practically solving the ACOPF

Q & A

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